1-1

(a) A rigid body is a solid body in which all the points in the body remain in fixed positions relative to each other; that is, a rigid body does not deform.

(b) In statics and dynamics, all bodies are considered rigid because small deformation, if any, is negligible in the static and dynamic analyses.

(c) Deformations of bodies are important in the study of strength of materials for two reasons: (1) the amount of deformation (even if small) is required in designing a structural member, (2) the deformation condition is needed in solving statically indeterminate problems.

1-2

(a) Strength of materials
(b) Statics
(c) Dynamics
(d) Strength of materials
(e) Statics
(f) Dynamics

1-3

The characteristics of a vector quantity are magnitude, direction, line of action, and point of application.

1-4

(a) Scalar
(b) Vector
(c) Vector
(d) Scalar
(e) Vector
(f) Scalar

1-5

A concurrent coplanar force system.

1-6

A nonconcurrent spatial force system.

1-7

Mass is a measure of a particle's (or a body's) inertia, i.e., its resistance to a change of motion. A body of greater mass has greater resistance to a change of motion, and hence less acceleration is caused by a given force.

1-8

Newton's third law asserts that every force is accompanied by an equal reactive force acting in the opposite direction. Thus the forces between two interactive bodies are always equal in magnitude and opposite in direction.

1-9

No, a force acting on a rigid body can be considered to act anywhere along its line of action according to the principle of transmissibility.

1-10

The principle of transmissibility cannot be applied if we are concerned with

(a) the internal forces in the body and
(b) the deformations of the body.

1-11

In a gravitational system, the unit of force (or weight) that is dependent on the gravitational attraction is chosen as one of the base units. In an absolute system, the unit of mass that is independent of the gravitational attraction, is chosen as one of the base units.

1-12

$W = mg = (5 \text{ slug})(32.2 \text{ ft/s}^2) = 161 \text{ lbs}$ ◀

1-13

$$m = \frac{W}{g} = \frac{500 \text{ lb}}{32.2 \text{ ft/s}^2}$$

$= 15.53 \text{ lb}\cdot\text{s}^2/\text{ft} = 15.53 \text{ slug}$ ◀

1-14

$W = mg = (10 \text{ kg})(9.81 \text{ m/s}^2)$

$= 98.1 \text{ kg}\cdot\text{m/s}^2 = 98.1 \text{ N}$ ◀

1-15

$$m = \frac{W}{g} = \frac{1000 \text{ N}}{9.81 \text{ m/s}^2}$$

$= 101.9 \text{ kg}$ ◀

1-16

$m = 10 \text{ Mg} = 10 \times 10^3 \text{ kg}$

$W = mg = (10 \times 10^3 \text{ kg})(9.81 \text{ m/s}^2)$

$= 98.1 \times 10^3 \text{ N} = 98.1 \text{ kN}$ ◀

1-17

$$m = \frac{W}{g} = \frac{100 \text{ N}}{9.81 \text{ m/s}^2} = 10.19 \text{ kg}$$ ◀

1-18

(a)

$$m = \frac{W}{g} = \frac{150 \text{ lb}}{32.2 \text{ ft/s}^2}$$

$= 4.66 \text{ slug}$ ◀

(b)

$W_m = mg_m = (4.66 \text{ lb}\cdot\text{s}^2/\text{ft})(5.30 \text{ ft/s}^2) = 24.7 \text{ lb}$ ◀

The mass 4.66 slugs is unchanged on the moon. ◀

1-19

(a) $6.38 \text{ Gg} = 6.38 \times 10^9 \text{ g}$

$= 6.38 \times 10^6 (10^3 \text{ g})$

$= 6.38 \times 10^6 \text{ kg}$ ◀

(b) $900 \text{ km} = 900 \times 10^3 \text{ m}$

$= 9 \times 10^5 \text{ m}$ ◀

(c) $3.76 \times 10^7 \text{ g} = 37.6 (10^6 \text{ g})$

$= 37.6 \text{ Mg}$ ◀

(d) $70 \text{ mm} = 70 \times 10^{-3} \text{ m}$

$= 0.070 \text{ m}$ ◀

(e) $23\,400 \text{ N} = 23.4 \times 10^3 \text{ N}$

$= 23.4 \text{ kN}$ ◀

1-20

$$v = 60\text{mph} = \left(60 \frac{\text{mi}}{\text{h}}\right)\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right)\left(\frac{1 \text{ h}}{60 \text{ min}}\right)$$

$= 5280 \text{ ft/min}$ ◀

1-21

$$v = \frac{100 \text{ m}}{9.82 \text{ s}} = (10.18 \text{ m/s})\frac{1 \text{ mph}}{0.4470 \text{ m/s}}$$

$= 22.78 \text{ mph}$ ◀

1-22

$$\gamma = \left(150 \frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{4.448 \text{ N}}{1 \text{ lb}}\right)\left(\frac{1 \text{ ft}^3}{0.02832 \text{ m}^3}\right)\left(\frac{1 \text{ kN}}{1000 \text{ N}}\right)$$

$= 23.6 \text{ kN/m}^3$ ◀

1-23

$$R = (6371 \text{ km})\left(\frac{1 \text{ mi}}{1.609 \text{ km}}\right) = 3960 \text{ mi}$$ ◀

1-24

(a)

$$9.81 \text{ m/s}^2 = (9.81 \text{ m/s}^2)\left(\frac{1 \text{ ft/s}^2}{0.3048 \text{ m/s}^2}\right)$$

$= 32.2 \text{ ft/s}^2$ ◀

(b)

$$100 \text{ MN/m}^2 = (100 \text{ MPa})\left(\frac{1 \text{ ksi}}{6.895 \text{ MPa}}\right)$$

$= 14.5 \text{ ksi}$ ◀

(c)

$$10 \text{ m/s} = (10 \text{ m/s})\left(\frac{1 \text{ km}}{1000 \text{ m}}\right)\left(\frac{3600 \text{ s}}{1 \text{ h}}\right)$$

$= 36 \text{ km/h}$

$$= (36 \text{ km/h})\left(\frac{1 \text{ mph}}{1.609 \text{ km/h}}\right)$$

$= 22.4 \text{ mph}$ ◀

1-25

(a)

$$200 \text{ lb}\cdot\text{ft} = (200 \text{ lb}\cdot\text{ft})\left(\frac{1.356 \text{ N}\cdot\text{m}}{1 \text{ lb}\cdot\text{ft}}\right)$$

$= 271 \text{ N}\cdot\text{m}$ ◀

(b)

$$60 \text{ mph} = (60 \text{ mph})\left(\frac{1.609 \text{ km/h}}{1 \text{ mph}}\right)$$

$= 96.5 \text{ km/h}$ ◀

(c)

$$100 \text{ hp} = (100 \text{ hp})\left(\frac{745.7 \text{ W}}{1 \text{ hp}}\right)$$

$= 74.6 \times 10^3 \text{ W} = 74.6 \text{ kW}$ ◀

1-26

$A = \pi r^2 = \pi(3.25 \text{ ft})^2 = 33.2 \text{ ft}^2$ ◀

1-27

$A = \pi r^2 = \pi(134 \text{ ft})^2$

$$= (56\,410 \text{ ft}^2)\left(\frac{1 \text{ acre}}{43\,560 \text{ ft}^2}\right)$$

$= 1.30 \text{ acres}$ ◀

1-28

$$y = v_o t + \frac{1}{2}gt^2$$

$$= (2.25 \text{ m/s})(30 \text{ s}) + \frac{1}{2}(9.81 \text{ m/s}^2)(30 \text{ s})^2$$

$$= 4482 \text{ m}$$ ◄

1-29

$$y = v_0 t + \frac{1}{2}gt^2$$

$$= (25.0 \text{ ft/s})(15 \text{ s}) + \frac{1}{2}(32.2 \text{ ft/s}^2)(15 \text{ s})^2$$

$$= 4000 \text{ ft}$$ ◄

1-30

$$y = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(1250 \text{ ft})}{32.2 \text{ ft/s}^2}}$$

$$= 8.81 \text{ s}$$ ◄

1-31

$$x = 3 \text{ in.} = (3 \text{ in.})\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right) = 0.25 \text{ ft}$$

$$F = kx = (100 \text{ lb/ft})(0.25 \text{ ft}) = 25 \text{ lb}$$ ◄

1-32

The unit consistent to m, kg, s is N (newton).

$k = 1.57 \text{ kN/m} = 1570 \text{ N/m}$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1570 \text{ N/m}}{30.5 \text{ kg}}}$$

$$= 1.14 \text{ cycles/s} = 1.14 \text{ Hz}$$ ◄

1-33

$$A = \frac{\pi}{4}(0.002 \text{ m})^2 = 3.14 \times 10^{-6} \text{ m}^2$$

$$E = 270 \text{ GN/m}^2 = 270 \times 10^9 \text{ N/m}^2$$

$$e = \frac{FL}{AE} = \frac{(400 \text{ N})(10.0 \text{ m})}{(3.14 \times 10^{-6} \text{ m}^2)(270 \times 10^9 \text{ N/m}^2)}$$

$$= 0.00472 \text{ m} = 4.72 \text{ mm}$$ ◄

1-34

$$v = \sqrt{2gh} = \sqrt{2(32.2 \text{ ft/s}^2)(125 \text{ ft})}$$

$$= 89.7 \text{ ft/s}$$ ◄

1-35

$$v = \sqrt{2gh} = \sqrt{2(9.81 \text{ m/s}^2)(40 \text{ m})}$$

$$= 28.0 \text{ m/s}$$ ◄

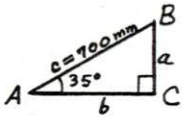
1-36

$$s = \frac{a+b+c}{2} = \frac{5.45+6.85+7.39}{2} = 9.85 \text{ ft}$$

$$A = \sqrt{(9.85)(9.85-5.45)(9.85-6.85)(9.85-7.39)}$$

$$= 17.9 \text{ ft}^2$$ ◄

1-37



$a = c \sin 35° = (700 \text{ mm}) \sin 35° = 402 \text{ mm}$ ◄

$b = c \cos 35° = (700 \text{ mm}) \cos 35° = 573 \text{ mm}$ ◄

Check:

$\sqrt{a^2 + b^2} = \sqrt{402^2 + 573^2} = 700 = c$ (Checks)

1-38

The other side is:

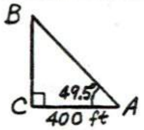
$\sqrt{15^2 - 10^2} = 11.2$ in.

∴ The shorter side is 10 in.

The angle between the hypotenuse and the shorter side is:

$\cos^{-1}\frac{10}{15} = 48.2°$ ◄

1-39

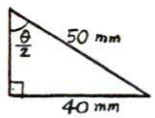


$\tan A = \frac{BC}{AC}$

$BC = AC \tan A = (400 \text{ ft}) \tan 49.5°$

$= 468 \text{ ft}$ ◄

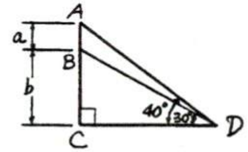
1-40



$\frac{\theta}{2} = \sin^{-1}\frac{40}{50} = 53.1°$

$\theta = 106.2°$ ◄

1-41



$AC = (50 \text{ m}) \tan 40° = 42.0 \text{ m}$

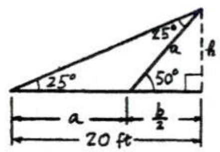
$BC = (50 \text{ m}) \tan 30° = 28.9 \text{ m}$

$a = AC - BC = 42.0 \text{ m} - 28.9 \text{ m}$

$= 13.1 \text{ m}$ ◄

$b = BC = 28.9 \text{ m}$ ◄

1-42



$h = 20 \tan 25° = 9.33 \text{ ft}$ ◄

$\frac{b}{2} = \frac{h}{\tan 50°} = \frac{9.33 \text{ ft}}{\tan 50°} = 7.82 \text{ ft}$

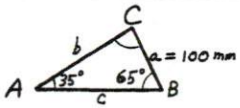
$b = 15.64 \text{ ft}$

$a = \frac{h}{\sin 50°} = \frac{9.33 \text{ ft}}{\sin 50°} = 12.18 \text{ ft}$ ◄

Check:

$a + \frac{b}{2} = 12.18 + 7.82 = 20$ (Checks)

1-43



$C = 180° - (35° + 65°) = 80°$ ◄

$\frac{b}{\sin 65°} = \frac{c}{\sin 80°} = \frac{100 \text{ mm}}{\sin 35°}$

From which

$b = 158 \text{ mm}$ ◄

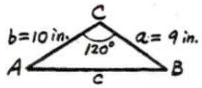
$c = 171.7 \text{ mm}$ ◄

Check:

$a = \sqrt{158^2 + 171.7^2 - 2(158)(171.7)\cos 35°}$

$= 100$ (Checks)

1-44



$A = 180° - (32° + 105°) = 43°$ ◄

$\frac{b}{\sin 32°} = \frac{c}{\sin 105°} = \frac{3.5 \text{ ft}}{\sin 43°}$

From which

$b = 2.72 \text{ ft}$ ◄

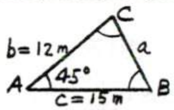
$c = 4.96 \text{ ft}$ ◄

Check:

$a = \sqrt{2.72^2 + 4.96^2 - 2(2.72)(4.96)\cos 43°}$

$= 3.5$ (Checks)

1-45



$a = \sqrt{12^2 + 15^2 - 2(12)(15)\cos 45°} = 10.7 \text{ m}$

$12^2 = 10.7^2 + 15^2 - 2(10.7)(15)\cos B$

$\cos B = 0.609$

$B = 52.5°$ ◄

$15^2 = 10.7^2 + 12^2 - 2(10.7)(12)\cos C$

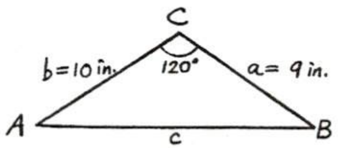
$\cos C = 0.1304$

$C = 82.5°$ ◄

Check:

$A + B + C = 45° + 52.5° + 82.5° = 180°$ (Checks)

1-46



$c = \sqrt{9^2 + 10^2 - 2\,(9)\,(10)\cos 120^\circ} = 16.46$ in.

$9^2 = 10^2 + 16.46^2 - 2(10)(16.46)\cos A$

$\cos A = 0.8807$

$A = 28.3^\circ$ ◀

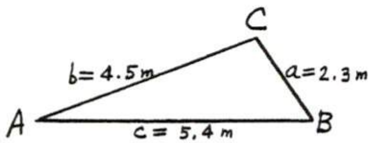
$10^2 = 9^2 + 16.46^2 - 2(9)(16.46)\cos B$

$\cos B = 0.8503$

$B = 31.7^\circ$ ◀

Check:
$A + B + C = 28.3^\circ + 31.7^\circ + 120^\circ = 180^\circ$ (Checks)

1-47



$2.3^2 = 4.5^2 + 5.4^2 - 2(4.5)(5.4)\cos A$

$\cos A = 0.9078$

$A = 24.8^\circ$ ◀

$4.5^2 = 5.4^2 + 2.3^2 - 2(5.4)(2.3)\cos B$

$\cos B = 0.5717$

$B = 55.1^\circ$ ◀

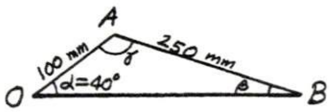
$5.4^2 = 2.3^2 + 4.5^2 - 2(2.3)(4.5)\cos C$

$\cos C = -0.1749$

$C = 100.1^\circ$ ◀

Check:
$A + B + C = 24.8^\circ + 55.1^\circ + 100.1^\circ = 180^\circ$ (Checks)

1-48



$$\frac{\sin\beta}{100} = \frac{\sin 40^\circ}{250}$$

$$\beta = \sin^{-1}\frac{100\sin 40^\circ}{250} = 14.9^\circ$$ ◀

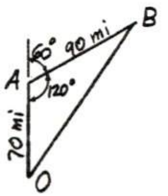
$\gamma = 180^\circ - (40^\circ + 14.9^\circ) = 125.1^\circ$

$$\frac{OB}{\sin 125.1^\circ} = \frac{250\text{mm}}{\sin 40^\circ}$$

$$OB = (250\text{mm})\frac{\sin 125.1^\circ}{\sin 40^\circ} = 318\text{mm}$$ ◀

Check:
$OB = \sqrt{100^2 + 250^2 - 2\,(100)\,(250)\cos 125.1^\circ}$
$= 318$ mm (Checks)

1-49



$$OB = \sqrt{70^2 + 90^2 - 2\,(70)\,(90)\cos 120^\circ}$$
$= 138.9$ mi ◀

1-50

$3x + 5y = -8$ (a)
$5x - 3y = 15$ (b)

3 x (a): $9x + 15y = -24$ (c)
5 x (b): $25x - 15y = 75$ (d)

(c) + (d): $34x = 51$

$x = 1.5$ ◀

Subs in (a): $5y = -8 - 3(1.5) = -12.5$

$y = -2.5$ ◀

1-51

$3.45x - 2.65y = 2.77$ (a)
$1.86x + 3.76y = 9.85$ (b)

(a)/2.65: $1.302x - y = 1.045$ (c)
(b)/3.76: $0.495x + y = 2.620$ (d)

(c) + (d): $1.797x = 3.665$

$x = 2.04$ ◀

Subs in (b): $3.76y = 9.85 - 1.86(2.04) = 6.06$

$y = 1.61$ ◀

1-52

$T\sin 10^\circ - P\sin 40^\circ = 0$ (a)
$T\cos 10^\circ - P\cos 40^\circ = 200$ lb (b)

(a)/sin40°: $T\sin 10^\circ/\sin 40^\circ - P = 0$ (c)
(b)/cos40°: $T\cos 10^\circ/\cos 40^\circ - P = 200\text{ lb}/\cos 40^\circ$ (d)

(d) - (c): $$T\left(\frac{\cos 10^\circ}{\cos 40^\circ} - \frac{\sin 10^\circ}{\sin 40^\circ}\right) = \frac{200\text{ lb}}{\cos 40^\circ}$$

$1.015T = 261$ lb

$T = 257$ lb ◀

Subs in (c): $$P = \frac{(257\text{ lb})\sin 10^\circ}{\sin 40^\circ} = 69.5\text{ lb}$$ ◀

1-53

$-0.429P + 0.231Q = 1920$ lb (a)
$-0.857P - 0.923Q - 0.923R = 2880$ lb (b)
$0.286P - 0.308Q - 0.385R = 2160$ lb (c)

(b)/0.923: $-0.928P - Q - R = 3120$ lb (d)
(c)/0.385: $0.743P - 0.8Q - R = 5610$ lb (e)

(e)-(d): $1.671P + 0.2Q = 2490$ lb (f)

(f)/0.2: $8.355P + Q = 12\,450$ lb (g)
(a)/0.231: $-1.857P + Q = 8312$ lb (h)

(g)-(h): $10.21P = 4138$ lb

$P = 405.3$ lb ◀

Subs in (g): $Q = 12\,450 - 8.355(405.3) = 9064$ lb ◀

Subs in (d): $R = -0.928(405.3) - 9064 - 3120$
$= -12\,560$ lb ◀

1-54

$-0.444x - 0.857y + 0.667z = 0$ (a)
$0.444x + 0.429y + 0.667z = 17$ kN (b)
$0.778x - 0.286y - 0.333z = 0$ (c)

(b)-(a): $0.888x + 1.286y = 17$ kN (d)
(a)+2(c): $1.112x - 1.429y = 0$ (e)

(d)/1.286: $0.6905x + y = 13.22$ kN (f)
(e)/1.429: $0.7782x - y = 0$ (g)

(f) + (g): $1.469x = 13.22$ kN

$x = 9.00$ kN ◀

Subs in (f): $y = 0.7782(9.00) = 7.00$ kN ◀

Subs in (a): $0.667z = 0.444(9.00) + 0.857(7.00)$
$= 10.00$

$z = 15.00$ kN ◀

1-55

$3x + 5y = -8$
$5x - 3y = 15$

$$D = \begin{vmatrix} 3 & 5 \\ 5 & -3 \end{vmatrix} = (3)(-3) - (5)(5) = -34$$

$$D_x = \begin{vmatrix} -8 & 5 \\ 15 & -3 \end{vmatrix} = (-8)(-3) - (15)(5) = -51$$

$$D_y = \begin{vmatrix} 3 & -8 \\ 5 & 15 \end{vmatrix} = (3)(15) - (5)(-8) = 85$$

$$x = \frac{D_x}{D} = \frac{-51}{-34} = 1.5$$ ◀

$$y = \frac{D_y}{D} = \frac{85}{-34} = -2.5$$ ◀

1-56

$3.45x - 2.65y = 2.77$
$1.86x + 3.76y = 9.85$

$$D = \begin{vmatrix} 3.45 & -2.65 \\ 1.86 & 3.76 \end{vmatrix} = 17.90$$

$$D_x = \begin{vmatrix} 2.77 & -2.65 \\ 9.85 & 3.76 \end{vmatrix} = 36.52$$

$$D_y = \begin{vmatrix} 3.45 & 2.77 \\ 1.86 & 9.85 \end{vmatrix} = 28.83$$

$$x = \frac{D_x}{D} = \frac{36.52}{17.90} = 2.04$$ ◀

$$y = \frac{D_y}{D} = \frac{28.83}{17.90} = 1.61$$ ◀

1-57

$$T \sin 10° - P \sin 40° = 0$$
$$T \cos 10° - P \cos 40° = 200 \text{ lb}$$

$$D = \begin{vmatrix} \sin 10° & -\sin 40° \\ \cos 10° & -\cos 40° \end{vmatrix} = (\sin 10°)(-\cos 40°) - (\cos 10°)(-\sin 40°) = 0.5$$

$$D_T = \begin{vmatrix} 0 & -\sin 40° \\ 200 \text{ lb} & -\cos 40° \end{vmatrix} = 0 - (200 \text{ lb})(-\sin 40°) = 128.5 \text{ lb}$$

$$D_P = \begin{vmatrix} \sin 10° & 0 \\ \cos 10° & 200 \text{ lb} \end{vmatrix} = (\sin 10°)(200 \text{ lb}) - 0 = 34.73 \text{ lb}$$

$$T = \frac{D_T}{D} = \frac{128.5 \text{ lb}}{0.5} = 257 \text{ lb} \quad ◀$$

$$P = \frac{D_P}{D} = \frac{34.73 \text{ lb}}{0.5} = 69.5 \text{ lb} \quad ◀$$

1-58

$$-0.429P + 0.231Q = 1920 \text{ lb} \quad (a)$$
$$-0.857P - 0.923Q - 0.923R = 2880 \text{ lb} \quad (b)$$
$$0.286P - 0.308Q - 0.385R = 2160 \text{ lb} \quad (c)$$

$$D = \left|\begin{array}{ccc|cc} -0.429 & 0.231 & 0 & -0.429 & 0.231 \\ -0.857 & -0.923 & -0.923 & -0.857 & -0.923 \\ 0.286 & -0.308 & -0.385 & 0.286 & -0.308 \end{array}\right.$$

$= (-0.429)(-0.923)(-0.385)+(0.231)(-0.923)(0.286) - (-0.308)(-0.923)(-0.429)-(-0.385)(-0.857)(0.231)$
$= -0.1677$

$$D_P = \left|\begin{array}{ccc|cc} 1920 & 0.231 & 0 & 1920 \text{ lb} & 0.231 \\ 2880 & -0.923 & -0.923 & 2880 \text{ lb} & -0.923 \\ 2160 & -0.308 & -0.385 & 2160 \text{ lb} & -0.308 \end{array}\right.$$

$= (1920)(-0.923)(-0.385) + (0.231)(-0.923)(2160 \text{ lb}) - (-0.308)(-0.923)(1920) - (-0.385)(2880)(0.231)$
$= -67.95 \text{ lb}$

$$P = \frac{D_P}{D} = \frac{-67.95 \text{ lb}}{-1.667} = 405.2 \text{ lb} \quad ◀$$

Subs in (a): $Q = [0.429(405.2 \text{ lb}) + 1920 \text{ lb}] / 0.231$
$= 9064 \text{ lb}$ ◀

Subs in (b): $R = [-0.857(405.2 \text{ lb}) - 0.923(9064 \text{ lb}) - 2880 \text{ lb}] / 0.923$
$= -12\,560 \text{ lb}$ ◀

1-59

$$-0.444x - 0.857y + 0.667z = 0 \quad (a)$$
$$0.444x + 0.429y + 0.667z = 17 \text{ kN} \quad (b)$$
$$0.778x - 0.286y - 0.333z = 0 \quad (c)$$

$$D = \left|\begin{array}{ccc|cc} -0.444 & -0.857 & 0.667 & -0.444 & -0.857 \\ 0.444 & 0.429 & 0.667 & 0.444 & 0.429 \\ 0.778 & -0.286 & -0.333 & 0.778 & -0.286 \end{array}\right.$$

$= (-0.444)(0.429)(-0.333) + (-0.857)(0.667)(0.778) + (0.667)(0.444)(-0.286) - (0.778)(0.429)(0.667) - (-0.286)(0.667)(-0.444) - (-0.333)(0.444)(-0.857)$
$= -0.9002$

$$D_x = \left|\begin{array}{ccc|cc} 0 & 0.857 & 0.667 & 0 & 0.857 \\ 17 & 0.429 & 0.667 & 17 & 0.429 \\ 0 & -0.286 & -0.333 & 0 & -0.286 \end{array}\right.$$

$= (0.667)(17)(-0.286) - (-0.333)(17)(-0.857)$
$= -8.094 \text{ kN}$

$$D_y = \left|\begin{array}{ccc|cc} -0.444 & 0 & 0.667 & -0.444 & 0 \\ 0.444 & 17 & 0.667 & 0.444 & 17 \\ 0.778 & 0 & -0.333 & 0.778 & 0 \end{array}\right.$$

$= (-0.444)(17)(-0.333) - (0.778)(17)(0.667)$
$= -6.308 \text{ kN}$

$$D_z = \left|\begin{array}{ccc|cc} -0.444 & -0.857 & 0 & -0.444 & -0.857 \\ 0.444 & 0.429 & 17 & 0.444 & 0.429 \\ 0.778 & -0.286 & 0 & 0.778 & -0.286 \end{array}\right.$$

$= (-0.857)(17)(0.778) - (-0.286)(17)(-0.444)$
$= -13.49 \text{ kN}$

$$x = \frac{D_x}{D} = \frac{-8.094 \text{ kN}}{-0.9002} = 8.99 \text{ kN} \quad ◀$$

$$y = \frac{D_y}{D} = \frac{-6.308 \text{ kN}}{-0.9002} = 7.01 \text{ kN} \quad ◀$$

$$z = \frac{D_z}{D} = \frac{-13.49 \text{ kN}}{-0.9002} = 14.99 \text{ kN} \quad ◀$$

Test Problems for Chapter 1

The following problems may either be given to students for them to practice their problem solving skills or be used as test problems. The answers are provided on the next page.

(1) Carry out the following unit conversion. Round off the results to a proper number of significant digits.

(*a*) 258.4 lb/in² to MN/m²

(*b*) 2.45 lb/ft³ to N/m³

(*c*) 258 N • m to lb • ft

(*d*) 1000 kg/m³ to slug/ft³

(2) Calculate the stress σ (force per unit area) in kip/in.² due to bending from the formula

$$\sigma = \frac{Mc}{I}$$

where the moment $M = 3.53$ kip • ft, the distance $c = 5.00$ in., and the moment of inertia $I = 136$ in.⁴. Round off the results to a proper number of significant digits.

(3) The three sides of a triangular lot are measured to be 147.3 ft, 87.9 ft, and 91.4 ft. Find the area of the lot. (Hint: Find one of the three interior angle and a height of the triangle. Then calculate the area by $A = bh/2$.) Check the answer by using the formula

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where a, b, c are the three sides of the triangle and $s = (a + b + c) / 2$.

(4) Solve the two simultaneous equations

$$P \cos 30° - 0.8\,Q = 0$$
$$P \sin 30° + 0.6\,Q = 100 \text{ lb}$$

for the two unknown forces P and Q by the method of addition or subtraction.

(5) Solve the three simultaneous linear equations

$$2x + 3y - z = 5$$
$$x - 4y - 5z = -22$$
$$-x + y + z = 4$$

for three unknowns by using Cramer's rule.

Solutions to Test Problems for Chapter 1

(1)

(a)

$$(258.4\ \text{lb/in.}^2)\left(\frac{6.895\ \text{kN/m}^2}{1\ \text{lb/in.}^2}\right)$$

$= 1782\ \text{kN/m}^2 = 1.782\ \text{MN/m}^2$ ◀

(b)

$$\left(\frac{2.45\ \text{lb}}{1\ \text{ft}^3}\right)\left(\frac{4.448\ \text{N}}{1\ \text{lb}}\right)\left(\frac{1\ \text{ft}^3}{0.02832\ \text{m}^3}\right)$$

$= 385\ \text{N/m}^2$ ◀

(c)

$$(258\ \text{N}\cdot\text{m})\left(\frac{1\ \text{lb}\cdot\text{ft}}{1.356\ \text{N}\cdot\text{m}}\right)$$

$= 190.3\ \text{lb}\cdot\text{ft}$ ◀

(d)

$$(1000\ \text{kg/m}^3)\left(\frac{1\ \text{slug}}{14.59\ \text{kg}}\right)\left(\frac{0.02832\ \text{m}^3}{1\ \text{ft}^3}\right)$$

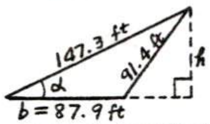
$= 1.941\ \text{slug/ft}^3$ ◀

(2)

$$\sigma = \frac{Mc}{I} = \frac{(3.53\ \text{kip}\cdot\text{ft})\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)(5.00\ \text{in.})}{136\ \text{in.}^4}$$

$= 1.56\ \text{ksi}$ ◀

(3)



$91.4^2 = 147.3^2 + 87.9^2 - 2(147.3)(87.9)\cos\alpha$

$\cos\alpha = 0.8136$

$\alpha = 35.5°$

$h = (147.3\ \text{ft})\sin 35.5° = 85.5\ \text{ft}$

$A = \frac{1}{2}bh = \frac{1}{2}(87.9\ \text{ft})(85.5\ \text{ft}) = 3760\ \text{ft}^2$ ◀

Check:

$$s = \frac{147.3 + 91.4 + 87.9}{2} = 163.3\ \text{ft}$$

$A = \sqrt{(163.3)(163.3 - 147.3)(163.3 - 91.4)(163.3 - 87.9)}$

$= 3760\ \text{ft}^2$ (Checks)

(4)

$P\cos 30° - 0.8Q = 0$ (a)

$P\sin 30° + 0.6Q = 100\ \text{lb}$ (b)

(a)/0.8: $1.25\,P\cos 30° - Q = 0$ (c)

(b)/0.6: $1.67P\sin 30° + Q = 167\ \text{lb}$ (d)

(c) + (d): $P(1.25\cos 30° + 1.67\sin 30°) = 167\ \text{lb}$

$$P = \frac{167}{1.25\cos 30° + 1.67\sin 30°} = 87.1\ \text{lb}$$ ◀

Substituting into Eq.(c) gives

$Q = 1.25(87.1)\cos 30° = 94.3\ \text{lb}$ ◀

(5)

$2x + 3y - z = 5$

$x - 4y - 5z = -22$

$-x + y + z = 4$

$$D = \begin{vmatrix} 2 & 3 & -1 \\ 1 & -4 & -5 \\ -1 & 1 & 1 \end{vmatrix}\begin{matrix} 2 & 3 \\ 1 & -4 \\ -1 & 1 \end{matrix}$$

$= (2)(-4)(1) + (3)(-5)(-1) + (-1)(1)(1) - (-1)(-4)(-1) - (1)(-5)(2) - (1)(1)(3) = 17$

$$D_x = \begin{vmatrix} 5 & 3 & -1 \\ -22 & -4 & -5 \\ 4 & 1 & 1 \end{vmatrix}\begin{matrix} 5 & 3 \\ -22 & -4 \\ 4 & 1 \end{matrix}$$

$= (5)(-4)(1) + (3)(-5)(4) + (-1)(-22)(1) - (4)(-4)(-1) - (1)(-5)(5) - (1)(-22)(3) = 17$

$$D_y = \begin{vmatrix} 2 & 5 & -1 \\ 1 & -22 & -5 \\ -1 & 4 & 1 \end{vmatrix}\begin{matrix} 2 & 5 \\ 1 & -22 \\ -1 & 4 \end{matrix}$$

$= (2)(-22)(1) + (5)(-5)(-1) + (-1)(1)(4) - (-1)(-22)(-1) - (4)(-5)(2) - (1)(1)(5) = 34$

$$D_z = \begin{vmatrix} 2 & 3 & 5 \\ 1 & -4 & -22 \\ -1 & 1 & 4 \end{vmatrix}\begin{matrix} 2 & 3 \\ 1 & -4 \\ -1 & 1 \end{matrix}$$

$= (2)(-4)(4) + (3)(-22)(-1) + (5)(1)(1) - (-1)(-4)(5) - (1)(-22)(2) - (4)(1)(3) = 51$

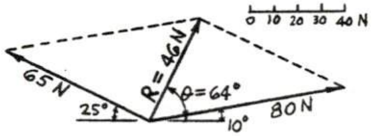
$$x = \frac{D_x}{D} = \frac{17}{17} = 1$$ ◀

$$y = \frac{D_y}{D} = \frac{34}{17} = 2$$ ◀

$$z = \frac{D_z}{D} = \frac{51}{17} = 3$$ ◀

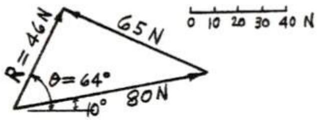
2-1

(a)



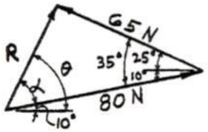
$\mathbf{R} = 46\ \text{N}\ \measuredangle\ 64°$ ◀

(b)



$\mathbf{R} = 46\ \text{N}\ \measuredangle\ 64°$ ◀

2-2



$R = \sqrt{80^2 + 65^2 - 2(80)(65)\cos 35°} = 45.9\ \text{N}$

$$\frac{\sin\alpha}{65} = \frac{\sin 35°}{45.9}$$

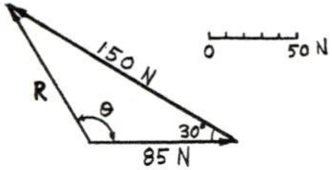
$$\alpha = \sin^{-1}\frac{65\sin 35°}{45.9} = 54.3°$$

$\theta = 54.3° + 10° = 64.3°$

$\mathbf{R} = 45.9\ \text{N}\ \measuredangle\ 64.3°$ ◀

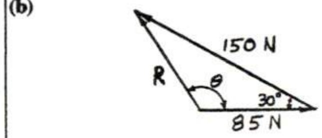
2-3

(a)



$\mathbf{R} = 88\ \text{N}$ $\theta = 121°$ ◀

(b)



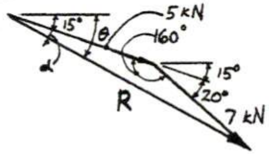
$R = \sqrt{150^2 + 85^2 - 2(150)(85)\cos 30°} = 87.4\text{N}$ ◀

$150^2 = 85^2 + 87.4^2 - 2(85)(87.4)\cos\theta$

$\cos\theta = -0.5134$

$\theta = 120.9°$ ◀

2-4



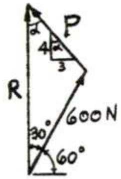
$R = \sqrt{5^2 + 7^2 - 2(5)(7)\cos 160°} = 11.82\ \text{kN}$ ◀

$$\frac{\sin\alpha}{7} = \frac{\sin 160°}{11.82}$$

$$\alpha = \sin^{-1}\frac{7\sin 160°}{11.82} = 11.7°$$

$\theta = \alpha + 15° = 26.7°$ ◀

2-5

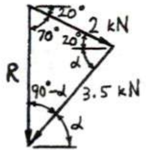


$$\alpha = \tan^{-1}\frac{3}{4} = 36.9°$$

$$\frac{P}{\sin 30°} = \frac{600}{\sin 36.9°}$$

$$P = \frac{600\sin 30°}{\sin 36.9°} = 500\ \text{lb}$$ ◀

2-6

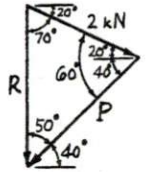


$$\frac{\sin(90° - \alpha)}{2} = \frac{\sin 70°}{3.5}$$

$$90° - \alpha = \sin^{-1}\left(\frac{2 \sin 70°}{3.5}\right) = 32.5°$$

$$\alpha = 90° - 32.5° = 57.5°$$ ◀

2-7

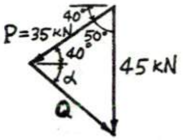


$$\frac{P}{\sin 70°} = \frac{2}{\sin 50°}$$

$$P = \frac{2 \sin 70°}{\sin 50°} = 2.45 \text{ kN}$$

P = 2.45 kN ⦩ 40° ◀

2-8



$$Q = \sqrt{35^2 + 45^2 - 2(35)(45)\cos 50°} = 35.0 \text{ kN}$$

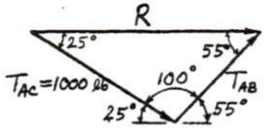
$$\frac{\sin(\alpha + 40°)}{45} = \frac{\sin 50°}{35}$$

$$\alpha + 40° = \sin^{-1}\left(\frac{45 \sin 50°}{35}\right) = 80.0°$$

$$\alpha = 80° - 40° = 40°$$

Q = 35.0 kN ⦪ 40° ◀

2-9



$$\frac{T_{AB}}{\sin 25°} = \frac{R}{\sin 100°} = \frac{1000 \text{ lb}}{\sin 55°}$$

$$T_{AB} = \frac{(1000 \text{ lb}) \sin 25°}{\sin 55°} = 516 \text{ lb}$$ ◀

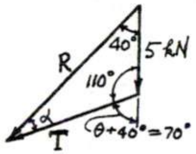
$$R = \frac{(1000 \text{ lb}) \sin 100°}{\sin 55°} = 1202 \text{ lb}$$ ◀

Check:

$$R = \sqrt{1000^2 + 516^2 - 2(1000)(516)\cos 100°}$$
$= 1202$ lb (Checks)

2-10

(a) $\theta = 30°$

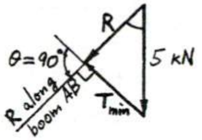


$$\alpha = 180° - (110° + 40°) = 30°$$

$$\frac{T}{\sin 40°} = \frac{5 \text{ kN}}{\sin 30°}$$

T = 6.43 kN ◀

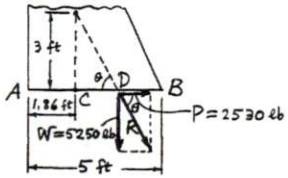
(b) Value of θ for T_{min}



T_{min} must be ⊥ to **R**

$\theta = 90°$ ◀

2-11



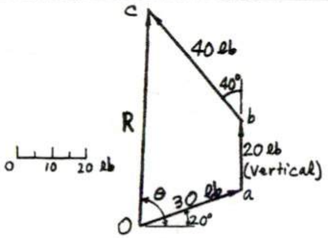
$$\theta = \tan^{-1}\frac{5250}{2530} = 64.3°$$

$$CD = \frac{3 \text{ ft}}{\tan 64.3°} = 1.44 \text{ ft}$$

$$BD = 5 - 1.86 - 1.44 = 1.70 \text{ ft} > \frac{5 \text{ ft}}{3} = 1.67 \text{ ft}$$ ◀

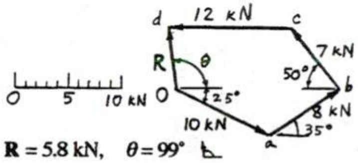
Thus the resultant passes through the middle one-third of the base. Hence compressive soil pressure exist over the entire base of the dam. (This is one of the required conditions for the safety of the dam.)

2-12



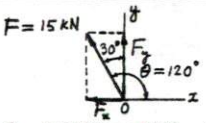
R = 61 lb, $\theta = 87.5°$ ◀

2-13



R = 5.8 kN, $\theta = 99°$ ◀

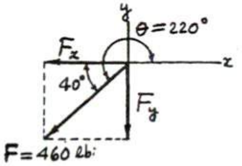
2-14



$F_x = F\cos\theta = (15 \text{ kN})\cos 120° = -7.5 \text{ kN}$ ← ◀

$F_y = F\sin\theta = (15 \text{ kN})\sin 120° = +12.99 \text{ kN}$ ↑ ◀

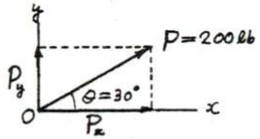
2-15



$F_x = F\cos\theta = (460 \text{ lb})\cos 220° = -352 \text{ lb}$ ← ◀

$F_y = F\sin\theta = (460 \text{ lb})\sin 220° = -296 \text{ lb}$ ↑ ◀

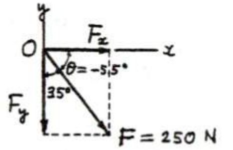
2-16



$P_x = P\cos\theta = (200 \text{ lb})\cos 30° = 173.2 \text{ lb}$ ◀

$P_y = P\sin\theta = (200 \text{ lb})\sin 30° = 100 \text{ lb}$ ◀

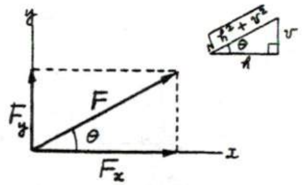
2-17



$F_x = F\cos\theta = (250 \text{ N})\cos(-55°) = 143.4 \text{ N}$ → ◀

$F_y = F\sin\theta = (250 \text{ N})\sin(-55°) = -205 \text{ N}$ ↑ ◀

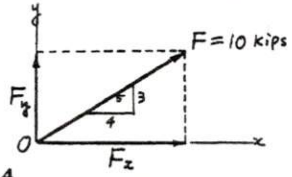
2-18



$$F_x = F\cos\theta = F\frac{h}{\sqrt{h^2 + v^2}}$$ ◀

$$F_y = F\sin\theta = F\frac{v}{\sqrt{h^2 + v^2}}$$ ◀

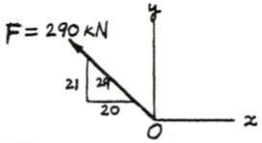
2-19



$F_x = +\frac{4}{5}$ (10 kips) = +8 kips →

$F_y = +\frac{3}{5}$ (10 kips) = +6 kips ↑

2-20

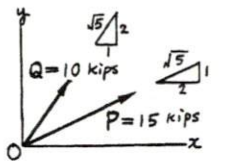


$\sqrt{21^2 + 20^2} = 29$

$F_x = -\frac{20}{29}$ (290 kN) = -200 kN ←

$F_y = +\frac{21}{29}$ (290 kN) = +210 kN ↑

2-21



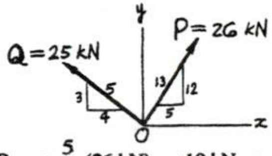
$P_x = \frac{2}{\sqrt{5}}(15)$ = +13.42 kips →

$P_y = \frac{1}{\sqrt{5}}(15)$ = +6.71 kips ↑

$Q_x = \frac{1}{\sqrt{5}}(10)$ = +4.47 kips →

$Q_y = \frac{2}{\sqrt{5}}(10)$ = +8.94 kips ↑

2-22



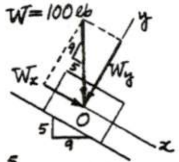
$P_x = +\frac{5}{13}$ (26 kN) = +10 kN →

$P_y = +\frac{12}{13}$ (26 kN) = +24 kN ↑

$Q_x = -\frac{4}{5}$ (25 kN) = -20 kN ←

$Q_y = +\frac{3}{5}$ (25 kN) = +15 kN ↑

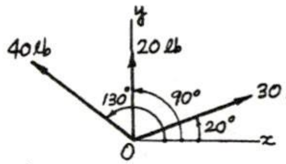
2-23



$W_x = +\frac{5}{\sqrt{5^2 + 9^2}}$(100 lb) = +48.6 lb ↘

$W_y = -\frac{9}{\sqrt{5^2 + 9^2}}$(100 lb) = -87.4 lb ↙

2-24



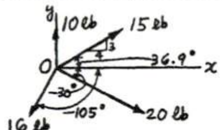
Force, F (lb)	Direction, θ	$F_x = F\cos\theta$ (lb)	$F_y = F\sin\theta$ (lb)
30	20°	28.2	10.3
20	90°	0	20.0
40	130°	-25.7	30.6
Σ		2.5	60.9

R_x = 2.5 lb R_y = 60.9 lb

$R = \sqrt{(2.5)^2 + (60.9)^2}$ = 61.0 lb

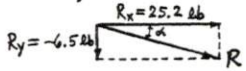
$\theta = \tan^{-1}\frac{60.9}{2.5}$ = 87.6°

2-25



Force, F(lb)	Direction, θ	$F_x = F\cos\theta$ (lb)	$F_y = F\sin\theta$ (lb)
15	36.9°	12.0	9.0
10	90°	0	10.0
20	-30°	17.3	-10.0
16	-105°	-4.1	-15.5
Σ		25.2	-6.5

R_x = 25.2 lb R_y = -6.5 lb

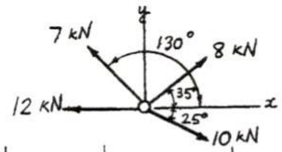


$R = \sqrt{(25.2)^2 + (6.5)^2}$ = 26.0 lb

$\alpha = \tan^{-1}\frac{6.5}{25.2}$ = 14.5°

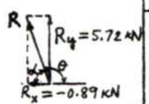
θ = -14.5°

2-26



Force F(kN)	Direction θ	$F_x = F\cos\theta$ (kN)	$F_y = F\sin\theta$ (kN)
8	35°	6.55	4.59
7	130°	-4.50	5.36
12	180°	-12.0	0
10	-25°	9.06	-4.23
Σ		-0.89	5.72

R_x = -0.89 kN R_y = 5.72 kN

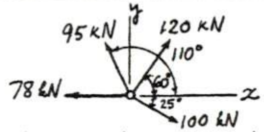


$R = \sqrt{(0.89)^2 + (5.72)^2}$ = 5.79 kN

$\alpha = \tan^{-1}\frac{5.72}{0.89}$ = 81.3°

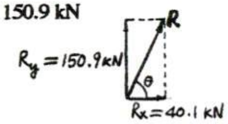
θ = 180° - 81.3° = 98.7°

2-27



Force F(kN)	Direction θ	$F_x = F\cos\theta$ (kN)	$F_y = F\sin\theta$ (kN)
100	-25°	90.6	-42.3
120	60°	60.0	103.9
95	110°	-32.5	89.3
78	180°	-78.0	0
Σ		40.1	150.9

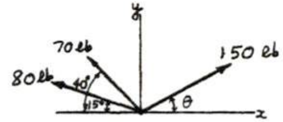
R_x = 40.1 kN R_y = 150.9 kN



$R = \sqrt{(40.1)^2 + (150.9)^2}$ = 156.1 kN

$\theta = \tan^{-1}\frac{150.9}{40.1}$ = 75.1°

2-28



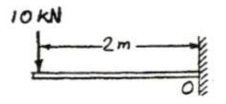
If the resultant is in the vertical direction, the x-component of the resultant must be zero. Thus,

R_x = 150 cos θ - 70 cos 40° - 80 cos 15° = 0

$\cos\theta = \frac{70\cos 40° + 80\cos 15°}{150}$ = 0.8726

θ = 29.2°

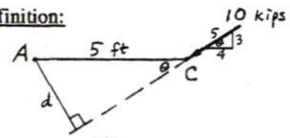
2-29



M_O = + (10 kN)(2m) = +20 kN • m ↺

2-30

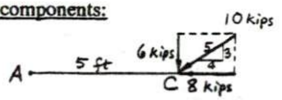
(a) By definition:



$d = 5\sin\theta = 5\left(\frac{3}{5}\right) = 3$ ft

$M_A = -(10 \text{ kips})(3 \text{ ft}) = -30$ kip• ft ↻ ◄

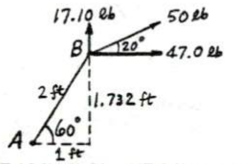
(b) Using components:



$M_A = -(6 \text{ kips})(5 \text{ ft}) + (8 \text{ kips})(0 \text{ ft})$

$= -30$ kip• ft ↻ ◄

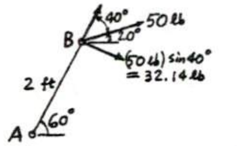
2-31



$M_A = (17.10 \text{ lb})(1 \text{ ft}) - (47.0 \text{ lb})(1.732 \text{ ft})$

$= -64.3$ lb• ft ↻ ◄

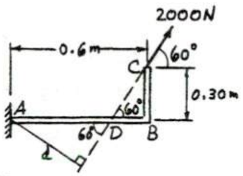
2-32



$M_A = -(32.14 \text{ lb})(2 \text{ ft}) = -64.3$ lb• ft ↻ ◄

2-33

(a) By definition:



$BD = \frac{0.30}{\tan 60°} = 0.173$m

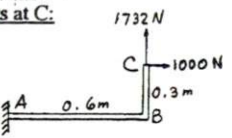
$AD = 0.6 - 0.173 = 0.427$m

$d = AD \sin 60° = (0.427\text{m}) \sin 60°$
$= 0.370$ m

$M_A = +(2000\text{N})(0.370\text{m})$

$= +740$ N• m ↺ ◄

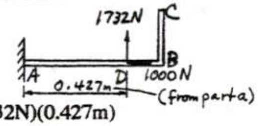
(b) By components at C:



$M_A = +(1732\text{N})(0.6\text{m}) - (1000\text{N})(0.3\text{m})$

$= +740$ N• m ↺ ◄

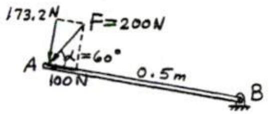
(c) By components at D:



$M_A = +(1732\text{N})(0.427\text{m})$

$= +740$ N• m ↺ ◄

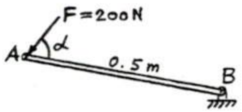
2-34



$M_B = +(173.2 \text{ N})(0.5 \text{ m}) + (100 \text{ N})(0)$

$= +86.6$ N• m ↺ ◄

2-35



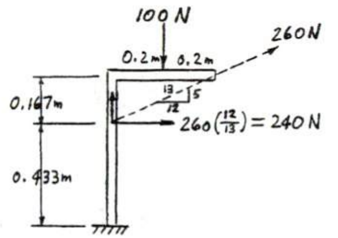
For M_B to be a maximum, the force **F** must be perpendicular to arm AB. Hence

$\alpha = 90°$ ◄

For $\alpha = 90°$, the maximum M_B is

$(M_B)_{max} = (200 \text{ N})(0.50 \text{ m}) = 100$ N • m ◄

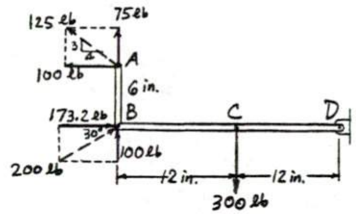
2-36



$M_O = -(100 \text{ N})(0.2 \text{ m}) - (240 \text{ N})(0.433 \text{ m})$

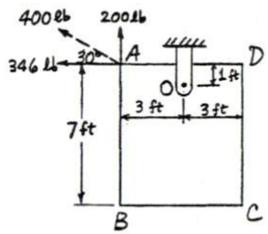
$= -124$ N• m ↺ ◄

2-37



$\Sigma M_D = +(100 \text{ lb})(6 \text{ in.}) - (75 \text{ lb})(24 \text{ in.})$
$-(100 \text{ lb})(24 \text{ in.}) + (300 \text{ lb})(12 \text{ in.})$
$= 0$ ◄

2-38



(a)

$M_O = +(346 \text{ lb})(1 \text{ ft}) - (200 \text{ lb})(3 \text{ ft})$

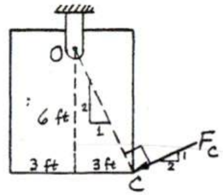
$= -254$ lb• ft ↻ ◄

(b) To produce a clockwise moment about O, the vertical force F_B must act upward.

$M_O = F_B (3 \text{ ft}) = 254$ lb • ft

$F_B = 84.7$ lb ↑ ◄

(c) The smallest force F_C which will produce the same M_O must be perpendicular to CO.

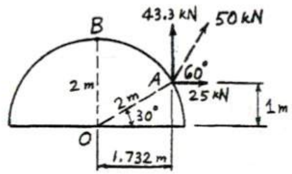


$CO = \sqrt{6^2 + 3^2} = 6.71$ ft

$F_C (6.71 \text{ ft}) = 254$ lb• ft

$F_C = 37.9$ lb ◄

2-39



(a)

$M_o = (43.3 \text{ kN})(1.732 \text{ m}) - (25 \text{ kN})(1 \text{ m})$

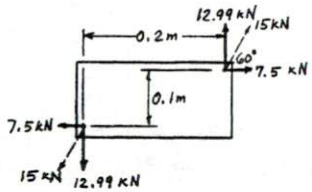
$= +50.0$ kN• m ↺ ◄

(b)

$M_B = (43.3 \text{ kN})(1.732 \text{ m}) + (25 \text{ kN})(2 \text{ m} - 1 \text{ m})$

$= +100.0$ kN• m ↺ ◄

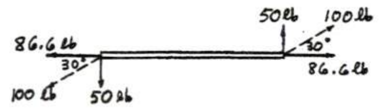
2-40



$M = +(12.99 \text{ kN})(0.2 \text{ m}) - (7.5 \text{ kN})(0.1 \text{ m})$

$= +1.848$ kN• m ↺ ◄

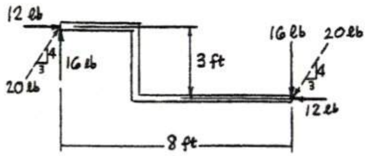
2-41



M = +(50 lb)(10 ft) + (86.6 lb)(0 ft)

= +500 lb • ft ◀

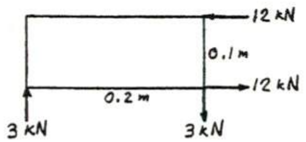
2-42



M = - (12 lb)(3 ft) - (16 lb)(8 ft)

= -164 lb • ft ◀

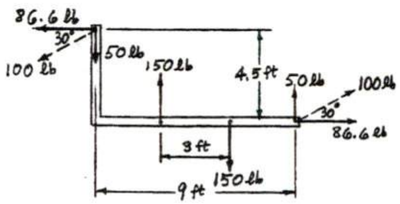
2-43



M = +(12 kN)(0.1 m) - (3 kN)(0.2 m)

= +0.6 kN • m ◀

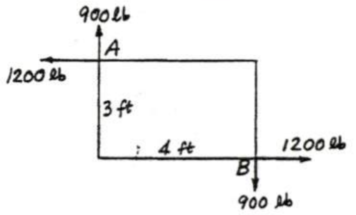
2-44



M = +(86.6 lb)(4.5 ft) + (50 lb)(9 ft)
- (150 lb)(3 ft)

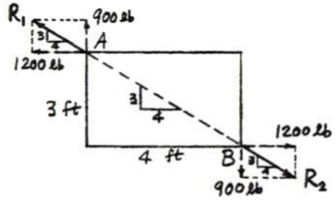
= +390 lb • ft ◀

2-45
(a)



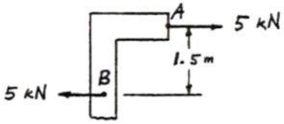
M = +(1200 lb)(3 ft) - (900 lb)(4 ft) = 0 ◀

(b)



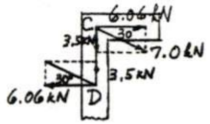
$\mathbf{R}_1$ and $\mathbf{R}_2$ are equal and opposite and both act along line AB, hence the moment is zero. ◀

2-46



M = -(5 kN)(1.5 m) = -7.5 kN • m

The force at D must be equal and opposite to the force at C, as shown.



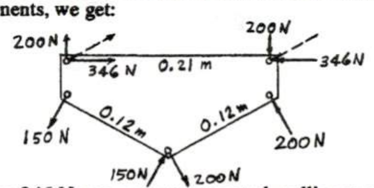
M = -(6.06 kN)(CD) = -7.5 kN • m

$$CD = \frac{7.5\ \text{kN}\cdot\text{m}}{6.06\ \text{kN}} = 1.24\ \text{m}$$ ◀

F_D = 7 kN ⦣ 30° ◀

2-47

Resolve the 400 N forces into horizontal and vertical components, we get:



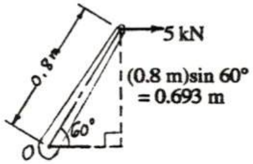
The two 346 N components are equal, collinear and opposite, they cancel each other out. The two vertical components form a couple. The resulatnt moment of the couples is

ΣM = (150 N)(0.12 m) + (200 N)(0.12 m)

- (200 N)(0.21 m) = 0 ◀

Hence the given system of forces is balanced.

2-48

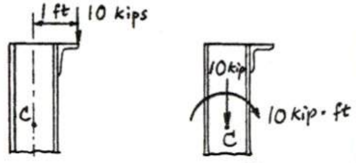


M_o = - (5 kN)(0.693 m) = -3.47 kN • m

Thus the force-couple at O is:

◀

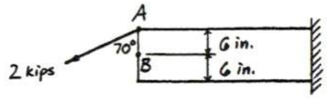
2-49



M_C = (10 kips)(1 ft) = 10 kip • ft

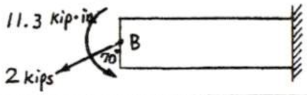
Thus the force-couple at C is shown above.

2-50



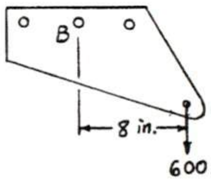
M_B = (2 kips)sin 70°(6 in.) = 11.3 kip • in.

Thus the force-couple at B is:



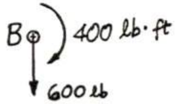
◀

2-51



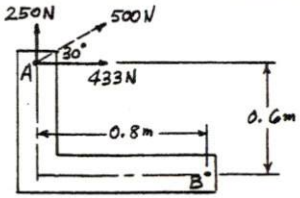
M_B = (600 lb)(8 in.) = 4800 lb • in. = 400 lb • ft

Thus the force-couple at B is:



◀

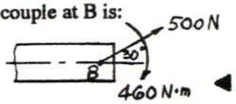
2-52



M_B = - (433 N)(0.6 m) - (250 N)(0.8 m)

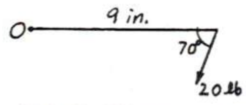
= -460 N • m

Thus the equivalent force-couple at B is:



◀

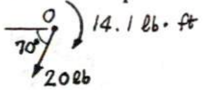
2-53



$M_o = -(20\text{ lb}\sin 70°)(9\text{ in.}) = -169.1\text{ lb}\cdot\text{in.}$

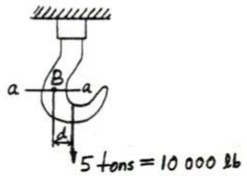
$= -14.1\text{ lb}\cdot\text{ft}$

The equivalent force-couple at 0 is:



◀

2-54

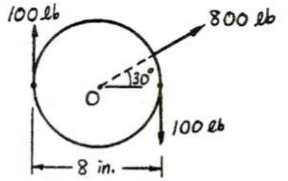


$M_B = (10\,000\text{ lb})d = 2500\text{ lb}\cdot\text{ft}$

$d = \dfrac{2500\text{ lb}\cdot\text{ft}}{10\,000\text{ lb}} = 0.25\text{ ft}$

$= 3\text{ in.}$ ◀

2-55



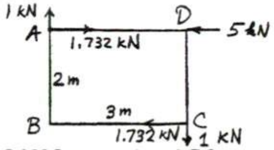
$\mathbf{R} = 800\text{ lb}\angle 30°$ ◀

$M_O = (100\text{ lb})(8\text{ in.}) = 800\text{ lb}\cdot\text{in.}$

$(800\text{ lb})\sin 30°(d) = 800\text{ lb}\cdot\text{in.}$

$d = 2$ in. to the left of 0 ◀

2-56



The two 2-kN forces at A and C form a couple. The resultant moment of the forces about A is

$\Sigma M_A = -(1\text{ kN})(3\text{ m}) - (1.732\text{ kN})(2\text{ m}) = -6.46\text{ kN}\cdot\text{m}$

Equating moment of R about A to the above moment:

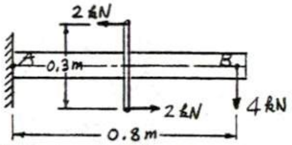
$(5\text{ kN})\,d = 6.46\text{ kN}\cdot\text{m}$

$d = \dfrac{6.46\text{ kN}\cdot\text{m}}{5\text{ kN}} = 1.292\text{ m}$

$\mathbf{R} = 5\text{ kN} \leftarrow$ ◀

At 1.292 m below A. ◀

2-57



$M_B = +(2\text{ kN})(0.3\text{ m}) = +0.6\text{ kN}\cdot\text{m}$

Equating moment of R about A to the above moment:

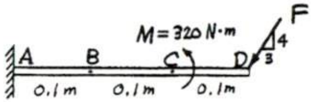
$(4\text{ kN})\,d = 0.6\text{ kN}\cdot\text{m}$

$d = \dfrac{0.6\text{ kN}\cdot\text{m}}{4\text{ kN}} = 0.15\text{ m}$

$\mathbf{R} = 4\text{ kN}\downarrow$ ◀

At 150 mm to the left of B. ◀

2-58



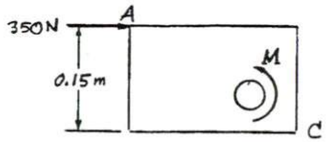
If the given system can be replaced by an equivalent single force at B, then $\Sigma M_B = 0$.

$\Sigma M_B = 320\text{ N}\cdot\text{m} - \frac{4}{5}F(0.2\text{m}) = 0$

From which

$F = 2000\text{ N} = 2\text{ kN}$ ◀

2-59

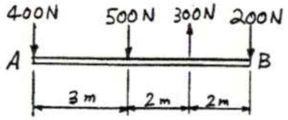


If the force-couple can be replaced by an equivalent force acting through the corner C, the moment ΣM_C must be zero. Thus,

$\Sigma M_C = -(350\text{N})(0.15\text{m}) + M = 0$

$M = +52.5\text{ N}\cdot\text{m}$ ◀

2-60



$R = \Sigma R_y = -400 - 500 + 300 - 200$

$= -800\downarrow$ ◀

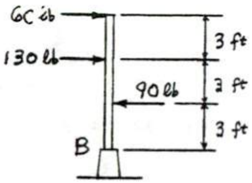
$\Sigma M_A = -(500)(3) + (300)(5) - (200)(7)$
$= -1400\text{ N}\cdot\text{m}$

Equating moment of R about A to the above moment:

$(800\text{ N})d = 1400$

$d = 1.75$ m to the right of A. ◀

2-61



$R_x = \Sigma F_x = 60 + 130 - 90 = 100\text{ lb}\rightarrow$ ◀

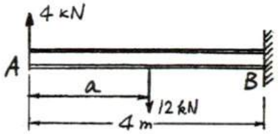
$\Sigma M_B = -(60)(9) - (130)(6) + (90)(3)$
$= -1050\text{ lb}\cdot\text{ft}$

Equating moment of R about A to the above moment:

$(100\text{ lb})d = 1050\text{ lb}\cdot\text{ft}$

$d = 10.5\text{ ft}$ (above B) ◀

2-62



(a) $a = 2\text{m}$

$R_y = \Sigma R_y = 4 - 12 = -8\text{kN}\downarrow$ ◀

$\Sigma M_A = -(12)(2) = -24\text{ kN}\cdot\text{m}$

$(8\text{ kN})d = 24\text{ kN}\cdot\text{m}$

$d = 3$ m (to the right of A) ◀

(b) $a = 3\text{ m}$

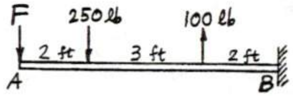
$R_y = \Sigma R_y = 4 - 12 = -8\text{ kN}\downarrow$ ◀

$\Sigma M_A = -(12)(3) = -36\text{ kN}\cdot\text{m}$

$(8\text{ kN})d = 36\text{ kN}\cdot\text{m}$

$d = 4.5$ m (to the right of A) ◀

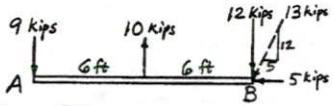
2-63



$\Sigma M_A = -(250)(2) + (100)(5) = 0$

Since $\Sigma M_A = 0$, the resultant of the three forces passes through A. ◀

2-64



$R_x = \Sigma R_x = -5\text{ kips}\leftarrow$

$R_y = \Sigma R_y = -9 + 10 - 12 = -11\text{ kips}\downarrow$

$R = \sqrt{11^2 + 5^2} = 12.1\text{ kips}$ ◀

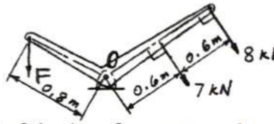
$\theta = \tan^{-1}\dfrac{11}{5} = 65.6°$ ◀

$\Sigma M_B = +(9)(12) - (10)(6) = +48\text{ kip}\cdot\text{ft}$

$(11\text{ kip})d = 48\text{ kip}\cdot\text{ft}$

$d = 4.36$ ft (to the left of B) ◀

2-65

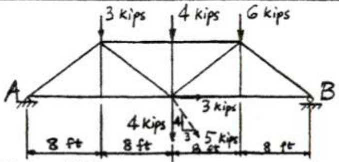


If the resultant of the three forces passes through O, the sum of moments of the three forces about O must be zero. Thus,

$\Sigma M_o = F(0.8 \text{ m} \times \cos 30°) - (7 \text{ kN})(0.6 \text{ m}) - (8 \text{ kN})(1.2 \text{ m}) = 0$

$F = 19.9 \text{ kN}$ ◀

2-66



$R_x = \Sigma F_x = +3 \text{ kips} \rightarrow$

$R_y = \Sigma F_y = -3 - 4 - 6 - 4 = -17 \text{ kips} \downarrow$

$R = \sqrt{3^2 + 17^2} = 17.26 \text{ kips}$ ◀

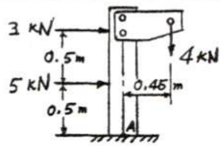
$\theta = \tan^{-1}\frac{17}{3} = 80.0°$ ◀

$\Sigma M_A = -(3)(8) - (4+4)(16) - (6)(24) = -296 \text{ kip}\cdot\text{ft}$

$(17 \text{ kips})d = 296 \text{ kip}\cdot\text{ft}$

$d = 17.41 \text{ ft}$ (to the right of A) ◀

2-67



$R_x = \Sigma F_x = 3 + 5 = 8 \text{ kN} \rightarrow$

$R_y = \Sigma F_y = -4 \text{ kN} \downarrow$

$R = \sqrt{8^2 + 4^2} = 8.94 \text{ kN}$ ◀

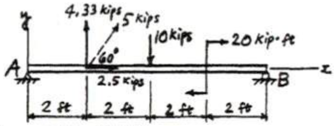
$\theta = \tan^{-1}\frac{4}{8} = 26.6°$ ◀

$\Sigma M_A = -(3)(1.0) - (5)(0.5) - (4)(0.45) = -7.3 \text{ kN}\cdot\text{m}$

$(8 \text{ kN})d = 7.3 \text{ kN}\cdot\text{m}$

$d = 0.913 \text{ m} = 91.3 \text{ mm}$ (above point A) ◀

2-68



$R_x = \Sigma F_x = +2.5 = 2.5 \text{ kips} \rightarrow$

$R_y = \Sigma F_y = +4.33 - 10 = -5.67 \text{ kips} \downarrow$

$R = \sqrt{(2.5)^2 + (5.67)^2} = 6.20 \text{ kips}$ ◀

$\theta = \tan^{-1}\frac{5.67}{2.5} = 66.2°$ ◀

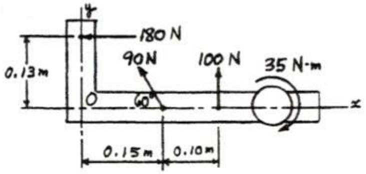
$\Sigma M_A = +(4.33)(2) - (10)(4) - 20 = -51.3 \text{ kip}\cdot\text{ft}$

$(5.67 \text{ kips})d = 51.3 \text{ kip}\cdot\text{ft}$

$d = 9.05 \text{ ft}$

$d - 8 = 9.05 - 8 = 1.05$ (to the right of B) ◀

2-69



$R_x = \Sigma F_x = -180 - 90 \cos 60° = -225 \text{ N} \leftarrow$

$R_y = \Sigma F_y = 90 \sin 60° + 100 = +177.9 \text{ N} \uparrow$

$R = \sqrt{(225)^2 + (177.9)^2} = 287 \text{ N}$ ◀

$\alpha = \tan^{-1}\left|\frac{177.9}{225}\right| = 38.3°$

$\theta = 180° - \alpha = 141.7°$ ◀

$\Sigma M_o = (180)(0.13) + (90 \sin 60°)(0.15) + (100)(0.25) - 35 = 25.1 \text{ N}\cdot\text{m}$

$R_y x = R_x y = 25.1 \text{ N}\cdot\text{m}$

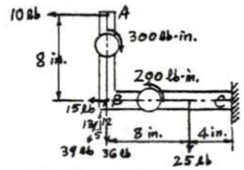
$x = \frac{25.1}{R_y} = \frac{25.1}{177.9}$

$= 0.112 \text{ m}$ to the right of O. ◀

$y = \frac{25.1}{R_x} = \frac{25.1}{225}$

$= 0.141 \text{ m}$ above O. ◀

2-70



$R_x = \Sigma F_x = -10 - 15 = -25 \text{ lb} \leftarrow$

$R_y = \Sigma F_y = -36 - 25 = -61 \text{ lb} \downarrow$

$R = \sqrt{(25)^2 + (61)^2} = 65.9 \text{ lb}$ ◀

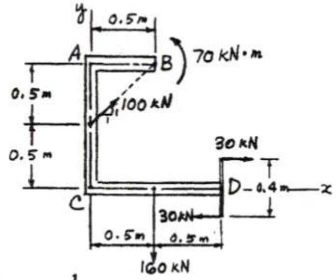
$\theta = \tan^{-1}\frac{61}{25} = 67.7°$ ◀

$\Sigma M_A = +(10)(8) + (36)(12) + (25)(4) - 300 + 200 = +512 \text{ lb}\cdot\text{in.}$

$(61 \text{ lb})\, d = 512 \text{ lb}\cdot\text{in.}$

$d = 8.39 \text{ in.}$ (to the left of C) ◀

2-71



$R_x = \sum F_x = +\frac{1}{\sqrt{2}}(100\text{kN}) = +70.7\text{kN} \rightarrow$

$R_y = \sum F_y = +\frac{1}{\sqrt{2}}(100\text{kN}) - 160\text{kN} = -89.3 \text{ kN} \downarrow$

$R = \sqrt{(70.7)^2 + (89.3)^2} = 114 \text{ kN}$ ◀

$\alpha = \tan^{-1}\frac{89.3}{70.7} = 51.6°$

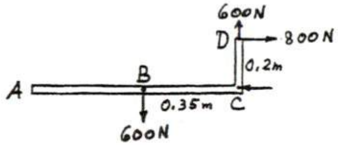
$\theta = -\alpha = -51.6°$ ◀

$\sum M_c = -\left[\frac{1}{\sqrt{2}}(100\text{kN})\right][0.5\text{m}] - (160\text{kN})(0.5 \text{ m}) + 70 \text{ kN}\cdot\text{m} - (30\text{kN})(0.4 \text{ m}) = -57.4 \text{ kN}\cdot\text{m}$

$(70.7 \text{ kN})\ \bar{y} = 57.4 \text{ kN}\cdot\text{m}$

$\bar{y} = 0.812 \text{ m}$ (above point C.) ◀

2-72



The forces form two couples and hence there is no force component for the resultant along any direction.

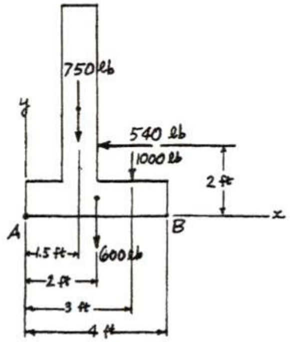
$\mathbf{R} = 0$ ◀

$\Sigma M = (600 \text{ N})(0.35 \text{ m}) - (800 \text{ N})(0.2 \text{ m}) = +50 \text{ N}\cdot\text{m}$

The given force system reduces to a couple. The moment of the couple is

$M = 50 \text{ N}\cdot\text{m}$ ◀

2-73



$R_x = \Sigma F_x = -540 \text{ lb} \leftarrow$

$R_y = \Sigma F_y = -750 - 600 - 1000 = -2350 \text{ lb} \downarrow$

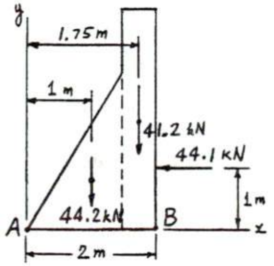
$\Sigma M_A = 750(1.5) + 600(2) + 1000(3) - 540(2) = 4245 \text{ lb}\cdot\text{ft}$

$2350(\bar{x}) = 4245$

$\bar{x} = 1.81 \text{ ft}$ ◀

Thus the resultant passes through the point on the base at 1.81 ft to the right of A which is within the middle one-third of the base. ◀

2-74



$R_x = \Sigma F_x = -44.1$ kN ←

$R_y = \Sigma F_y = -44.2 - 41.2 = -85.4$ kN ↓

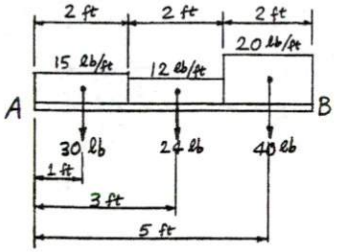
$\Sigma M_A = -44.2(1) - 41.2(1.75) + 44.1(1)$
$= -72.2$ kN • m ↻

$85.4(\bar{x}) = 72.2$

$\bar{x} = 0.845$ m ◀

Thus the resultant passes through the point on the base at 0.845 m to the right of A which is within the middle one-third of the base. ◀

2-75



$R = \Sigma F_y = -30 - 24 - 40 = -94$ lb

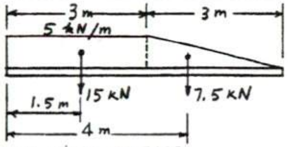
R $= 94$ lb ↓ ◀

$\Sigma M_A = -30(1) - 24(3) - 40(5)$
$= -302$ lb • ft ↻

$94(\bar{x}) = 302$

$\bar{x} = 3.21$ ft to the right of A ◀

2-76



$R = \Sigma F_y = 15 + 7.5 = 22.5$ kN

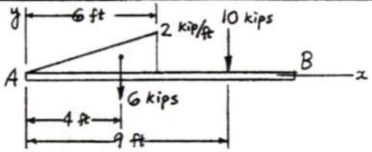
R $= 22.5$ kN ↓ ◀

$\Sigma M_A = -15(1.5) - 7.5(4)$
$= -52.5$ kN • m ↻

$22.5(\bar{x}) = 52.5$

$\bar{x} = 2.33$ m to the right of A ◀

2-77



$R = \Sigma F_y = -6 - 10 = -16$ kips

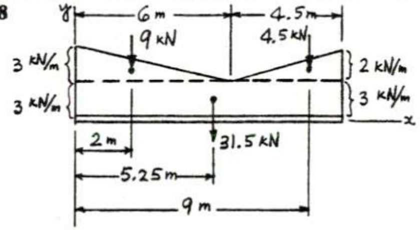
R $= 16$ kips ↓ ◀

$\Sigma M_A = -6(4) - 10(9) = -114$ kip • ft ↻

$16(\bar{x}) = 114$

$\bar{x} = 7.13$ ft to the right of A ◀

2-78



$R = \Sigma F_y = -9 - 31.5 - 4.5 = -45$ kN

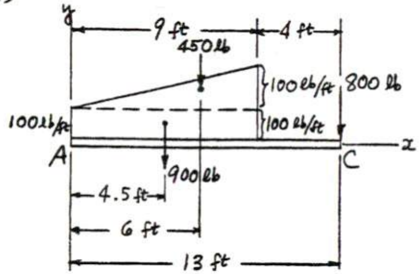
R $= 45$ kN ↓ ◀

$\Sigma M_A = -9(2) - 31.5(5.25) - 4.5(9)$
$= -224$ kN • m ↻

$45(\bar{x}) = 224$

$\bar{x} = 4.98$ m to the right of A ◀

2-79



$R = \Sigma F_y = -900 - 450 - 800 = -2150$ lb

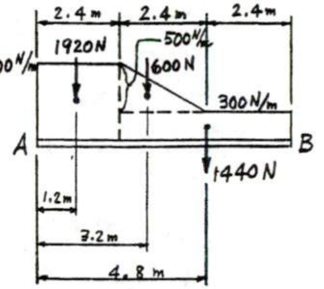
R $= 2150$ lb ↓ ◀

$\Sigma M_A = -900(4.5) - 450(6) - 800(13)$
$= -17\,150$ lb • ft ↻

$2150(\bar{x}) = 17\,150$

$\bar{x} = 7.98$ ft to the right of A ◀

2-80



$R = \Sigma F_y = -1920 - 600 - 1440 = -3960$ N

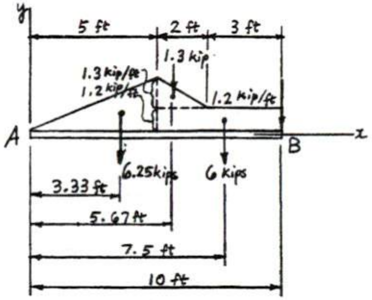
R $= 3960$ N ↓ ◀

$\Sigma M_A = -1920(1.2) - 600(3.2) - 1440(4.8)$
$= -11\,140$ N • m ↻

$3960(\bar{x}) = 11\,140$

$\bar{x} = 2.81$ m to the right of A ◀

2-81



$R = \Sigma F_y = -6.25 - 1.3 - 6 - 6 = -19.55$ kips

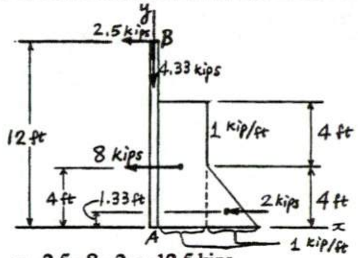
R $= 19.55$ kips ↓ ◀

$\Sigma M_A = -6.25(3.33) - 1.3(5.67) - 6(7.5) - 6(10)$
$= -133.2$ kip • ft ↻

$19.55(\bar{x}) = 133.2$

$\bar{x} = 6.81$ ft to the right of A ◀

2-82



$R_x = \Sigma F_x = -2.5 - 8 - 2 = -12.5$ kips

$R_y = \Sigma F_y = -4.33$ kips

$R = \sqrt{(12.5)^2 + (4.33)^2} = 13.33$ kips ◀

$\alpha = \tan^{-1}\left|\frac{4.33}{12.5}\right| = 19.1°$

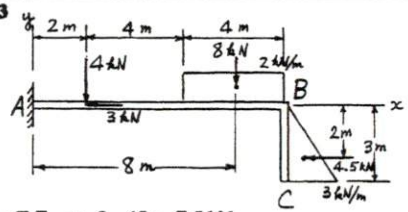
$\theta = 180° + \alpha = 199.1°$ ◀

$\Sigma M_A = 2.5(12) + 8(4) + 2(1.33) = 64.66$ kip • ft ↺

$12.5(\bar{y}) = 64.66$

$\bar{y} = 5.17$ ft above point A ◀

2-83



$R_x = \Sigma F_x = -3 - 45 = -7.5$ kN
$R_y = \Sigma F_y = -4 - 8 = -12$ kN

$$R = \sqrt{(7.5)^2 + (12)^2} = 14.5 \text{ kN}$$ ◄

$$\alpha = \tan^{-1}\left|\frac{12}{7.5}\right| = 58.0^\circ$$

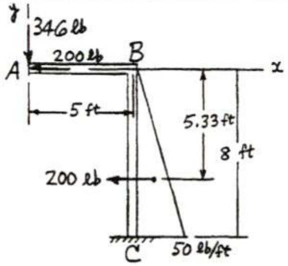
$$\theta = 180^\circ + \alpha = 238.0^\circ$$ ◄

$\Sigma M_A = -4(2) - 8(8) - 4.5(2) = -81$ kN • m

$12(\bar{x}) = 81$

$\bar{x} = 6.75$ m to the right of point A ◄

2-84



$R_x = \Sigma F_x = -200 - 200 = -400$ lb
$R_y = \Sigma F_y = -346$ lb

$$R = \sqrt{(400)^2 + (346)^2} = 529 \text{ lb}$$ ◄

$$\alpha = \tan^{-1}\left|\frac{346}{400}\right| = 40.9^\circ$$

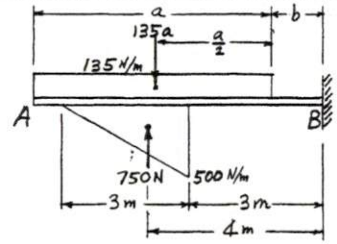
$$\theta = 180^\circ + \alpha = 220.9^\circ$$ ◄

$\Sigma M_A = -200(5.33) = -1066$ lb • ft

$346(\bar{x}) = 1066$

$\bar{x} = 3.08$ ft to the right of point A ◄

2-85



For a zero resultant force we must have

$135a = 750$

$a = 5.56$ ft ◄

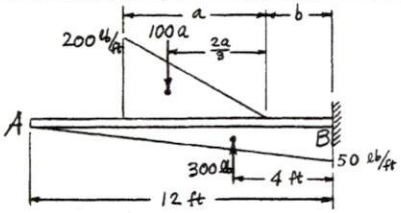
For a zero resultant couple, the moment of the forces about any point must be zero. Taking moment about B we write

$$\Sigma M_B = 135a\left(b + \frac{a}{2}\right) - 750(4) = 0$$

$$135(5.56)\left(b + \frac{5.56}{2}\right) = 750(4)$$

$b = 1.22$ ft ◄

2-86



For a resultant **R** = 200 lb ↓ we must have

$100a - 300 = 200$

$a = 5$ ft ◄

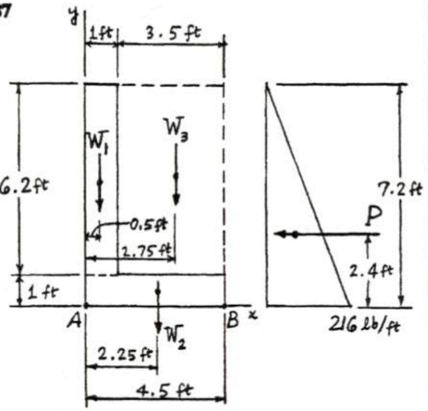
For the resultant to be located at the mid-span we must have

$$(+\ )\ M_B:\ 100a\left(\frac{2a}{3} + b\right) - 300(4) = 200(6)$$

$$100(5)\left(\frac{10}{3} + b\right) = 200(6) + 300(4)$$

$b = 1.47$ ft ◄

2-87



$W_1 = (1\text{ ft})(6.2\text{ ft})(1\text{ ft})(150\text{ lb/ft}^3) = 930$ lb
$W_2 = (4.5\text{ ft})(1\text{ ft})(1\text{ ft})(150\text{ lb/ft}^3) = 675$ lb
$W_3 = (3.5\text{ ft})(6.2\text{ ft})(1\text{ ft})(100\text{ lb/ft}^3) = 2170$ lb

$$P = \frac{1}{2}(216\text{ lb/ft})(7.2\text{ ft}) = 778\text{ lb}$$

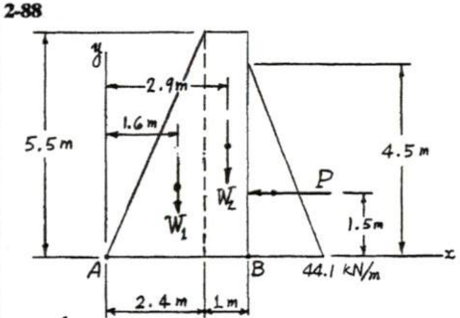
$R_x = -P = -778$ lb
$R_y = -W_1 - W_2 - W_3 = -930 - 675 - 2170 = -3775$ lb
$\Sigma M_A = -W_1(0.5) - W_2(2.25) - W_3(2.75) + P(2.4)$
$= -930(0.5) - 675(2.25) - 2170(2.75) + 778(2.4)$
$= -6084$ lb • ft

$3775(\bar{x}) = 6084$

$\bar{x} = 1.61$ ft ◄

Thus the resultant passes through the point $\bar{x} = 1.61$ ft to the right of A, which is within the middle one-third of the base AB. So that compressive soil pressure exist over the entire base.

2-88



$$W_1 = \frac{1}{2}(2.4\text{ m})(5.5\text{ m})(1\text{ m})(23.6\text{ kN/m}^3) = 156\text{ kN}$$

$W_2 = (1\text{ m})(5.5\text{ m})(1\text{ m})(23.6\text{ kN/m}^3) = 130$ kN

$$P = \frac{1}{2}(44.1\text{ kN/m})(4.5\text{ m}) = 99.2\text{ kN}$$

$R_x = -P = -99.2$ kN
$R_y = -W_1 - W_2 = -286$ kN
$\Sigma M_A = -W_1(1.6) - W_2(2.9) + P(1.5)$
$= -(156)(1.6) - (130)(2.9) + (99.2)(1.5)$
$= -478$ kN • m

$286(\bar{x}) = 478$

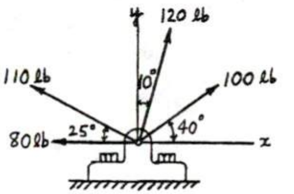
$\bar{x} = 1.67$ m ◄

Thus the resultant passes through the point $\bar{x} = 1.67$ m to the right of A, which is within the middle one-third of the base AB. So that compressive soil pressure exist over the entire base.

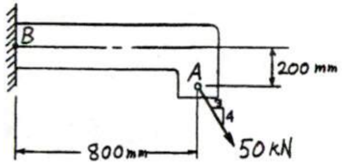
Test Problems for Chapter 2

The following problems may either be given to students for them to practice their problem solving skills or be used as test problems. The answers are provided on the next page.

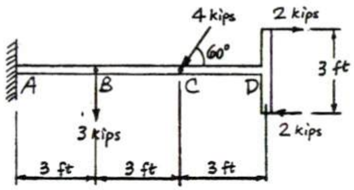
(1) Determine the magnitude and direction of the resultant of the concurrent coplanar force system shown.



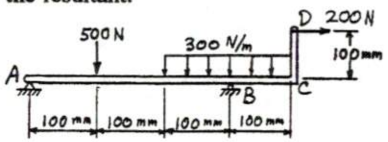
(2) Replace the 50-lb force at *A* by an equivalent force-couple at *B*.



(3) Find the resultant of the force system acting on the bracket shown. Specify the magnitude, direction, and point of application along *AD* of the resultant.

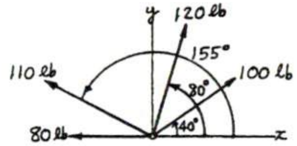


(4) Find the resultant of the loads applied to the beam shown. The loads include two concentrated loads and a uniform load. Specify the magnitude, direction, and point of application along *AC* of the resultant.



Solutions to Test Problems for Chapter 2

(1)



F (lb)	θ	Fx = F cos θ (lb)	Fy = F sin θ (lb)
100	40°	76.6	64.3
120	80°	20.8	118.2
110	155°	-99.7	46.5
80	180°	-80.0	0
Σ		-82.3	229.0

$R_x = -82.3$ lb $R_y = -229.0$ lb

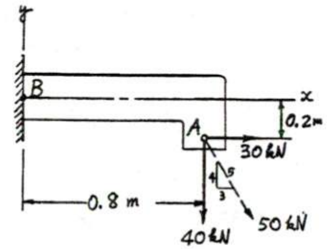
$R = \sqrt{(82.3)^2 + (229)^2} = 243$ lb

$\alpha = \tan^{-1}\left|\frac{229.0}{-82.3}\right| = 70.2°$

The resultant is in the second quadrant:

$\theta = 180° - \alpha = 109.8°$

(2)

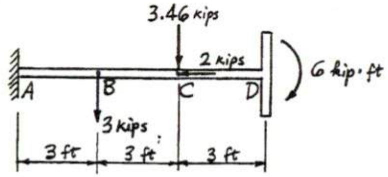


$M_B = -40(0.8) + 30(0.2) = -26.0$ kN • m ↻

The equivalent force-couple at B is:

F = 50 kN (4/3/5) M = 26.0 kN • m ↻

(3)



$R_x = -2$ kips ←

$R_y = -3 - 3.46 = -6.46$ kips ↓

$R = \sqrt{(2)^2 + (6.46)^2} = 6.76$ kips

$\alpha = \tan^{-1}\left|\frac{-6.46}{-2}\right| = 73.0°$

The resultant is in the third quadrant:

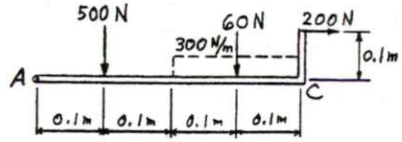
$\theta = 180° + \alpha = 253°$

$\Sigma M_A = -3(3) - 3.46(6) - 6 = -35.8$ kip • ft ↻

M_A(↻+): $6.46(\bar{x}) = 35.8$

$\bar{x} = 5.54$ ft to the right of A

(4)



$R_x = 200$ N →

$R_y = -500 - 60 = -560$ N ↓

$R = \sqrt{(200)^2 + (560)^2} = 595$ N

$\alpha = \tan^{-1}\left|\frac{-560}{200}\right| = 70.5°$

The resultant is in the fourth quadrant:

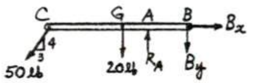
$\theta = -\alpha = -70.5°$

$\Sigma M_A = -500(0.1) - 60(0.3) - 200(0.1) = -88$ N • m ↻

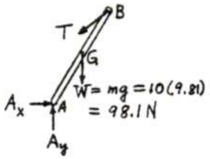
(↻+): $560(\bar{x}) = 88$

$\bar{x} = 0.157$ m = 157 mm to the right of A

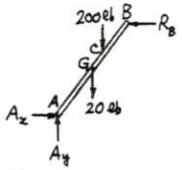
3-1



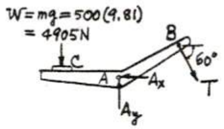
3-2



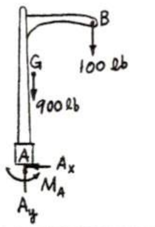
3-3



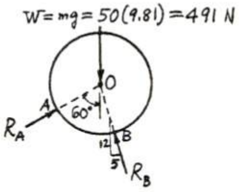
3-4



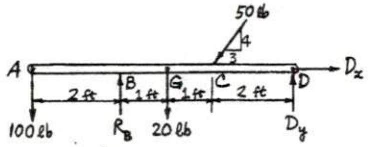
3-5



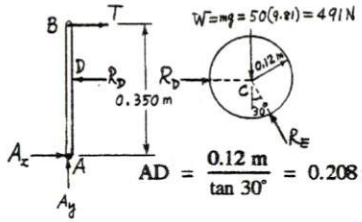
3-6



3-7

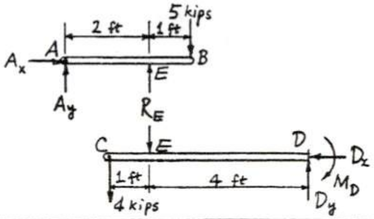


3-8

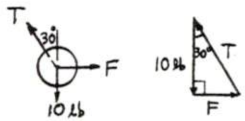


$$AD = \frac{0.12\text{ m}}{\tan 30^\circ} = 0.208\text{ m}$$

3-9

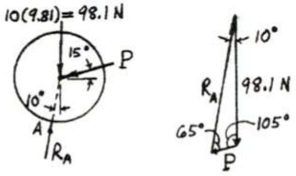


3-10



F = 10 tan 30° = 5.77 lb ◀

3-11

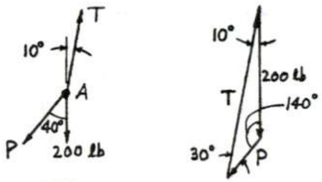


$$\frac{P}{\sin 10^\circ} = \frac{98.1}{\sin 65^\circ}$$

$$P = \frac{98.1 \sin 10^\circ}{\sin 65^\circ} = 18.8\text{ N} \quad ◀$$

3-12

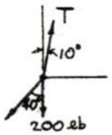
(a)



$$\frac{P}{\sin 10^\circ} = \frac{200}{\sin 30^\circ}$$

$$P = \frac{200 \sin 10^\circ}{\sin 30^\circ} = 69.5\text{ lb} \quad ◀$$

(b)



$$\Sigma F_x = T \sin 10^\circ - P \sin 40^\circ = 0$$

$$T = \frac{\sin 40^\circ}{\sin 10^\circ} P$$

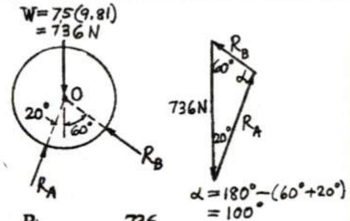
$$\Sigma F_y = T \cos 10^\circ - P \cos 40^\circ - 200 = 0$$

$$\left(\frac{\sin 40^\circ}{\sin 10^\circ} \cos 10^\circ - \cos 40^\circ\right) P = 200$$

from which

P = 69.5 lb ◀

3-13

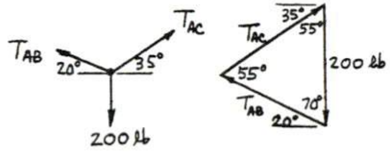


$$\frac{R_A}{\sin 60^\circ} = \frac{R_B}{\sin 20^\circ} = \frac{736}{\sin 100^\circ}$$

$$R_A = \frac{736 \sin 60^\circ}{\sin 100^\circ} = 647\text{ N} \quad ◀$$

$$R_B = \frac{736 \sin 20^\circ}{\sin 100^\circ} = 256\text{ N} \quad ◀$$

3-14



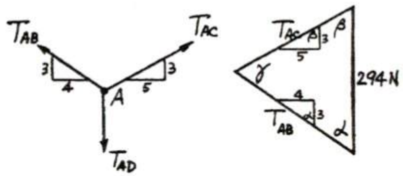
$$\frac{T_{AB}}{\sin 55^\circ} = \frac{T_{AC}}{\sin 70^\circ} = \frac{200\text{ lb}}{\sin 55^\circ}$$

from which we get

$T_{AB} = 200$ lb ◀

$T_{AC} = 229$ lb ◀

3-15



$T_{AD} = W = mg = 30 \times 9.81 = 294$ N

$$\alpha = \tan^{-1}\frac{4}{3} = 53.1^\circ$$

$$\beta = \tan^{-1}\frac{5}{3} = 59.0^\circ$$

$$\gamma = 180^\circ - (53.1^\circ + 59.0^\circ) = 67.9^\circ$$

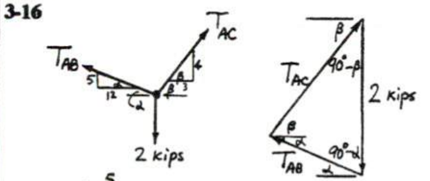
$$\frac{T_{AB}}{\sin\beta} = \frac{T_{AC}}{\sin\alpha} = \frac{294}{\sin\gamma}$$

$$T_{AB} = \frac{294 \sin 59.0^\circ}{\sin 67.9^\circ} = 272\text{ N} \quad ◀$$

$$T_{AC} = \frac{294 \sin 53.1^\circ}{\sin 67.9^\circ} = 254\text{ N} \quad ◀$$

$T_{AD} = W = 294$ N ◀

3-16



$$\alpha = \tan^{-1}\frac{5}{12} = 22.6^\circ$$

$$\beta = \tan^{-1}\frac{4}{3} = 53.1^\circ$$

$$\frac{T_{AB}}{\sin(90^\circ - \beta)} = \frac{T_{AC}}{\sin(90^\circ - \alpha)} = \frac{2\ \text{kips}}{\sin(\alpha + \beta)}$$

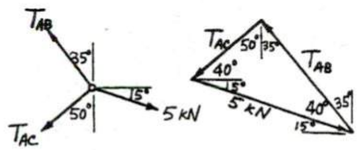
$$\frac{T_{AB}}{\sin 36.9^\circ} = \frac{T_{AC}}{\sin 67.4^\circ} = \frac{2\ \text{kips}}{\sin 75.4^\circ}$$

From which we get

$T_{AB} = 1.239$ kips ◀

$T_{AC} = 1.905$ kips ◀

3-17



$$\frac{T_{AB}}{\sin 55^\circ} = \frac{T_{AC}}{\sin 40^\circ} = \frac{5\ \text{kN}}{\sin 85^\circ}$$

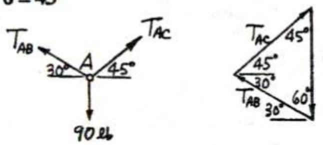
From which we get

$T_{AB} = 4.11$ kN ◀

$T_{AC} = 3.23$ kN ◀

3-18

(a) $\theta = 45^\circ$



$$\frac{T_{AB}}{\sin 45^\circ} = \frac{T_{AC}}{\sin 60^\circ} = \frac{90}{\sin 75^\circ}$$

$$T_{AB} = \frac{90 \sin 45^\circ}{\sin 75^\circ} = 65.9\ \text{lb}$$ ◀

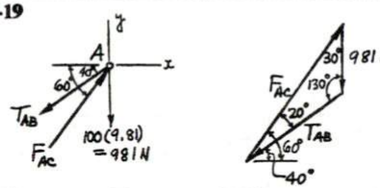
$$T_{AC} = \frac{90 \sin 60^\circ}{\sin 75^\circ} = 80.7\ \text{lb}$$ ◀

(b)

For T_{AC} to be a minimum,

$\theta = 90^\circ - 30^\circ = 60^\circ$ ◀

3-19



$$\frac{T_{AB}}{\sin 30^\circ} = \frac{F_{AC}}{\sin 130^\circ} = \frac{981\ \text{N}}{\sin 20^\circ}$$

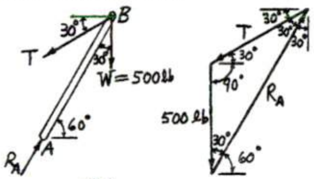
From which we get

$T_{AB} = 1430$ N ◀

$T_{AC} = 2200$ N ◀

3-20

The three forces meet at B.



$$\frac{R_A}{\sin 120^\circ} = \frac{500}{\sin 30^\circ}$$

from which we get

$R_A = 866$ lb ∡ 60° ◀

3-21

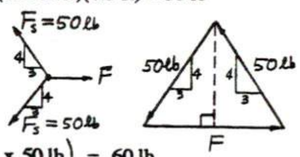
Deformed length of spring

$$AB = \sqrt{2^2 + 1.5^2} = 2.5\ \text{ft}$$

Sring deformation $x = 2.5 - 2 = 0.5$ ft

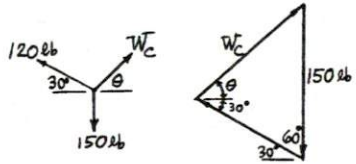
Tensile force in spring:

$F_s = kx = (100\ \text{lb/ft})(0.5\ \text{ft}) = 50$ lb



$$F = 2\left(\frac{3}{5} \times 50\ \text{lb}\right) = 60\ \text{lb}$$ ◀

3-22



$$W_C = \sqrt{150^2 + 120^2 - 2(150)(120)\cos 60^\circ}$$

$= 137.5$ lb ◀

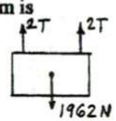
$$\frac{\sin(\theta + 30^\circ)}{150} = \frac{\sin 60^\circ}{137.5}$$

$\theta + 30^\circ = 70.9^\circ$

$\theta = 40.9^\circ$ ◀

3-23

The tension is T throughout the cable. Hence the free-body diagram is

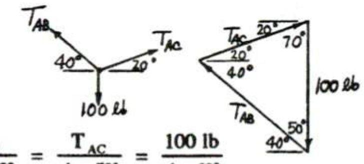


For equilibrium, we must have

$4T = 1962$ N

$T = 491$ N ◀

3-24



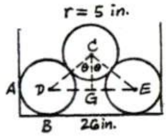
$$\frac{T_{AB}}{\sin 70^\circ} = \frac{T_{AC}}{\sin 50^\circ} = \frac{100\ \text{lb}}{\sin 60^\circ}$$

From which we get

$T_{AB} = 108.5$ lb ◀

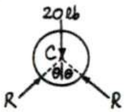
$T_{AC} = 88.5$ lb ◀

3-25



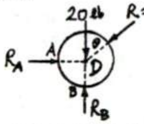
$CD = CE = 2r = 10$ in.
$DE = 26$ in. $- 2r = 16$ in.
$DG = DE/2 = 8$ in.

$$\theta = \sin^{-1}\frac{DG}{CD} = \sin^{-1}\frac{8}{10} = 53.1^\circ$$



$\Sigma F_y = 2R\cos\theta - 20 = 0$

$$R = \frac{20}{2\cos 53.1^\circ} = 16.66\ \text{lb}$$



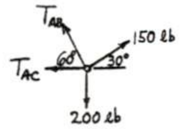
$\Sigma F_x = R_A - 16.66 \sin 53.1^\circ = 0$

$R_A = 13.32$ lb ◀

$\Sigma F_y = R_B - 20 - 16.66 \cos 53.1^\circ = 0$

$R_B = 30.0$ lb ◀

3-26



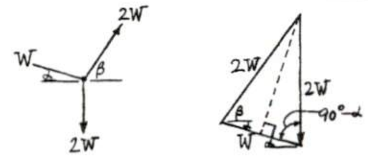
$\Sigma F_y = T_{AB} \sin 60° + 150 \sin 30° - 200 = 0$

$T_{AB} = 144.3$ lb ◀

$\Sigma F_x = -T_{AC} - 144.3 \sin 60° + 150 \cos 30° = 0$

$T_{AC} = 57.8$ lb ◀

3-27



Since the force triangle is an isosclels, we have

$W = 2(2W) \cos (90° - \alpha)$

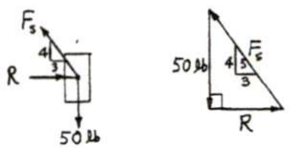
$W = 4W \sin \alpha$

$\alpha = \sin^{-1} 0.25 = 14.48°$ ◀

$\alpha + \beta = 90° - \alpha,$

$\beta = 90° - 2\alpha = 61.0°$ ◀

3-28



$\frac{F_s}{50} = \frac{5}{4}$

$F_s = 62.5$ lb

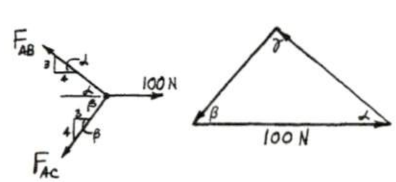
Elongation of Spring

$x = \sqrt{18^2 + 24^2} - 18 = 12 \text{ in.} = 1 \text{ ft}$

Spring Constant

$k = \frac{F_s}{x} = \frac{62.5 \text{ lb}}{1 \text{ ft}} = 62.5 \text{ lb / ft}$ ◀

3-29



$\alpha = \tan^{-1} \frac{3}{4} = 36.9°$

$\beta = \tan^{-1} \frac{4}{3} = 53.1°$

$\gamma = 180° - (\alpha + \beta) = 90°$

$F_{AB} = (100 \text{ N}) \cos 36.9° = 80 \text{ N}$

$F_{AC} = (100 \text{ N}) \cos 53.1° = 60 \text{ N}$

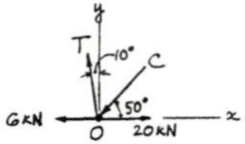
$x_{AB} = \frac{F_{AB}}{k_{AB}}$

$= \frac{80 \text{ N}}{250 \text{ N / m}} = 0.32 \text{ m}$ ◀

$x_{AC} = \frac{F_{AC}}{k_{AC}}$

$= \frac{60 \text{ N}}{200 \text{ N / m}} = 0.3 \text{ m}$ ◀

3-30



$\Sigma F_y = T \cos 10° - C \sin 50° = 0$

$T = 0.778 C$ (a)

$\Sigma F_x = -T \sin 10° - C \cos 50° + 20 - 6 = 0$ (b)

Substituting (a) into (b) gives

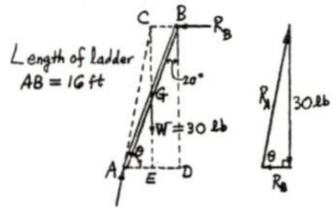
$(-0.778 \sin 10° - \cos 50°) C = -14$

$C = 18 \text{ kN}$ ◀

$T = 0.778 (18 \text{ kN}) = 14 \text{ kN}$ ◀

3-31

(a) The ladder is a three-force body. Two of the forces W and R_B meet at C, the third force R_A must also pass through C.



$CE = BD = 16 \cos 20° = 15.04$ ft

$AD = 16 \sin 20° = 5.47$ ft

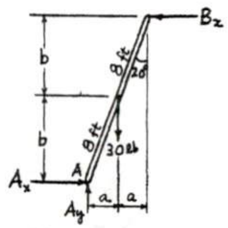
$AE = ED = \frac{1}{2} AD = 2.735$ ft

$\theta = \tan^{-1} \frac{CE}{AE} = \tan^{-1} \frac{15.04}{2.735} = 79.7°$

$R_A = \frac{30 \text{ lb}}{\sin 79.7°} = 30.5 \text{ lb} \measuredangle 79.7°$ ◀

$R_B = \frac{30 \text{ lb}}{\tan 79.7°} = 5.45 \text{ lb} \leftarrow$ ◀

(b)



$a = (8 \text{ ft}) \sin 20° = 2.74$ ft

$b = (8 \text{ ft}) \cos 20° = 7.52$ ft

$\Sigma M_B = B_x(2 \times 7.52) - 30(2.74) = 0$

$B_x = 5.47 \text{ lb} \leftarrow$ ◀

$\Sigma F_x = A_x - 5.47 = 0$

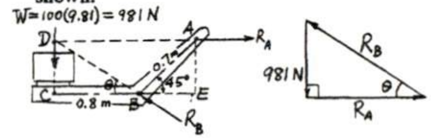
$A_x = 5.47 \text{ lb} \rightarrow$ ◀

$\Sigma F_y = A_y - 30 = 0$

$A_y = 30 \text{ lb} \uparrow$ ◀

3-32

(a) The three forces W, R_A, and R_B meet at D as shown.



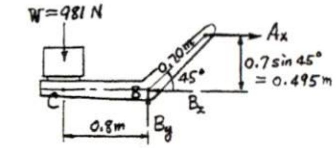
$CD = AE = AB \sin 45° = (0.70 \text{ m}) \sin 45° = 0.495 \text{ m}$

$\theta = \tan^{-1} \frac{CD}{BC} = \tan^{-1} \frac{0.495}{0.8} = 31.7°$

$R_A = \frac{981 \text{ N}}{\tan 31.7°} = 1590 \text{ N} \rightarrow$ ◀

$R_B = \frac{981 \text{ N}}{\sin 31.7°} = 1870 \text{ N} \measuredangle 31.7°$ ◀

(b)



$\Sigma M_B = -A_x(0.495) + 981(0.8) = 0$

$A_x = 1590 \text{ N} \rightarrow$ ◀

$\Sigma F_x = -B_x + 1590 = 0$

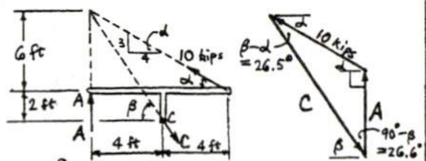
$B_x = 1590 \text{ N} \leftarrow$ ◀

$\Sigma F_y = B_y - 981 = 0$

$B_y = 981 \text{ N} \uparrow$ ◀

3-33

(a)



$\alpha = \tan^{-1} \frac{3}{4} = 36.9°$

$\beta = \tan^{-1} \frac{8}{4} = 63.4°$

$\frac{A}{\sin 26.5°} = \frac{C}{\sin 126.9°} = \frac{10 \text{ kips}}{\sin 26.6°}$ (Cont'd)

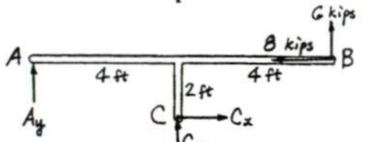
3-33 (Cont)

From which we get

A = 10.0 kips ↑ ◀

C = 17.9 kips 63.4° ◀

(b)



$\Sigma M_C = -A_y(4) + 8(2) + 6(4) = 0$

A_y = 10 kips ↑ ◀

$\Sigma F_x = C_x - 8 = 0$

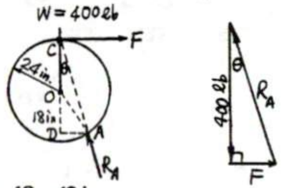
C_x = 8 kips → ◀

$\Sigma F_y = C_y + 10 + 6 = 0$

C_y = -16 kips ↓ ◀

3-34

(a) The three forces W, F, and R_A meet at C.



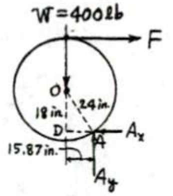
CD = 24 + 18 = 42 in.

$AD = \sqrt{24^2 - 18^2} = 15.87$ in.

$\theta = \tan^{-1}\dfrac{AD}{CD} = \tan^{-1}\dfrac{15.87 \text{ in.}}{42 \text{ in.}} = 20.7°$

F = (400 lb)tan θ = (400 lb)tan 20.7° = 151 lb ◀

(b)

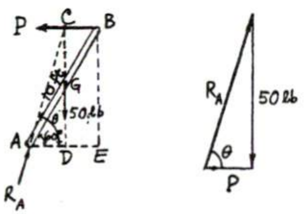


$\Sigma M_A = -F(24 + 18) + 400(15.87) = 0$

F = 151 lb → ◀

3-35

(a)



CD = BE = AB sin 60° = (10 ft) sin 60° = 8.66 ft

AE = AB cos 60° = (10 ft) cos 60° = 5 ft

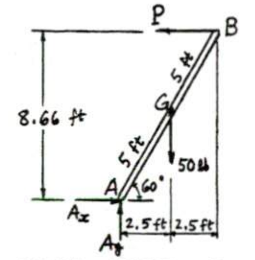
$AD = DE = \dfrac{1}{2}AE = 2.5$ ft

$\theta = \tan^{-1}\dfrac{CD}{AD} = \tan^{-1}\dfrac{8.66}{2.5} = 73.9°$

$P = \dfrac{50}{\tan\theta} = \dfrac{50}{\tan 73.9°} = 14.4$ lb ◀

$R_A = \dfrac{50}{\sin\theta} = \dfrac{50}{\sin 73.9°} = 52.0$ lb ∠ 73.9° ◀

(b)



$\Sigma M_A = P(8.66) - 50(2.5) = 0$

P = 14.4 lb ← ◀

$\Sigma F_x = A_x - 14.41 = 0$

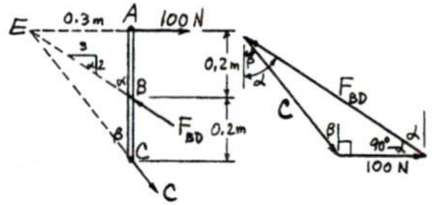
A_x = 14.4 lb → ◀

$\Sigma F_y = A_y - 150 = 0$

A_y = 50 lb ↑ ◀

3-36

(a) BD is a two-force body. ABC is a three-force body.



$\alpha = \tan^{-1}\dfrac{0.3}{0.2} = 56.3°$

$\beta = \tan^{-1}\dfrac{0.3}{0.4} = 36.9°$

$\dfrac{F_{BD}}{\sin(90° + \beta)} = \dfrac{C}{\sin(90° - \alpha)} = \dfrac{100}{\sin(\alpha - \beta)}$

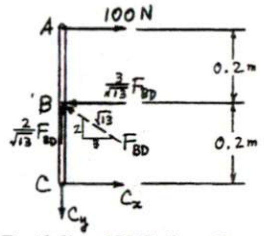
$\dfrac{F_{BD}}{\sin 126.9°} = \dfrac{C}{\sin 33.7°} = \dfrac{100}{\sin 19.4°}$

From which we get

F_{BD} = 241 N (C) ◀

C = 167.0 N ∠ 36.9° ◀

(b)



$\sum M_C = \dfrac{3}{\sqrt{13}}F_{BD}(0.2) - 100(0.4) = 0$

F_{BD} = 240 N ◀

$\sum F_x = C_x - \dfrac{3}{\sqrt{13}}(240) + 100 = 0$

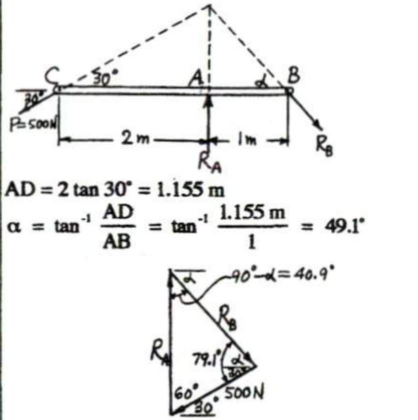
C_x = 100 N ◀

$\sum F_y = -C_y + \dfrac{2}{\sqrt{13}}(240) = 0$

C_y = 133 N ◀

3-37

(a) The three force P, R_A, and R_B meet at D.



AD = 2 tan 30° = 1.155 m

$\alpha = \tan^{-1}\dfrac{AD}{AB} = \tan^{-1}\dfrac{1.155 \text{ m}}{1} = 49.1°$

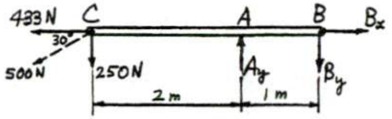
$\dfrac{R_A}{\sin 79.1°} = \dfrac{R_B}{\sin 60°} = \dfrac{500 \text{ N}}{\sin 40.9°}$

From which we get

R_A = 750 N ↑ ◀

R_B = 661 N ∠ 41.9° ◀

(b)



$\sum M_B = -A_y(1) + 250(3) = 0$

A_y = 750 N ↑ ◀

$\sum F_x = B_x - 433 = 0$

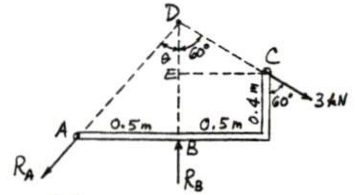
B_x = 433 N → ◀

$\sum F_y = -B_y - 250 + 750 = 0$

B_y = 500 N ↓ ◀

3-38

(a) The three forces meet at D.



$$DE = \frac{0.5}{\tan 60^\circ} = 0.289 \text{ m}$$

BD = 0.4 + 0.289 = 0.689 m

$$\theta = \tan^{-1}\frac{AB}{BD} = \tan^{-1}\frac{0.5}{0.689} = 36^\circ$$

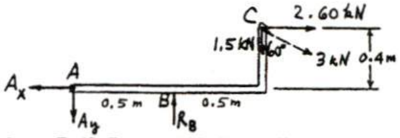
$\alpha = 180^\circ - (60^\circ + \theta) = 84^\circ$

$$\frac{R_A}{\sin 60^\circ} = \frac{R_B}{\sin\alpha} = \frac{3\text{ kN}}{\sin\theta}$$

$$R_A = \frac{(3\text{ kN})\sin 60^\circ}{\sin 36^\circ} = 4.42\text{ kN} \angle 36^\circ$$ ◀

$$R_B = \frac{(3\text{ kN})\sin 84^\circ}{\sin 36^\circ} = 5.08\text{ kN} \uparrow$$ ◀

(b)



$\sum M_A = B_y(0.5) - 2.60(0.4) - 1.5(1.0) = 0$

$B_y = 5.08$ kN ↑ ◀

$\sum F_x = -A_x + 2.60 = 0$

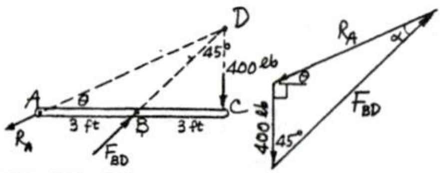
$A_x = 2.60$ kN ← ◀

$\sum F_y = -A_y + 5.08 - 1.5 = 0$

$A_y = 3.58$ kN ↓ ◀

3-39

(a) Member BD is a two-force body. Member ABC is a three-force body. The three forces meet at D.



CD = BC = 3 ft

$$\theta = \tan^{-1}\frac{CD}{AC} = \tan^{-1}\frac{3}{6} = 26.6^\circ$$

$\alpha = 180^\circ - (116.6^\circ + 45^\circ) = 18.4^\circ$

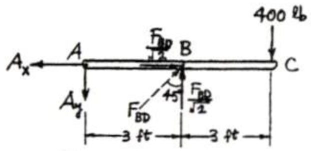
$$\frac{R_A}{\sin 45^\circ} = \frac{F_{BD}}{\sin 116.6^\circ} = \frac{400\text{ lb}}{\sin 18.4^\circ}$$

From which we get

$R_A = 896$ lb $\angle$ 26.6° ◀

$R_D = F_{BD} = 1133$ lb $\angle$ 45° ◀

(b)



$$\sum M_A = \frac{F_{AB}}{\sqrt{2}}(3) - 400(6) = 0$$

$F_{AB} = 1130$ lb (C) ◀

$$\sum F_x = -A_x + \frac{F_{AB}}{\sqrt{2}} = 0$$

$A_x = 800$ lb ← ◀

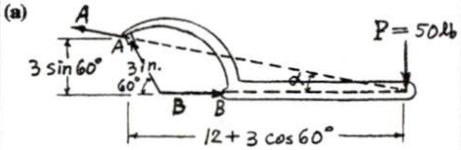
$$\sum F_y = -A_y + \frac{1130}{\sqrt{2}} - 400 = 0$$

$A_y = 400$ lb ↓ ◀

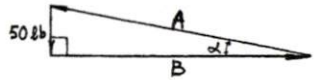
$R_D = F_{AB} = 1130$ lb $\angle$ 45° ◀

3-40

(a)



$$\alpha = \tan^{-1}\frac{3\sin 60^\circ}{12 + 3\cos 60^\circ} = 10.89^\circ$$



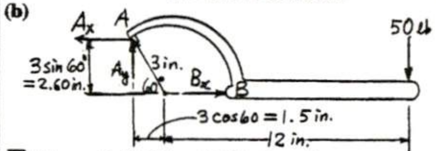
$$B = \frac{50}{\tan\alpha} = 260 \text{ lb}$$

B = 260 lb → ◀

$$A = \frac{50}{\sin\alpha} = 265 \text{ lb}$$

A = 265 lb ∠ 10.89° ◀

(b)



$\sum M_A = B_x(2.60) - 50(13.5) = 0$

$B_x = 260$ lb → ◀

$\sum F_x = -A_x + 260 = 0$

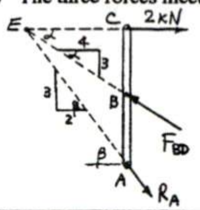
$A_x = 260$ lb ← ◀

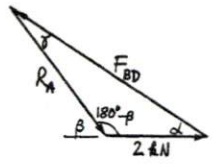
$\sum F_y = A_y - 50 = 0$

$A_y = 50$ lb ↑ ◀

3-41

(a) The three forces meet at E.





$$\alpha = \tan^{-1}\frac{3}{4} = 36.9^\circ$$

$$\beta = \tan^{-1}\frac{3}{2} = 56.3^\circ$$

$\gamma = 180^\circ - (123.7^\circ + 36.9) = 19.4^\circ$

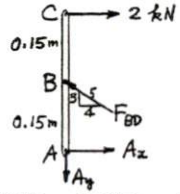
$$\frac{R_A}{\sin 36.9^\circ} = \frac{F_{BD}}{\sin 123.7^\circ} = \frac{2\text{ kN}}{\sin 19.4^\circ}$$

From which we get

$R_A = 3.62$ kN ◀

$R_{BD} = F_{BD} = 5.00$ kN ◀

(b)



$$\sum M_A = \frac{4}{5}F_{BD}(0.15) - 2(0.30) = 0$$

$F_{BD} = 5.00$ kN

$R_D = F_{BD} = 5.00$ kN ◀

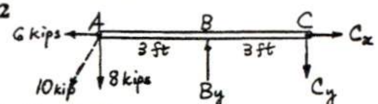
$$\sum F_x = A_x + 2 - \frac{4}{5}(5.00) = 0$$

$A_x = 2.00$ kN → ◀

$$\sum F_y = -A_y + \frac{3}{5}(5.00) = 0$$

$A_y = 3.00$ kN ↓ ◀

3-42



$\Sigma F_x = C_x - 6 = 0$

$C_x = 6$ kips → ◀

$\Sigma M_C = -B_y(3) + 8(6) = 0$

$B_y = 16$ kips ↑ ◀

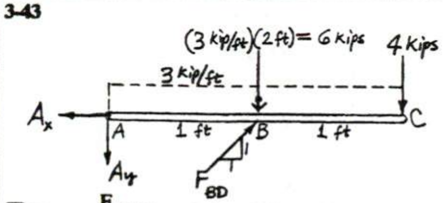
$\Sigma M_B = C_y(3) + 8(3) = 0$

$C_y = 8$ kips ↓ ◀

Check:

$\Sigma F_y = -8 + 16 - 8 = 0$ (Checks)

3-43



$\sum M_A = \frac{F_{BD}}{\sqrt{2}}(1) - 6(1) - 4(2) = 0$

$F_{BD} = 19.8$ kips

$R_D = F_{BD} = 19.8$ kips ◀

$\sum F_x = -A_x + \frac{19.8}{\sqrt{2}} = 0$

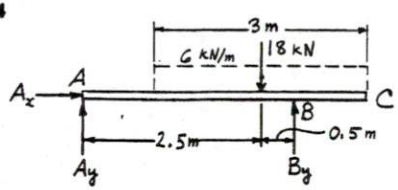
$A_x = 14.0$ kips ← ◀

$\sum F_y = -A_y + \frac{19.8}{\sqrt{2}} - 6 - 4 = 0$

$A_y = 4.00$ kips ↓ ◀

Check:
$\sum M_B = 4.00(1) - 4(1) = 0$ (Checks)

3-44



$\sum F_x = A_x + 0 = 0$

$A_x = 0$ ◀

$\sum M_B = -A_y(3\text{ m}) + (18\text{ kN})(0.5\text{ m}) = 0$

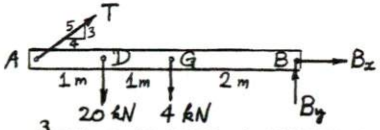
$A_y = 3$ kN ↑ ◀

$\sum M_A = B_y(3\text{ m}) - (18\text{ kN})(2.5\text{ m}) = 0$

$B_y = 15$ kN ↑ ◀

Check:
$\sum F_y = 3 + 15 - 18 = 0$ (Checks)

3-45



$\Sigma M_B = -\frac{3}{5}T(4\text{ m}) + (20\text{ kN})(3\text{ m}) + (4\text{ kN})(2\text{ m}) = 0$

$T = 28.33$ kN ◀

$\Sigma F_x = \frac{4}{5}(28.33) - B_x = 0$

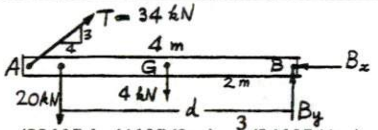
$B_x = 22.7$ kN ← ◀

$\Sigma M_A = B_y(4\text{ m}) - (4\text{ kN})(2\text{ m}) - (20\text{ kN})(1\text{ m}) = 0$

$B_y = 7$ kN ↑ ◀

Check:
$\Sigma F_y = \frac{3}{5}(28.33) + 7 - 20 - 4 = 0$ (Checks)

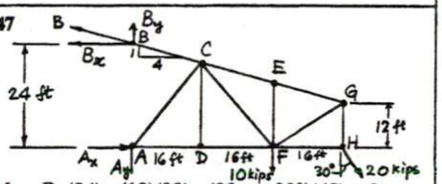
3-46



$\Sigma M_B = (20\text{ kN})d - (4\text{ kN})(2\text{ m}) - \frac{3}{5}(34\text{ kN})(4\text{ m}) = 0$

$d = 3.68$ m ◀

3-47



$\Sigma M_A = B_x(24) - (10)(32) - (20\cos 30°)(48) = 0$

$B_x = 48.0$ kips ← ◀

$B_y = B_x/4 = 12$ kips ↑ ◀

$\Sigma M_B = A_x(24) - (10)(32) - (20\cos 30°)(48) + (20\sin 30°)(24) = 0$

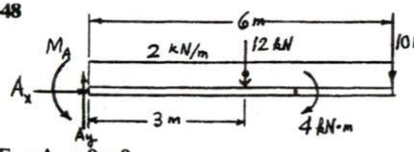
$A_x = 38.0$ kips → ◀

$\Sigma F_y = A_y + B_y - 10 - 20\cos 30° = 0$

$A_y = 15.3$ kips ↑ ◀

Check:
$\Sigma F_x = 38 - 48.0 + 20\sin 30° = 0$ (Checks)

3-48



$\Sigma F_x = A_x + 0 = 0$

$A_x = 0$

$\Sigma F_y = A_y - 12 - 10 = 0$

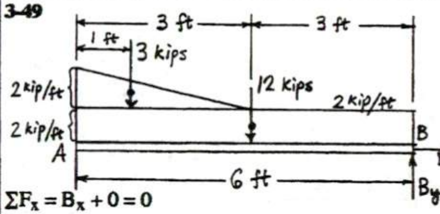
$A_y = 22$ kips

$A = 22$ kN ↑ ◀

$\Sigma M_A = M_A - (12)(3) - (10)(6) - 4 = 0$

$M_A = 100$ kN·m ◀

3-49



$\Sigma F_x = B_x + 0 = 0$

$B_x = 0$ ◀

$\Sigma M_A = B_y(6) - (3)(1) - (12)(3) = 0$

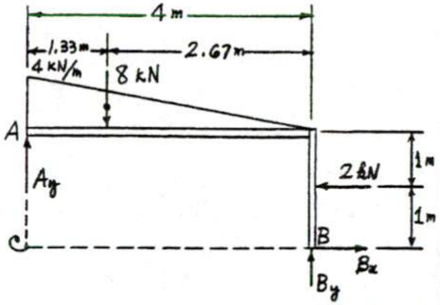
$B_y = 6.5$ kips ↑ ◀

$\Sigma M_B = -A_y(6) + (3)(5) + (12)(3) = 0$

$A_y = 8.5$ kips ↑ ◀

Check:
$\Sigma F_y = 6.5 + 8.5 - 3 - 12 = 0$ (Checks)

3-50



$\Sigma F_x = B_x - 2 = 0$

$B_x = 2$ kN → ◀

$\Sigma M_C = B_x(4) + (2)(1) - (8)(1.33) = 0$

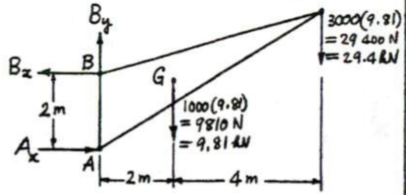
$B_y = 2.16$ kN ↑ ◀

$\Sigma M_B = -A_y(4) + (8)(2.67) + (2)(1) = 0$

$A_y = 5.84$ kN ↑ ◀

Check:
$\Sigma F_y = 5.84 + 2.16 - 8 = 0$ (Checks)

3-51



$\sum M_B = A_x(2) - (9.81)(2) - (29.4)(6) = 0$

$A_x = 98.0$ kN → ◀

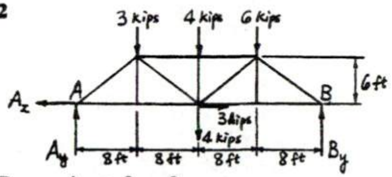
$\sum F_x = -B_x + 98.0 = 0$

$B_x = 98.0$ kN ← ◀

$\sum F_y = B_y - 9.81 - 29.4 = 0$

$B_y = 39.2$ kN ↑ ◀

3-52



$\sum F_x = -A_x + 3 = 0$

$A_x = 3$ kips ← ◀

$\sum M_B = -A_y(32) + (3)(24) + (4+4)(16) + (6)(8) = 0$

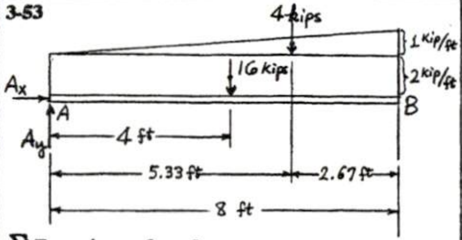
$A_y = 7.75$ kips ↑ ◀

$\sum M_A = B_y(32) - (3)(8) - (4+4)(16) - (6)(24) = 0$

$B_y = 9.25$ kips ↑ ◀

Check:
$\sum F_y = 7.75 + 9.25 - 3 - 4 - 6 - 4 = 0$ (Checks)

3-53



$\sum F_x = A_x + 0 = 0$

$A_x = 0$ ◀

$\sum M_B = -A_y(8) + (16)(4) + (4)(2.67) - 20 = 0$

$A_y = 6.84 \text{ kips} \uparrow$ ◀

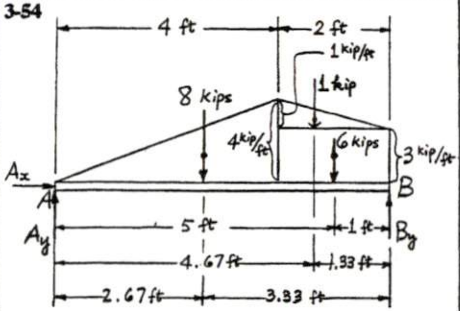
$\sum M_A = B_y(8) - (16)(4) - (4)(5.33) - 20 = 0$

$B_y = 13.17 \text{ kips} \uparrow$ ◀

Check:

$\sum F_y = 6.84 + 13.17 - 16 - 4 = 0.01$ (Checks)

3-54



$\sum F_x = A_x + 0 = 0$

$A_x = 0$ ◀

$\sum M_B = -A_y(6) + (8)(3.33) + (1)(1.33) + (6)(1) = 0$

$A_y = 5.66 \text{ kips} \uparrow$ ◀

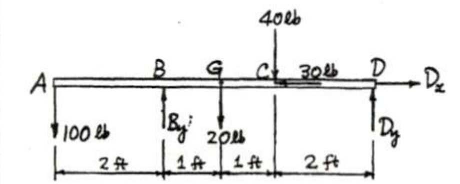
$\sum M_A = B_y(6) - (8)(2.67) - (1)(4.67) - (6)(5) = 0$

$B_y = 9.34 \text{ kips} \uparrow$ ◀

Check:

$\sum F_y = 5.66 + 9.34 - 8 - 1 - 6 = 0$ (Checks)

3-55



$\sum F_x = D_x - 30 = 0$

$D_x = 30 \text{ lb} \rightarrow$ ◀

$\sum M_B = D_y(4) + (100)(2) - (20)(1) - (40)(2) = 0$

$D_y = -25 \text{ lb} \downarrow$ ◀

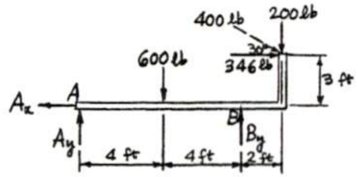
$\sum M_D = -B_y(4) + (100)(6) + (20)(3) + (40)(2) = 0$

$B_y = 185 \text{ lb} \uparrow$ ◀

Check:

$\sum F_y = -100 + 185 - 20 - 40 - 25 = 0$ (Checks)

3-56



$\sum F_x = A_x + 346 = 0$

$A_x = 346 \text{ lb} \leftarrow$ ◀

$\sum M_B = -A_y(8) + (600)(4) - (346)(3) - (200)(2) = 0$

$A_y = 120 \text{ lb} \uparrow$ ◀

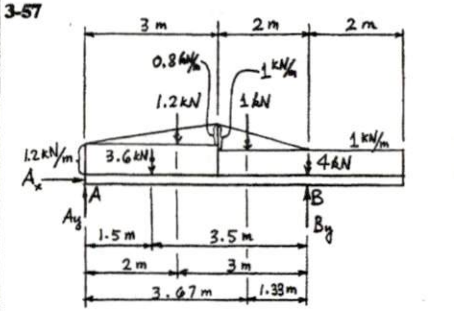
$\sum M_A = B_y(8) - (600)(4) - (346)(3) - (200)(10) = 0$

$B_y = 680 \text{ lb} \uparrow$ ◀

Check:

$\sum F_y = 120 - 600 + 680 - 200 = 0$ (Checks)

3-57



$\sum F_x = A_x + 0 = 0$

$A_x = 0$ ◀

$\sum M_B = -A_y(5) + (3.6)(3.5) + (1.2)(3) + (1.2)(3) + (1)(1.33) = 0$

$A_y = 3.51 \text{ kN} \uparrow$ ◀

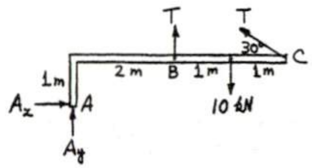
$\sum M_A = B_y(5) - (3.6)(1.5) - (1.2)(2) - (1)(3.67) - (4)(5) = 0$

$B_y = 6.29 \text{ kN} \uparrow$ ◀

Check:

$\sum F_y = 3.51 + 6.29 - 3.6 - 1.2 - 1 - 4 = 0$ (Checks)

3-58



$\sum M_A = T \sin 30°(4) + T \cos 30°(1) + T(2) - (10 \text{ kN})(3) = 0$

$T = 6.17 \text{ kN}$ ◀

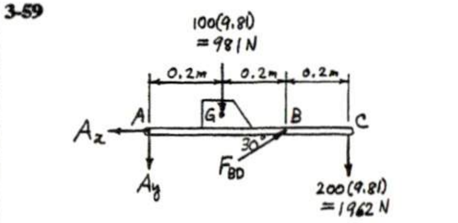
$\sum F_x = A_x - (6.17) \cos 30° = 0$

$A_x = 5.34 \text{ kN} \rightarrow$ ◀

$\sum F_y = A_y + 6.17 + 6.17 \sin 30° - 10 = 0$

$A_y = 0.745 \text{ kN} \uparrow$ ◀

3-59



$\sum M_A = F_{BD} \sin 30°(0.4) - (981)(0.2) - (1962)(0.6) = 0$

$F_{BD} = 6870 \text{ N}$

$\mathbf{R}_D = F_{BD} = 6870 \text{ N} \measuredangle 30°$ ◀

$\sum M_B = A_y(0.4) + (981)(0.2) - (1962)(0.2) = 0$

$A_y = 491 \text{ N} \downarrow$ ◀

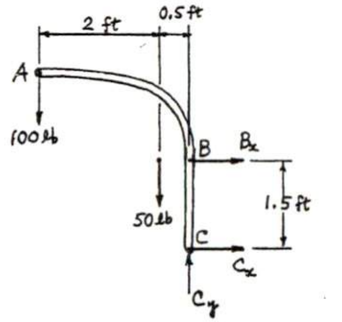
$\sum F_x = -A_x + 6870 \cos 30° = 0$

$A_x = 5950 \text{ N} \leftarrow$ ◀

Check

$\sum F_y = -491 - 981 + 6870 \sin 30° - 1962 = 0$ (Checks)

3-60



$\sum M_C = -B_x(1.5 \text{ ft}) + (100 \text{ lb})(2.5 \text{ ft}) + (50 \text{ lb})(0.5 \text{ ft}) = 0$

$B_x = 183.3 \text{ lb} \leftarrow$ ◀

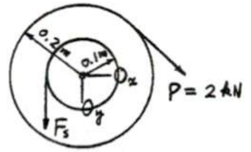
$\sum F_x = -C_x + 183.3 \text{ lb} = 0$

$C_x = 183.3 \text{ lb} \leftarrow$ ◀

$\sum F_y = C_y - 100 \text{ lb} - 50 \text{ lb} = 0$

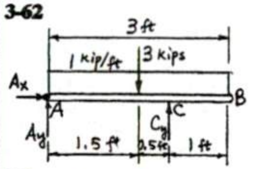
$C_y = 150 \text{ lb} \uparrow$ ◀

3-61



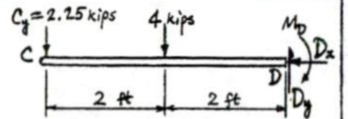
$$\sum M_o = F_s(0.1\text{ m}) - (2\text{ kN})(0.2\text{ m}) = 0$$
$$F_s = 4\text{ kN}$$
$$x = \frac{F_s}{k} = \frac{4\text{ kN}}{20\text{ kN/m}} = 0.2\text{ m}$$
$$= 200\text{ mm} \blacktriangleleft$$

3-62



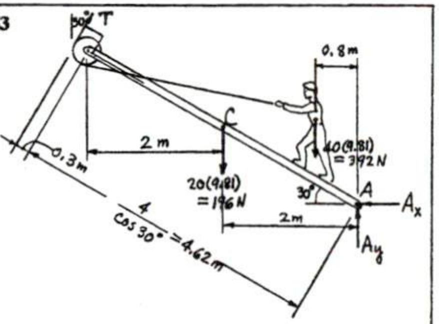
$$\sum F_x = A_x + 0 = 0$$
$$A_x = 0 \blacktriangleleft$$
$$\sum M_C = -A_y(2) + (3)(0.5) = 0$$
$$A_y = 0.75\text{ kips}\uparrow \blacktriangleleft$$
$$\sum M_A = C_y(2) - (3)(1.5) = 0$$
$$C_y = 2.25\text{ kips}\uparrow \blacktriangleleft$$

Check:
$$\sum F_y = 0.75 + 2.25 - 3 = 0 \quad \text{(Checks)}$$



$$\sum F_x = -D_x + 0 = 0$$
$$D_x = 0 \blacktriangleleft$$
$$\sum F_y = D_y - 2.25 - 4 = 0$$
$$D_y = 6.25\uparrow \blacktriangleleft$$
$$\sum M_D = -M_D + (2.25)(4) + (4)(2) = 0$$
$$M_D = 17\text{ kip}\cdot\text{ft} \blacktriangleleft$$

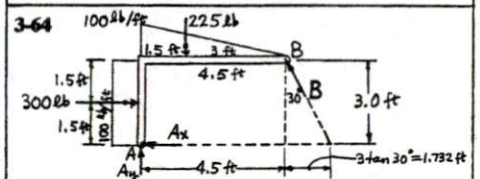
3-63



$$\sum M_A = -T(0.3 + 4.62) + (196)(2) + (392)(0.8) = 0$$
$$T = 143\text{ N} \blacktriangleleft$$
$$\sum F_x = -A_x + 143\sin 30^\circ = 0$$
$$A_x = 71.5\text{ N}\leftarrow \blacktriangleleft$$
$$\sum F_y = A_y + 143\cos 30^\circ - 196 - 392 = 0$$
$$A_y = 464\text{ N}\uparrow \blacktriangleleft$$

Check:
$$\sum M_C = -143\left(0.3 + \frac{4.62}{2}\right) - (392)(2 - 0.8)$$
$$-71.5(2\tan 30^\circ) + 464(2) = 0 \quad \text{(Checks)}$$

3-64



Resolving B into components at D, we write

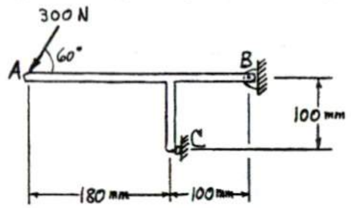
$$\Sigma M_A = B_y(4.5 + 1.732) - (300)(1.5) - (225)(1.5) = 0$$
$$B_y = 126.4\text{ lb}$$
$$B = \frac{B_y}{\cos 30^\circ} = \frac{126.4\text{ lb}}{\cos 30^\circ} = 146\text{ lb} \measuredangle 60^\circ \blacktriangleleft$$
$$\Sigma M_D = -A_y(4.5 + 1.732) - 300(1.5) + 225(4.732) = 0$$
$$A_y = 98.6\text{ lb}\uparrow \blacktriangleleft$$
$$\Sigma F_x = -A_x + 300 - 146\sin 30^\circ = 0$$
$$A_x = 227\text{ lb}\leftarrow \blacktriangleleft$$

Check:
$$\Sigma F_y = -98.6 - 225 + 146\cos 30^\circ = 0 \quad \text{(Checks)}$$

Test Problems for Chapter 3

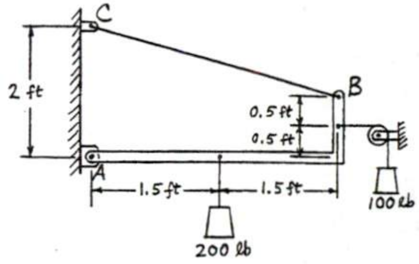
The following problems may either be given to students for them to practice their problem solving skills or be used as test problems. The answers are provided on the next page.

(1) A T-shaped bracket supports a 300-N load as shown. Determine the reactions for the hinge support at *B* and the roller support at *C*. Treat the bracket as a three-force body and solve the problem by using a force-triangle.

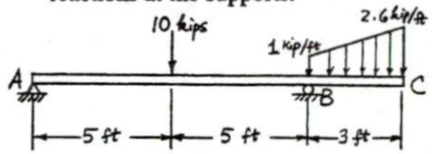


(2) Solve Problem 1 using equilibrium equations.

(3) The L-shaped bracket is supported by a cable *BC* and a hinge at *A* as shown. Determine the tension in cable *BC* and the reaction for the hinge support at *A* due to the two weights applied as shown.

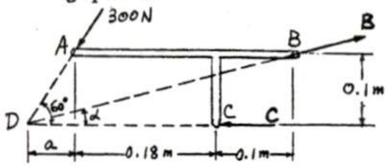


(4) The overhanging beam is subjected to the concentrated load and the distributed load shown. Determine the reactions at the supports.



Solutions to Test Problems for Chapter 3

(1) The given 300 N force and Reaction C intersect at D. The third force, the reaction B, must also pass through point D.

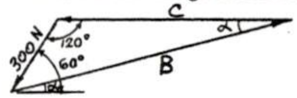


$$a = \frac{AE}{\tan 60^\circ} = \frac{0.1\text{ m}}{\tan 60^\circ} = 0.0577\text{ m}$$

$DF = 0.0577\text{ m} + 0.28\text{ m} = 0.338\text{ m}$

$$\alpha = \tan^{-1}\frac{BF}{DF} = \tan^{-1}\frac{0.1}{0.338} = 16.5^\circ$$

The three forces form a closed triangle as shown:



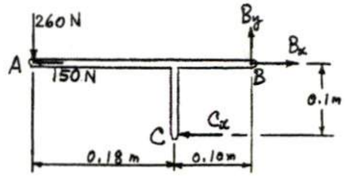
$$\frac{B}{\sin 120^\circ} = \frac{C}{\sin(60^\circ - \alpha)} = \frac{300\text{ N}}{\sin\alpha}$$

$$\frac{B}{\sin 120^\circ} = \frac{C}{\sin 43.5^\circ} = \frac{300\text{ N}}{\sin 16.5^\circ}$$

$B = 915\text{ N} \angle 16.5^\circ$ ◀

$C = 728\text{ N} \leftarrow$ ◀

(2)



$\Sigma M_B = -C_x(0.1\text{ m}) + (260\text{ N})(0.28\text{ m}) = 0$

$C_x = 728\text{ N}$ ◀

$\Sigma F_x = B_x - 150 - 728 = 0$

$B_x = 878\text{ N}$

$\Sigma F_y = B_y - 260\text{ N} = 0$

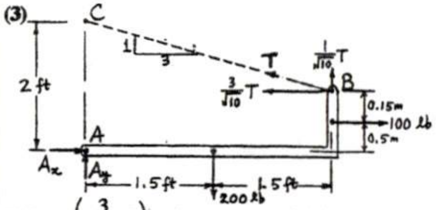
$B_y = 260\text{ N}$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{878^2 + 260^2} = 916\text{ N}$$

$$\theta = \tan^{-1}\frac{B_y}{B_x} = \tan^{-1}\frac{260}{878} = 16.5^\circ$$

$B = 916\text{ N} \angle 16.5^\circ$ ◀

(3)



$$\Sigma M_A = \left(\frac{3}{\sqrt{10}}T\right)(2\text{ ft}) - (100\text{ lb})(0.5\text{ ft})$$

(at C) $\qquad - (200\text{ lb})(1.5\text{ ft}) = 0$

$T = 184.5\text{ lb}$ ◀

$\Sigma M_C = A_x(2\text{ ft}) + (100\text{ lb})(1.5\text{ ft})$

$\qquad - (200\text{ lb})(1.5\text{ ft}) = 0$

$A_x = 75\text{ lb}$ ◀

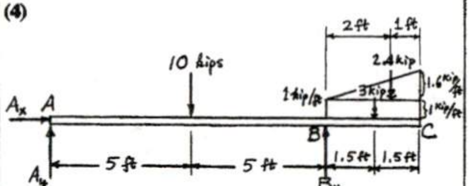
$$\Sigma F_y = A_y + \frac{1}{\sqrt{10}}(184.5\text{ lb}) - 200\text{ lb} = 0$$

$A_y = 141.7\text{ lb}$ ◀

Check:

$$\Sigma F_x = 75 + 100 - \frac{3}{\sqrt{10}}(184.5\text{ lb}) = -0.03 \quad \text{(Checks)}$$

(4)



$\Sigma F_x = A_x + 0 = 0$

$A_x = 0$ ◀

$\Sigma M_B = -A_y(10\text{ ft}) + (10\text{ kips})(5\text{ ft})$

$\qquad -(3\text{ kips})(1.5\text{ ft}) - (2.4\text{ kips})(2\text{ ft}) = 0$

$A_y = 4.07\text{ kips}$ ◀

$\Sigma M_A = B_y(10) - (10)(5\text{ ft})$

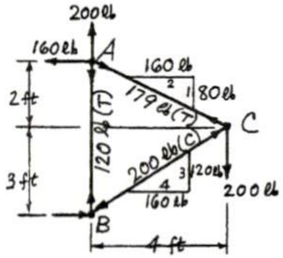
$\qquad -(3\text{ kips})(11.5\text{ ft}) - (2.4\text{ kips})(12\text{ ft}) = 0$

$B_y = 11.33\text{ kips}$ ◀

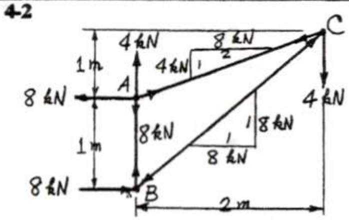
Check:

$\Sigma F_y = 4.07 + 11.33 - 10 - 3 - 2.4 = 0 \quad$ (Checks)

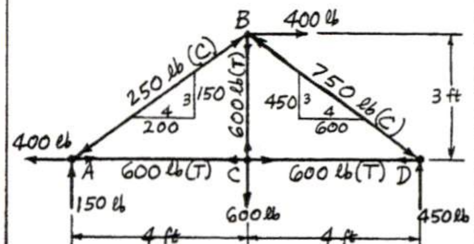
4-1



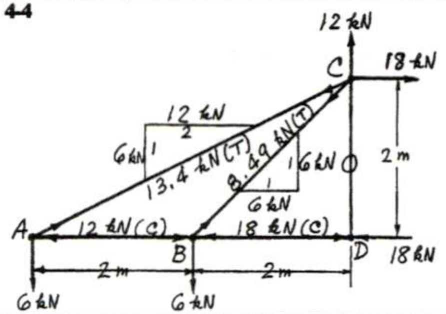
4-2



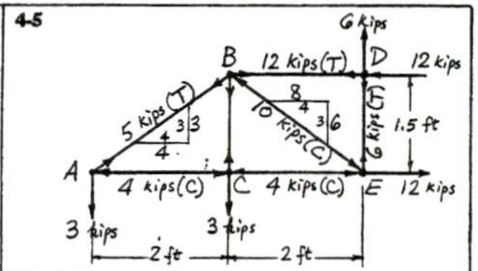
4-3



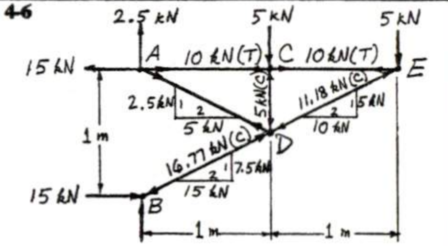
4-4



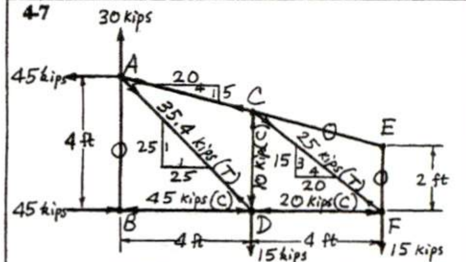
4-5



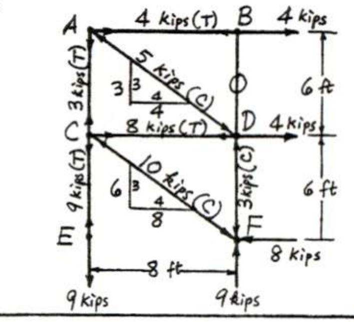
4-6



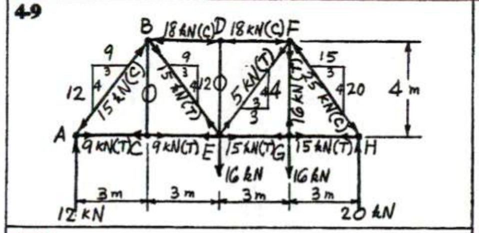
4-7



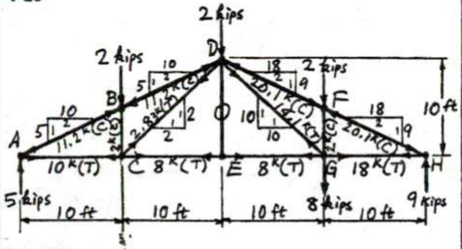
4-8



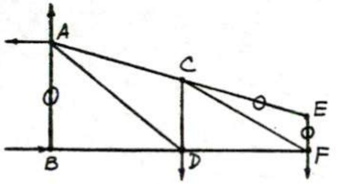
4-9



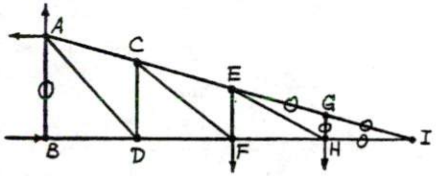
4-10



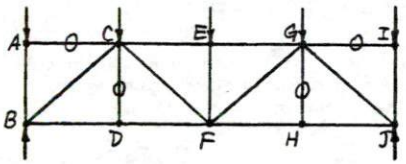
4-11



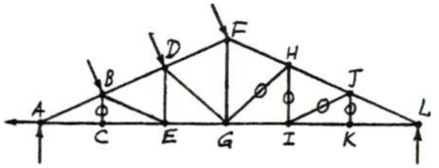
4-12



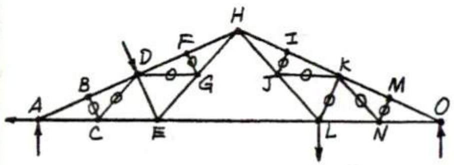
4-13



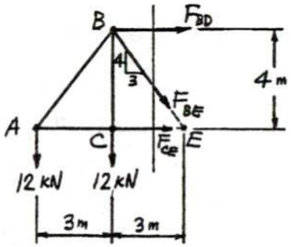
4-14



4-15



4-16



$\sum M_E = -F_{BD}(4) + 12(6) + 12(3) = 0$

$F_{BD} = +27$ kN (T) ◀

$\sum F_y = -\frac{4}{5}F_{BE} - 12 - 12 = 0$

$F_{BE} = -30$ kN (C) ◀

$\sum M_B = F_{CE}(4) + 12(3) = 0$

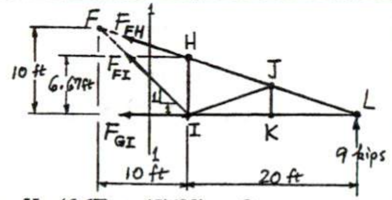
$F_{CE} = -9$ kN (C) ◀

Check:

$\sum F_x = F_{BD} + \frac{3}{5}F_{BE} + F_{CE}$

$= 27 + \frac{3}{5}(-30) + (-9) = 0$ (Checks)

4-17



$\sum M_I = H_{FH}(6.67) + (9)(20) = 0$
(at H)

$H_{FH} = -27$ kips

$V_{FH} = \frac{1}{3}H_{FH} = -9$ kips

$F_{FH} = \sqrt{(27)^2 + (9)^2} = 28.5$ kips (C) ◀

$\sum M_L = -V_{FI}(20) + 0 = 0$
(at I)

$V_{FI} = 0, \quad F_{FI} = 0$ ◀

$\sum M_F = -F_{GI}(10) + (9)(30) = 0$

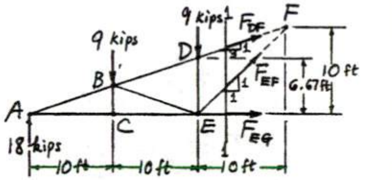
$F_{GI} = +27$ kips (T) ◀

Check:

$\sum F_x = -H_{FH} - H_{FI} - F_{GI} = -(-27) - 0 - 27 = 0$

$\sum F_y = V_{FH} - V_{FI} + 9 = -9 + 0 + 9 = 0$

4-18



$\sum M_E = -H_{DF}(6.67) - (18)(20) + (9)(10) = 0$
(at D)

$H_{DF} = -40.5$ kips

$V_{DF} = \frac{1}{3}H_{DF} = -13.5$ kips

$F_{DF} = \sqrt{(40.5)^2 + (13.5)^2} = 42.7$ kips (C) ◀

$\sum M_A = V_{EF}(20) - (9)(10) - (9)(20) = 0$
(at E)

$V_{EF} = +13.5$ kips

$H_{EF} = V_{EF} = 13.5$ kips

$F_{EF} = \sqrt{(13.5)^2 + (13.5)^2} = 19.1$ kips (T) ◀

$\sum M_F = F_{EG}(10) - (18)(30) + (9)(20) + (9)(10) = 0$

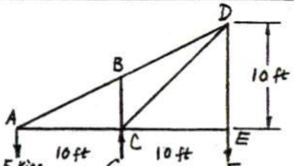
$F_{EG} = +27$ kips (T) ◀

Check:

$\sum F_x = H_{DF} + H_{EF} + F_{EG} = -40.5 + 13.5 + 27 = 0$

$\sum F_y = V_{DF} - V_{EF} - 9 - 9 + 18$
$= -13.5 + 13.5 - 9 - 9 + 18 = 0$

4-19

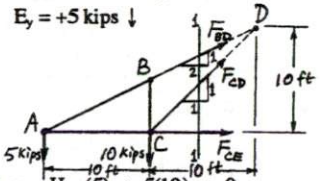


$\sum M_E = -C_y(10) + 5(20) = 0$

$C_y = +10$ kips ↑

$\sum F_y = -5 + 10 - E_y = 0$

$E_y = +5$ kips ↓



$\sum M_C = -H_{BD}(5) + 5(10) = 0$
(at B)

$H_{BD} = +10$ kips, $V_{BD} = \frac{1}{2}(10) = 5$ kips

$F_{BD} = \sqrt{(10)^2 + (5)^2} = 11.2$ kips (T) ◀

$\sum M_A = V_{CD}(10) + 10(10) = 0$
(at C)

$V_{CD} = -10$ kips, $H_{CD} = -10$ kips

$F_{CD} = \sqrt{(10)^2 + (10)^2} = 14.1$ kips (C) ◀

$\sum M_D = F_{CE}(10) + 5(20) - 10(10) = 0$

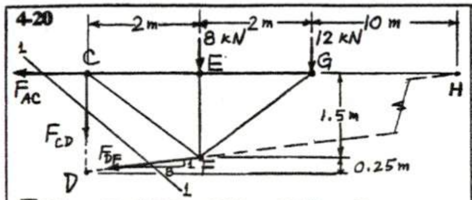
$F_{CE} = 0$ ◀

Check:

$\sum F_x = H_{BD} + H_{CD} + F_{CE} = 10 - 10 + 0 = 0$

$\sum F_y = V_{BD} + V_{CD} - 5 + 10 = 5 - 10 - 5 + 10 = 0$

4-20



$\sum M_D = F_{AC}(1.75) - 8(2) - 12(4) = 0$

$F_{AC} = +36.6$ kN (T) ◄

$\sum M_H = F_{CD}(14) + 8(12) + 12(10) = 0$

$F_{CD} = -15.43$ kN (C) ◄

$\sum M_C = -H_{BF}(1.75) - 8(2) - 12(4) = 0$
(at D)

$H_{DF} = -36.6$ kN, $V_{DF} = \frac{1}{8}(-36.6) = -4.57$ kN

$F_{DF} = \sqrt{(36.6)^2 + (4.57)^2} = 36.9$ kN (C) ◄

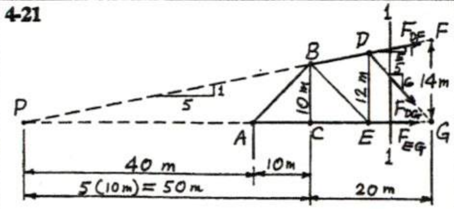
Check:

$\sum F_x = -F_{AC} - H_{DF} = -(+36.6) - (-36.6) = 0$

$\sum F_y = -F_{CD} - V_{DF} - 8 - 12 = 0$

$= -(-15.43) - (-4.57) - 8 - 12 = 0$

4-21



$\sum M_G = -H_{DF}(14) - (35)(30) = 0$
(at F)

$H_{DF} = -75$ kN, $V_{DF} = \frac{1}{5}H_{DF} = -15$ kN

$F_{DF} = \sqrt{(75)^2 + (15)^2} = 76.5$ kN (C) ◄

$\sum M_P = -V_{DG}(70) + (35)(40) = 0$
(at G)

$V_{DG} = +20$ kN,

$H_{DG} = \frac{5}{6}V_{DG} = +16.7$ kN

$F_{DG} = \sqrt{(20)^2 + (16.7)^2} = 26.0$ kN (T) ◄

$\sum M_D = F_{FG}(12) - (35)(20) = 0$

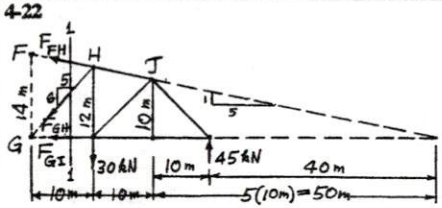
$F_{FG} = +58.3$ kN (T) ◄

Check:

$\sum F_x = H_{DF} + H_{DG} + F_{EG} = -75 + 16.7 + 58.3 = 0$

$\sum F_y = V_{DF} - V_{DG} + 35 = -15 - 20 + 35 = 0$

4-22



$\sum M_G = H_{FH}(14) - (30)(10) + (45)(30) = 0$
(at F)

$H_{FH} = -75$ kN, $V_{FH} = \frac{1}{5}H_{FH} = -15$ kN

$F_{FH} = \sqrt{(75)^2 + (15)^2} = 76.5$ kN (C) ◄

$\sum M_Q = V_{GH}(70) + (30)(60) - (45)(40) = 0$

$V_{GH} = 0$

$F_{GH} = 0$ ◄

$\sum M_H = -F_{GI}(12) + (45)(20) = 0$

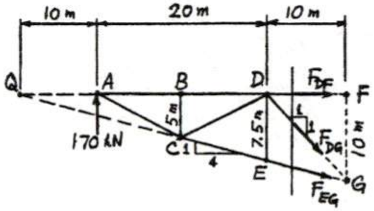
$F_{GI} = +75$ kN (T) ◄

Check:

$\sum F_x = -H_{FH} - H_{GH} - F_{GI} = -(-75) - 0 - 75 = 0$

$\sum F_y = V_{FH} - V_{GH} - 30 + 45 = -15 - 0 - 30 + 45 = 0$

4-23



$\sum M_G = -F_{DF}(10) - (170)(30) = 0$

$F_{DF} = -510$ kN (C) ◄

(Cont'd)

4-23 (Cont)

$\sum M_Q = -V_{DG}(30) + (170)(10) = 0$
(at D)

$V_{DG} = +56.7$ kN, $H_{DG} = V_{DG} = +56.7$ kN

$F_{DG} = \sqrt{(56.7)^2 + (56.7)^2} = 80.2$ kN (T) ◄

$\sum M_D = H_{EG}(7.5) - (170)(20) = 0$
(at E)

$H_{EG} = +453.3$ kN,

$V_{EG} = \frac{1}{4}H_{EG} = +113.3$ kN

$F_{EG} = \sqrt{(453.3)^2 + (113.3)^2} = 467$ kN (T) ◄

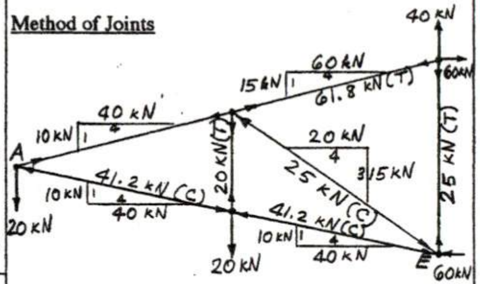
Check:

$\sum F_x = F_{DF} + H_{DG} + H_{EG} = -510 + 56.7 + 453.3 = 0$

$\sum F_y = -V_{DG} - V_{EG} + 170 = -56.7 - 113.3 + 170 = 0$

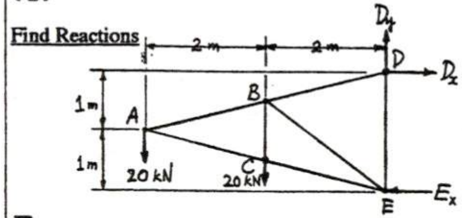
With the reactions and the horizontal component H_{CE} found, the method of joints can be used to determine the forces on all members. The joints can be analyzed in the following sequence: D, E, C, and B. Equilibrium conditions of joint A will provide useful checks.

Method of Joints



4-24

Find Reactions



$\sum M_D = -E_x(2) + 20(4) + 20(2) = 0$

$E_x = 60$ kN ←

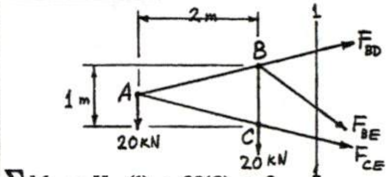
$\sum F_x = D_x - E_x = D_x - 60 = 0$

$D_x = 60$ kN →

$\sum F_y = D_y - 20 - 20 = 0$

$D_y = 40$ kN ↑

Method of Section

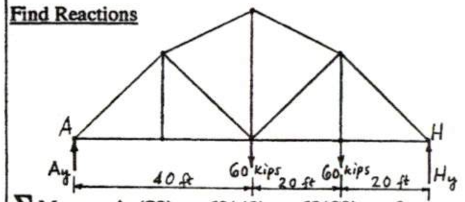


$\sum M_B = H_{CE}(1) + 20(2) = 0$
(at C)

$H_{CE} = -40$ kN (C)

4-25

Find Reactions



$\sum M_H = -A_y(80) + 60(40) + 60(20) = 0$

$A_y = 45$ kips

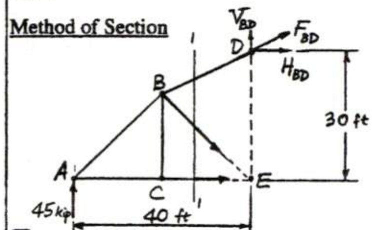
$\sum M_A = H_y(80) - 60(40) - 60(60) = 0$

$H_y = 75$ kips

Check:

$\sum F_y = 45 + 75 - 60 - 60 = 0$ (Checks)

Method of Section



$\sum M_E = -H_{BD}(30) - 45(40) = 0$

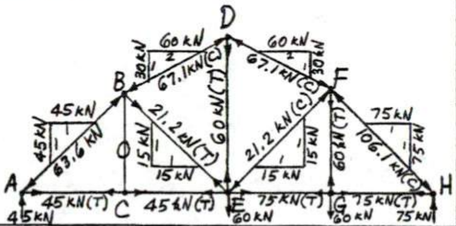
$H_{BD} = -60$ kips

(Cont'd)

4-25 (Cont)

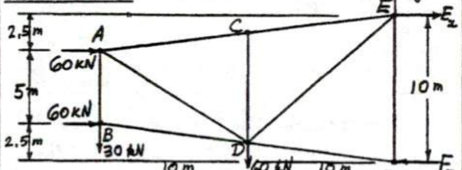
With the reactions and the horizontal component H_{BD} found, the method of joints can be used to determine the forces on all members. The joints can be analyzed in the following sequence: A, C, B, D, E, G, and H. Equilibrium conditions of joint F will provide useful checks.

Method of Joints



4-26

Find Reactions



$\sum M_B = -F_x(10) + 30(20) + 60(10) + 60(2.5) + 60(7.5) = 0$

$F_x = 180$ kN ←

$\sum M_F = -E_x(10) + 30(20) + 60(10) - 60(7.5) - 60(2.5) = 0$

$E_x = 60$ kN →

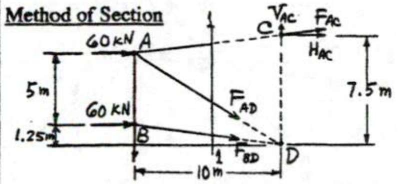
$\sum F_y = E_y - 30 - 60 = 0$

$E_y = 90$ kN ↑

Check:

$\sum F_x = 60 + 60 + 60 - 180 = 0$ (Checks)

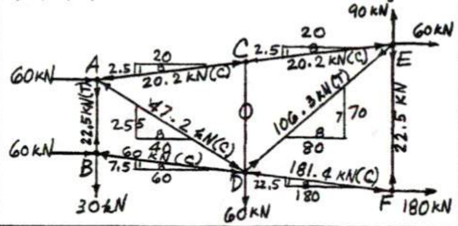
Method of Section



$\sum M_D = -H_{AC}(7.5) + 30(10) - 60(6.25) - 60(1.25) = 0$ (at C)

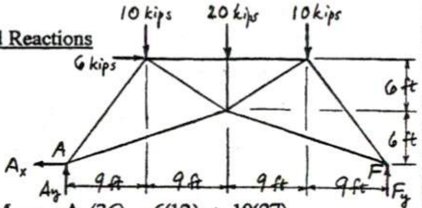
$H_{AC} = -20$ kN (C)

Method of Joints: Joints are analyzed in the sequence: B, A, C, E, and F. The checks are obtained at joint D.



4-27

Find Reactions



$\sum M_F = -A_y(36) - 6(12) + 10(27) + 20(18) + 10(9) = 0$

$A_y = 18$ kips ↑

$\sum M_A = F_y(36) - 6(12) - 10(9) - 20(18) - 10(27) = 0$

$F_y = 22$ kips ↑

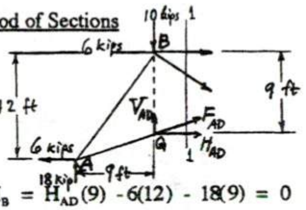
$\sum F_x = -A_x + 6 = 0$

$A_x = +6$ kips ←

Check:

$\sum F_y = 18 + 22 - 10 - 20 - 10 = 0$ (Checks)

Method of Sections

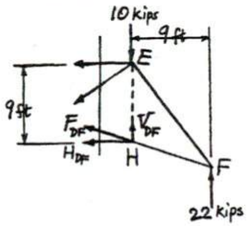


$\sum M_B = H_{AD}(9) - 6(12) - 18(9) = 0$ (at G)

$H_{AD} = +26$ kips (T)

(Cont'd)

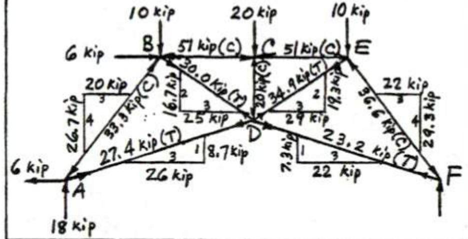
4-27 (Cont)



$\sum M_E = -H_{DF}(9) + 22(9) = 0$ (at H)

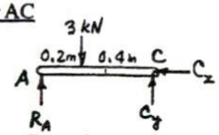
$H_{DF} = +22$ kips (T)

Method of Joints: Joints are analyzed in the sequence: A, B, C, F, and E. The checks are obtained at joint D.



4-28

Member AC



$\sum F_x = -C_x + 0 = 0$

$C_x = 0$

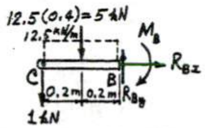
$\sum M_C = -R_A(0.6) + (3)(0.4) = 0$

$R_A = +2$ kN ↑ ◀

$\sum M_A = C_y(0.6) - (3)(0.2) = 0$

$C_y = +1$ kN

Member CB



$\sum M_x = R_{Bx} + 0 = 0$

$R_{Bx} = 0$ ◀

$\sum M_y = R_{Bx} - 1 - 5 = 0$

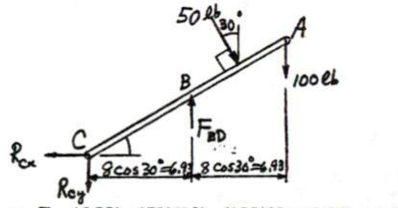
$R_{Bx} = +6$ kN ↑ ◀

$\sum M_B = -M_B + (1)(0.4) + (5)(0.2) = 0$

$M_B = +1.4$ kN • m ↻ ◀

4-29

Member ABC: Note that BD is a two-force member.



$\sum M_C = F_{BD}(6.93) - (50)(12) - (100)(2 \times 6.93) = 0$

$F_{BD} = 287$ lb (C) ◀

$\sum F_x = -R_{Cx} + 50 \sin 30° = 0$

$R_{Cx} = +25$ lb ← ◀

$\sum M_B = R_{Cy}(6.93) - (25)(8 \sin 30°) - (50)(4) - (100)(6.93) = 0$

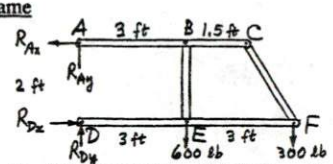
$R_{Cy} = 143.3$ lb ↓ ◀

Check:

$\sum F_y = -143.3 + 287 - 50 \cos 30° - 100 = 0$ (Checks)

4-30

Entire Frame



$\sum M_D = R_{Ax}(2) - (600)(3) - (300)(6) = 0$

$R_{Ax} = +1800$ lb ← ◀

(Cont'd)

4-30 (Cont)

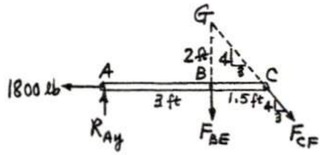
$\sum F_x = R_{Dx} - 1800 = 0$

$R_{Dx} = 1800\text{ lb} \rightarrow$

$\sum F_y = R_{Ay} + R_{Dy} - 600 - 300 = 0$

$R_{Ay} + R_{Dy} = 900\text{ lb}$ (a)

Member ABC: Note that BE and CF are two-force members.



$\sum F_x = \frac{3}{5}F_{CF} - 1800 = 0$

$F_{CF} = +3000\text{ lb (T)}$

$\sum M_G = -R_{Ay}(3) + (1800)(2) = 0$

$R_{Ay} = -1200\text{ lb} \downarrow$

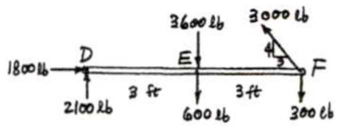
From Eq. (a):

$R_{Dy} = 900 - (-1200) = +2100\text{ lb} \uparrow$

$\sum F_y = -F_{BE} - 1200 - \frac{4}{5}(3000) = 0$

$F_{BE} = -3600\text{ lb (C)}$

Member DEF (For checking)

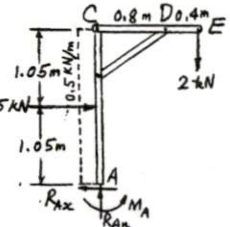


$\sum F_x = 1800 - \frac{3}{5}(3000) = 0$ (Checks)

$\sum F_y = 2100 - 3600 - 600 - \frac{4}{5}(3000) - 300 = 0$ (Checks)

4-31

Entire Frame



$\sum F_x = -R_{Ax} + 1.05 = 0$

$R_{Ax} = 1.05\text{ kN} \rightarrow$

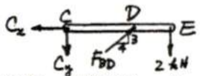
$\sum F_y = R_{Ay} - 2 = 0$

$R_{Ay} = 2\text{ kN} \uparrow$

$\sum M_A = M_A - (2)(1.2) - (1.05)(1.05) = 0$

$M_A = 3.50\text{ kN} \cdot \text{m}$ ↺

Member CDE Note that BD is a two-force member.



$\sum M_C = \left(\frac{3}{5}F_{BD}\right)(0.8) - (2)(1.2) = 0$

$F_{BD} = +5\text{ kN (C)}$

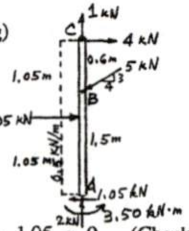
$\sum M_D = C_y(0.8) - (2)(0.4) = 0$

$C_y = +1\text{ kN}$

$\sum F_x = -C_x + \frac{4}{5}(5) = 0$

$C_x = +4\text{ kN}$

Member ABC (For checking)

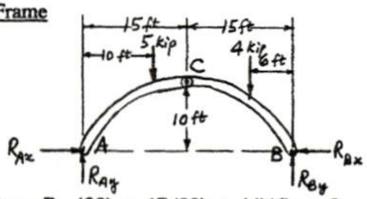


$\sum F_x = 4 - \frac{4}{5}(5) + 1.05 - 1.05 = 0$ (Checks)

$\sum F_y = 1 - \frac{3}{5}(5) + 2 = 0$ (Checks)

4-32

Entire Frame



$\sum M_B = -R_{Ay}(30) + (5)(20) + (4)(6) = 0$

$R_{Ay} = 4.13\text{ kips} \uparrow$

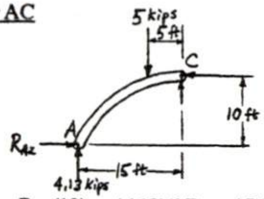
$\sum F_y = R_{By} + 4.13 - 5 - 4 = 0$

$R_{By} = 4.87\text{ kips} \uparrow$

$\sum F_x = R_{Ax} - R_{Bx} = 0$

$R_{Bx} = R_{Ax}$ (a)

Member AC



$\sum M_C = R_{Ax}(10) - (4.13)(15) + (5)(5) = 0$

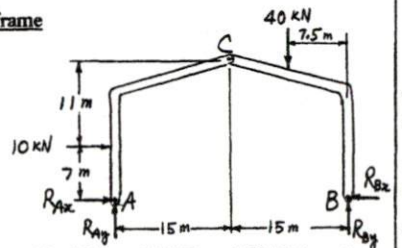
$R_{Ax} = 3.70\text{ kips} \rightarrow$

From Eq. (a):

$R_{Bx} = 3.70\text{ kips} \leftarrow$

4-33

Entire Frame



$\sum M_B = -R_{Ay}(30) - (10)(7) + (40)(7.5) = 0$

$R_{Ay} = 7.67\text{ kN} \uparrow$

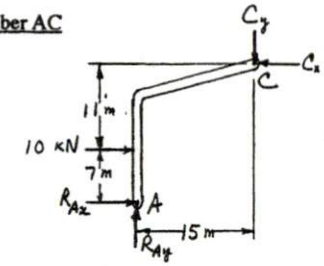
$\sum F_y = R_{By} + 7.67 - 40 = 0$

$R_{By} = 32.33\text{ kN} \uparrow$

$\sum F_x = R_{Ax} - R_{Bx} + 10 = 0$

$R_{Bx} = R_{Ax} + 10$ (a)

Member AC



$\sum M_C = R_{Ax}(18) - (7.67)(15) + (10)(11) = 0$

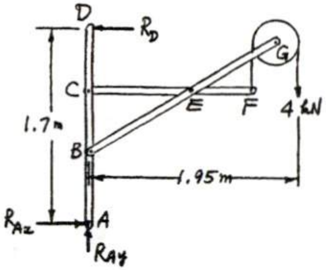
$R_{Ax} = 0.28\text{ kN} \rightarrow$

From Eq. (a):

$R_{Bx} = 0.28 + 10 = 10.28\text{ kN} \leftarrow$

4-34

Entire Frame



$\sum M_A = R_D(1.7) - (4)(1.95) = 0$

$R_D = 4.59\text{ kN} \cdot \text{m} \leftarrow$

$\sum F_x = R_{Ax} - 4.59 = 0$

$R_{Ax} = +4.59\text{ kN} \rightarrow$

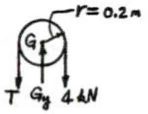
$\sum F_y = R_{Ay} - 4 = 0$

$R_{Ay} = +4\text{ kN} \uparrow$

(Cont'd)

4-34 (Cont)

Pulley



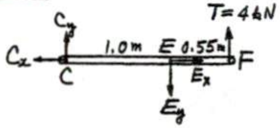
$\sum M_G = T(0.2) - (4)(0.2) = 0$

$T = +4$ kN ◀

$\sum F_y = G_y - 4 - 4 = 0$

$G_y = +8$ kN ◀

Member CEF



$\sum M_E = -C_y(1.0) + (4)(0.55) = 0$

$C_y = +2.2$ kN ◀

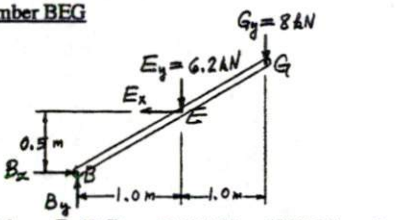
$\sum F_y = -E_y + 2.2 + 4 = 0$

$E_y = +6.2$ kN ◀

$\sum F_x = E_x - C_x = 0$

$E_x = C_x$ (a)

Member BEG



$\sum M_B = E_x(0.5) + (6.2)(1.0) - (8)(1.75) = 0$

$E_x = +15.6$ kN ◀

From Eq.(a):

$C_x = +15.6$ kN ◀

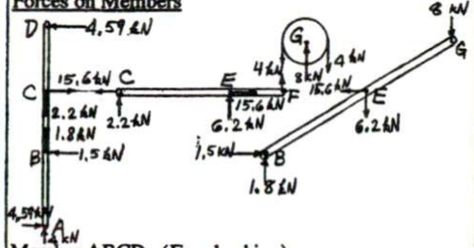
$\sum F_x = B_x - 15.6 = 0$

$B_x = +15.6$ kN ◀

$\sum F_y = B_y + 6.2 - 8 = 0$

$B_y = +1.8$ kN ◀

Forces on Members



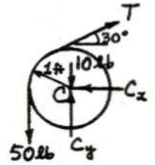
Member ABCD (For checking)

$\sum F_x = -4.59 + 15.6 - 15.6 + 4.59 = 0$ (Checks)

$\sum F_y = -2.2 - 1.8 + 4 = 0$ (Checks)

4-35

Pulley



$\sum M_C = -T(1) + 50(1) = 0$

$T = 50$ lb

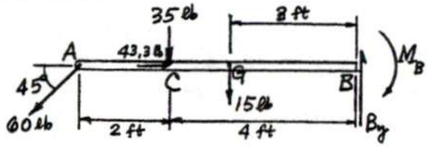
$\sum F_x = -C_x - 50 \cos 30° = 0$

$C_x = +43.3$ lb → ◀

$\sum F_y = C_y - 50 - 10 + 50 \sin 30° = 0$

$C_y = +35$ lb ↑ ◀

Member ACB



$\sum F_y = B_y - 60 \sin 45° - 35 - 15 = 0$

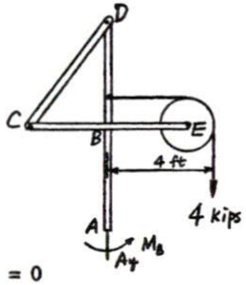
$B_y = 92.4$ lb ↑ ◀

$\sum M_B = -M_B + 60 \cos 45°(6) + 35(4) + 15(3) = 0$

$M_B = 440$ lb-ft ↻ ◀

4-36

Entire Frame



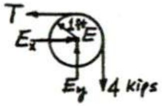
$\sum F_y = A_y - 4 = 0$

$A_y = 4$ kips ↑ ◀

$\sum M_A = M_A - 4(4) = 0$

$M_A = +16$ kip • ft ◀

Pulley



$\sum M_E = T(1) - 4(1) = 0$

$T = +4$ kips ◀

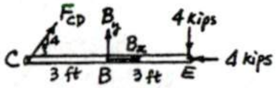
$\sum F_x = E_x - 4 = 0$

$E_x = +4$ kips ◀

$\sum F_y = E_y - 4 = 0$

$E_y = +4$ kips ◀

Member CDE Note that CD is a two-force member.



$\sum M_B = -\frac{4}{5}F_{CD}(3) - (4)(3) = 0$

$F_{CD} = -5$ kips (C) ◀

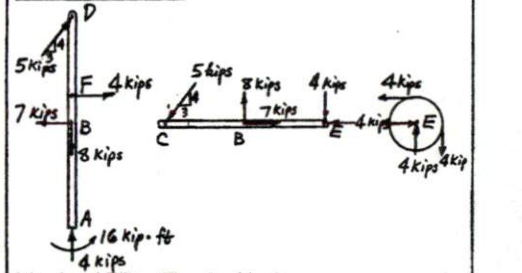
$\sum F_x = B_x + \frac{3}{5}(-5) - 4 = 0$

$B_x = +7$ kips ◀

$\sum F_y = B_y + \frac{4}{5}(-5) - 4 = 0$

$B_y = +8$ kips ◀

Forces on Members



Member ABD (For checking)

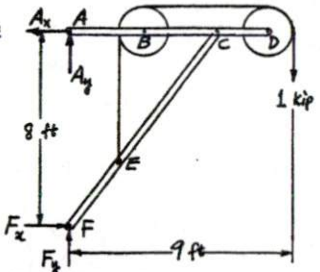
$\sum F_x = \frac{3}{5}(5) + 4 - 7 = 0$ (Checks)

$\sum F_y = \frac{4}{5}(5) - 8 + 4 = 0$ (Checks)

$\sum M_A = -\frac{3}{5}(5)(8) - 4(5) + 7(4) + 16 = 0$ (Checks)

4-37

Entire Frame



$\sum M_F = A_x(8) - (1)(9) = 0$

$A_x = \frac{9}{8}$ kips ← ◀

$\sum F_x = F_x - \frac{9}{8} = 0$

$F_x = \frac{9}{8}$ kips → ◀

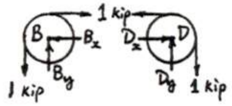
(Cont'd)

4-37 (Cont)

$\sum F_y = A_y + F_y - 1 = 0$

$A_y + F_y = 1$ (a)

Pulleys



By inspection:

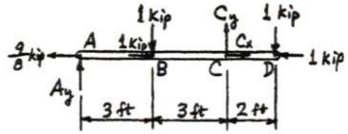
$B_x = 1$ kip ◀

$B_y = 1$ kip ◀

$D_x = 1$ kip ◀

$D_y = 1$ kip ◀

Member ABCD



$\sum M_C = -A_y(6) + (1)(3) - 1(2) = 0$

$A_y = +\frac{1}{6}$ kip ↑ ◀

From (a):

$F_y = 1 - \frac{1}{6} = \frac{5}{6}$ kip ↑ ◀

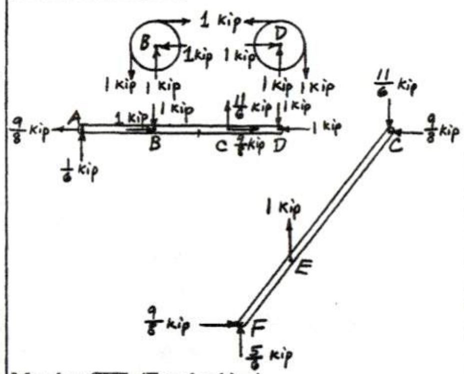
$\sum F_x = -\frac{9}{8} + 1 + C_x - 1 = 0$

$C_x = +\frac{9}{8}$ kips ◀

$\sum F_y = C_y + \frac{1}{6} - 1 - 1 = 0$

$C_y = +\frac{11}{6}$ kips ◀

Forces on Members



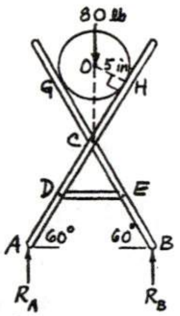
Member CEF (For checking)

$\sum F_x = \frac{9}{8} - \frac{9}{8} = 0$ (Checks)

$\sum F_y = \frac{5}{6} + 1 - \frac{11}{6} = 0$ (Checks)

4-38

Entire Frame



$CH = \frac{OH}{\tan 30^\circ} = \frac{5}{\tan 30^\circ} = 8.66$ in.

Due to Symmetry, $R_A = R_B$

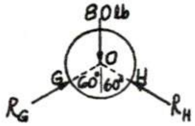
$\sum F_y = R_A + R_B - 80 = 0$

$R_A = R_B = 40$ lb

(Cont'd)

4-38 (Cont)

Log

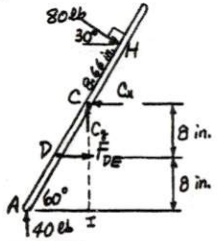


Due to Symmetry, $R_G = R_H$

$\sum F_y = 2R_G \cos 60^\circ - 80 = 0$

$R_G = R_H = \frac{80}{2 \cos 60^\circ} = 80$ lb

Member ADCH Note that DE is a two-force member.



$AI = \frac{16}{\tan 60^\circ} = 9.24$ in.

$\sum M_C = F_{DE}(8) - (80)(8.66) - (40)(9.24) = 0$

$F_{DE} = 132.8$ lb (T) ◀

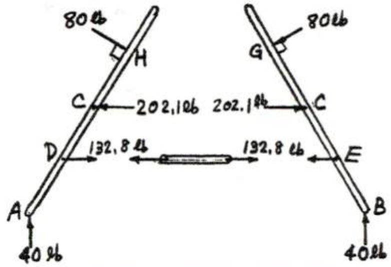
$\sum F_x = -C_x + 80 \cos 30^\circ + 132.8 = 0$

$C_x = 202.1$ lb ◀

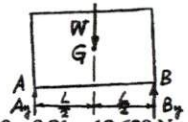
$\sum F_y = C_y - 80 \sin 30^\circ + 40 = 0$

$C_y = 0$ ◀

Forces on Members



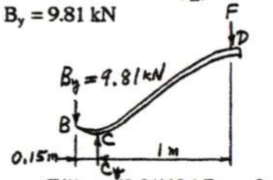
4-39



$W = mg = 2000 \times 9.81 = 19\,620\text{ N} = 19.62$ kN

$\sum M_A = B_y(L) - (19.62)\left(\frac{L}{2}\right) = 0$

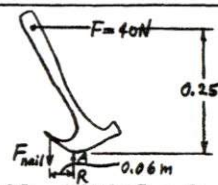
$B_y = 9.81$ kN



$\sum M_C = -F(1) + (9.81)(0.15) = 0$

$F = 1.47$ kN ◀

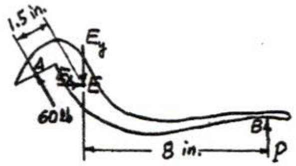
4-40



$\sum M_A = F_{nail}(0.06) - (40)(0.25) = 0$

$F_{nail} = 167$ N ◀

4-41



(a) $\sum M_E = P(8) - (60)(1.5) = 0$

$P = 11.25$ lb ◀

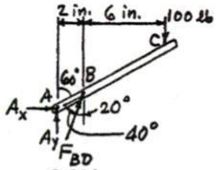
(b) $\sum F_x = E_x - 60 \sin 30^\circ = 0$

$E_x = 30$ lb ◀

$\sum F_y = -E_y + 60 \cos 30^\circ + 11.25 = 0$

$E_y = 63.2$ lb ◀

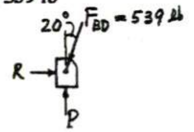
4-42



$$AB = \frac{2}{\sin 60^\circ} = 2.31 \text{ in.}$$

$$\sum M_A = F_{BD} \sin 40^\circ(2.31) - (100)(8) = 0$$

$$F_{BD} = 539 \text{ lb}$$

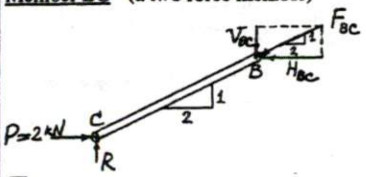


$$\sum F_y = P - 539 \cos 20^\circ = 0$$

$$P = 506 \text{ lb}$$ ◀

4-43

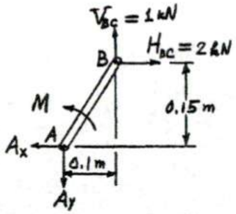
Member BC (a two force member)



$$\sum F_x = -H_{BC} + 2 = 0$$

$$H_{BC} = 2 \text{ kN}, \quad V_{BC} = \frac{1}{2}H_{BC} = 1 \text{ kN}$$

Member AB

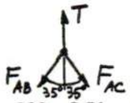


$$\sum M_A = M - (2)(0.15) + (1)(0.10) = 0$$

$$M = 0.2 \text{ kN} \cdot \text{m} \circlearrowleft$$ ◀

4-44

Joint A



$$T = W = mg = 200 \times 9.81 = 1962 \text{ N}$$

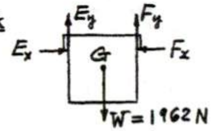
$$\sum F_x = F_{AC} \sin 35^\circ - F_{AB} \sin 35^\circ = 0$$

$$F_{AB} = F_{AC}$$

$$\sum F_y = -F_{AB} \cos 35^\circ - F_{AC} \cos 35^\circ + 1962 = 0$$

$$F_{AB} = F_{AC} = 1198 \text{ N}$$ ◀

The Block

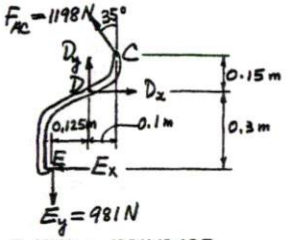


Due to Symmetry, $E_y = F_y$

$$\sum F_y = E_y + F_y - 1962 = 0$$

$$E_y = F_y = 981 \text{ N}$$ ◀

Tong CDE



$$\sum M_D = -E_x(0.3) + (981)(0.125)$$
$$+ (1198 \sin 35^\circ)(0.15) + (1198 \cos 35^\circ)(0.1) = 0$$

$$E_x = 1079 \text{ N}$$ ◀

$$\sum F_x = D_x - 1079 - 1198 \sin 35^\circ = 0$$

$$D_x = 1766 \text{ N}$$ ◀

$$\sum F_y = D_y + 1198 \cos 35^\circ - 981 = 0$$

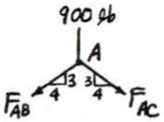
$$D_y = 0$$ ◀

Check:

$$\sum M_C = 1766(0.15) + (981)(0.225)$$
$$- (1079)(0.45) = 0$$ (Checks)

4-45

Joint A



$$\sum F_x = \frac{4}{5}F_{AC} - \frac{4}{5}F_{AB} = 0$$

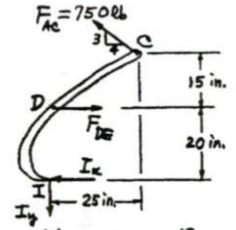
$$F_{AB} = F_{AC}$$

$$\sum F_y = -\frac{3}{5}F_{AB} - \frac{3}{5}F_{AC} + 900 = 0$$

$$-\frac{3}{5}F_{AC} - \frac{3}{5}F_{AC} + 900 = 0$$

$$F_{AC} = 750 \text{ lb}$$

Tong CDI



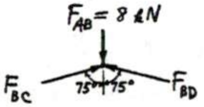
$$\sum M_I = -F_{DE}(20) + \left(\frac{4}{5}x750\right)(35) + \left(\frac{3}{5}x750\right)(25) = 0$$

$$F_{DE} = 1613 \text{ lb (T)}$$ ◀

4-46

Note that AB, BC, and BD are all two-force members.

Joint B



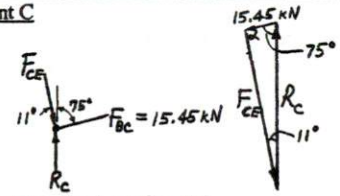
$$\sum F_x = F_{BC} \sin 75^\circ - F_{BD} \sin 75^\circ = 0$$

$$F_{BC} = F_{BD}$$

$$\sum F_y = F_{BC} \cos 75^\circ + F_{BD} \cos 75^\circ - 8 = 0$$

$$F_{BC} = F_{BD} = 15.45 \text{ kN}$$

Joint C



$$\alpha = 180^\circ - (75^\circ + 11^\circ) = 94^\circ$$

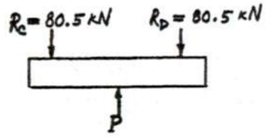
$$\frac{R_C}{\sin 94^\circ} = \frac{15.45}{\sin 11^\circ}$$

$$R_C = 80.5 \text{ kN}$$

Due to Symmetry:

$$R_D = R_C = 80.5 \text{ kN}$$

Press Head



$$\sum F_y = P - 80.5 - 80.5 = 0$$

$$P = 161 \text{ kN}$$ ◀

The ratio of the compressive force to the resultant pressure is

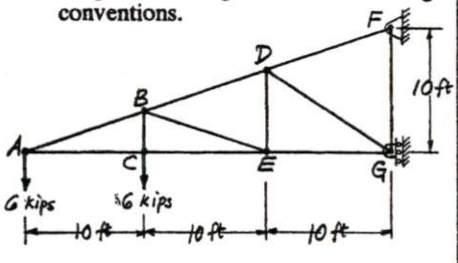
$$\frac{161}{8} \approx 20$$

Hence the resultant pressure force is magnified about 20 times.

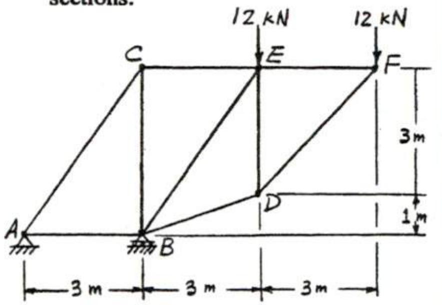
Test Problems for Chapter 4

The following problems may either be given to students for them to practice their problem solving skills or be used as test problems. The answers are provided on the next page.

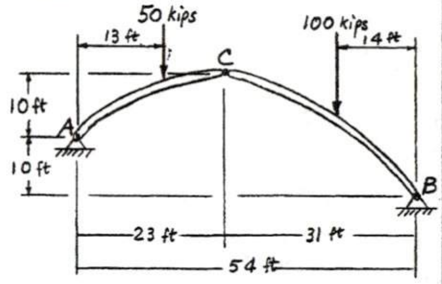
(1) Determine the forces in all members of the truss shown using the method of joints. Indicate the results on a truss diagram using the arrow sign conventions.



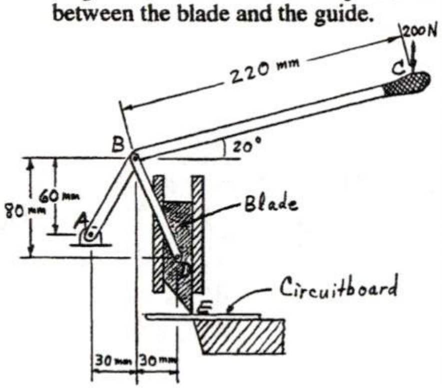
(2) Determine the forces in members CE, BE, and BD by using the method of sections.



(3) The three-hinged arch ACB is subjected to the loads shown. Determine the reaction components at supports A and B.

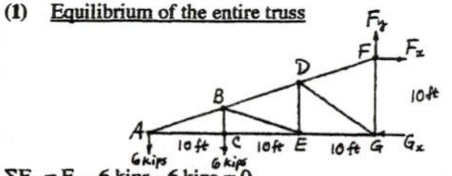


(4) The shear shown is used to cut circuitboards. For the position shown, determine the vertical force exerted on the circuitboard by the blade at E. Neglect friction forces on the joints and between the blade and the guide.



Solutions to Test Problems for Chapter 4

(1) Equilibrium of the entire truss



$\Sigma F_y = F_y - 6 \text{ kips} - 6 \text{ kips} = 0$

$F_y = 12 \text{ kips} \uparrow$

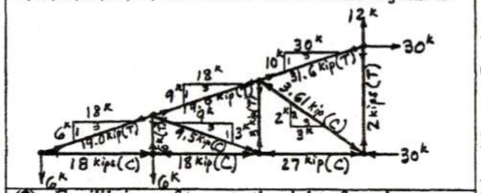
$\Sigma M_F = -G_x (10 \text{ ft}) + (6 \text{ kips})(30 \text{ ft}) + (6 \text{ kips})(20 \text{ ft}) = 0$

$G_x = 30 \text{ kips} \leftarrow$

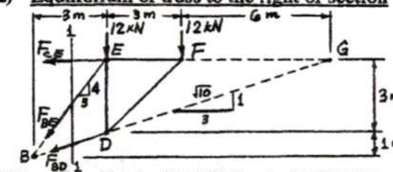
$\Sigma F_x = F_x - 30 \text{ kips} = 0$

$F_x = 30 \text{ kips} \rightarrow$

Method of joints is applied in the following sequence: A, C, F, G, D, B, and checks can be made at joint E.



(2) Equilibrium of truss to the right of section



$\Sigma M_B = F_{CE} (4 \text{ m}) - (12 \text{ kN})(3 \text{ m}) - (12 \text{ kN})(6 \text{ m}) = 0$

$F_{CE} = 27 \text{ kN (T)}$ ◀

$\Sigma M_G = V_{BE} (9 \text{ m}) + (12 \text{ kN})(9 \text{ m}) + (12 \text{ kN})(6 \text{ m}) = 0$
(at E)

$V_{BE} = -20 \text{ kN}$

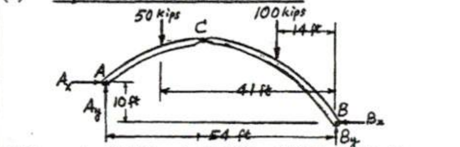
$F_{BE} = \frac{5}{4} V_{BE} = 25 \text{ kN (C)}$ ◀

$\Sigma M_E = -H_{BD} (3 \text{ m}) - (12 \text{ kN})(3 \text{ m}) = 0$
(at D)

$H_{BD} = -12 \text{ kN}$

$F_{BD} = \frac{\sqrt{10}}{3} H_{BD} = 12.65 \text{ kN (C)}$ ◀

(3) Equilibrium of the entire arch



$\Sigma M_B = -A_x (10 \text{ ft}) - A_y (54 \text{ ft}) + (50 \text{ kips})(41 \text{ ft}) + (100 \text{ kips})(14 \text{ ft}) = 0$

$10 A_x + 54 A_y = 3450$ (a)

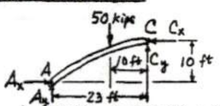
$\Sigma F_x = A_x - B_x = 0$

$A_x = B_x$ (b)

$\Sigma F_y = A_y + B_y - 50 \text{ kips} - 100 \text{ kips} = 0$

$B_y = 150 - A_y$ (c)

Equilibrium of AC part



$\Sigma M_C = A_x (10 \text{ ft}) - A_y (23 \text{ ft}) + (50 \text{ kips})(10 \text{ ft}) = 0$

$10 A_x - 23 A_y = -500$ (d)

(a) - (d): $77 A_y = 3950$

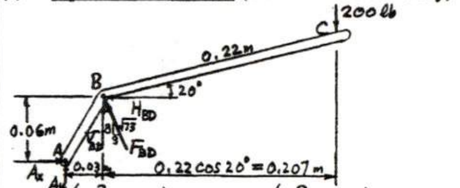
$A_y = 51.3 \text{ kips} \uparrow$ ◀

From (a): $A_x = \dfrac{3450 - (54)(51.3)}{10} = 68.0 \text{ kips} \rightarrow$ ◀

From (b): $B_x = 68.0 \text{ kips} \leftarrow$ ◀

From (c): $B_y = 150 - 51.3 = 98.7 \text{ kips} \uparrow$ ◀

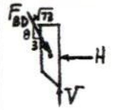
(4) Equilib. of the handle (BD is a two-force body)



$\Sigma M_A = \left(\frac{3}{\sqrt{73}} F_{BD}\right)(0.06 \text{ m}) + \left(\frac{8}{\sqrt{73}} F_{BD}\right)(0.03 \text{ m}) - (200 \text{ N})(0.03 \text{ m} + 0.207 \text{ m}) = 0$

$F_{BD} = 963 \text{ N}$

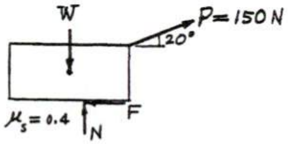
Equilibrium of the blade



$\Sigma F_y = V - \frac{8}{\sqrt{73}} F_{BD} = 0$

$V = \frac{8}{\sqrt{73}} (963 \text{ N}) = 902 \text{ N}$ ◀

5-1



$W = mg = 50 \times 9.81 = 491$ N

$\sum F_y = N + 150 \sin 20° - 491 = 0$

$N = 440$ N

$\sum F_x = -F + 150 \cos 20° = 0$

$F = 141$ N

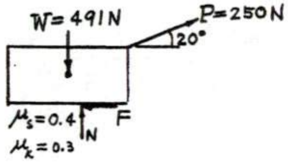
$F_m = \mu_s N = (0.4)(440) = 176$ N

$F < F_m$

The block is at rest and the friction force is

$F = 141$ N ◀

5-2



$\sum F_y = N + 250 \sin 20° - 491 = 0$

$N = 405$ N

$\sum F_x = -F + 250 \cos 20° = 0$

$F = 235$ N

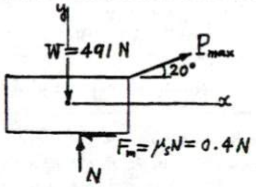
$F_m = \mu_s N = (0.4)(405) = 162$ N

$F > F_m$

Hence the block is in motion and the friction force is the kinematic friction force equal to:

$F_k = \mu_k N = (0.3)(405) = 122$ N ◀

5-3



$\sum F_x = P_{max} \cos 20° - 0.4N = 0$ (a)

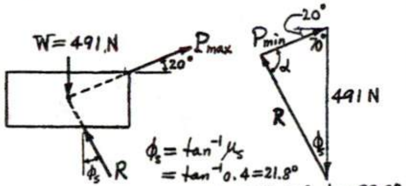
$\sum F_y = P_{max} \sin 20° + N - 491 = 0$ (b)

(a) + 0.4 × (b):

$P_{max} (\cos 20° + 0.4 \sin 20°) - 0.4(491) = 0$

$P_{max} = 182$ N ◀

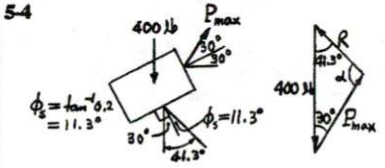
To avoid solving simultaneous equations, P_{max} may be obtained by solving the force triangle as shown in the following:



$$\frac{P_{max}}{\sin 21.8°} = \frac{491}{\sin 88.2°}$$

$P_{max} = 182$ N ◀

5-4

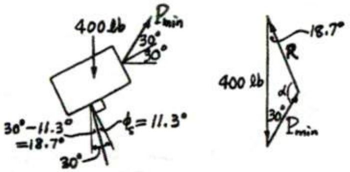


$\alpha = 180° - (30° + 41.3°) = 108.7°$

$$\frac{P_{max}}{\sin 41.3°} = \frac{400}{\sin 108.7°}$$

$P_{max} = 279$ lb ◀

5-5



$\phi_s = \tan^{-1} \phi_s = \tan^{-1} 0.20 = 11.3°$

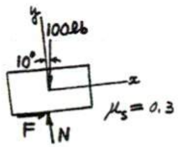
$\alpha = 180° - (30° + 18.7°) = 131.3°$

$$\frac{P_{min}}{\sin 18.7°} = \frac{400}{\sin 131.3°}$$

$P_{min} = 170.7$ lb ◀

5-6

(a) $\alpha = 10°$



$\sum F_y = N - 100 \cos 10° = 0$

$N = 98.5$ lb

$\sum F_x = F - 100 \sin 10° = 0$

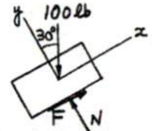
$F = 17.4$ lb

$F_m = \mu_s N = (0.3)(98.5) = 29.6$ lb

$F < F_m$

Hence the block is at rest. ◀

(b) $\alpha = 30°$



$\sum F_y = N - 100 \cos 30° = 0$

$N = 86.6$ lb

$\sum F_x = F - 100 \sin 30° = 0$

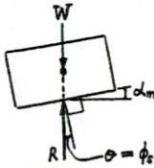
$F = 50$ lb

$F_m = \mu_s N = (0.3)(86.6) = 26.0$ lb

$F > F_m$

Hence the block will slide down. ◀

5-7



When the crate is on the verge of motion, the maximum friction force F_m develops. Thus

$\theta = \phi_s = \tan^{-1} \mu_s$

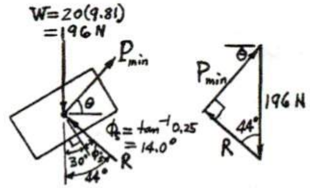
The weight is balanced by R. The weight is in the vertical direction, the reaction R must be also in the vertical direction to satisfy the equilibrium condition. Thus

$\theta = \alpha_{max}$

Therefore

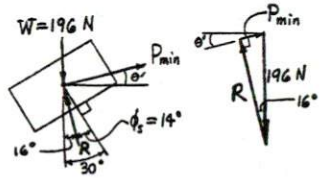
$\alpha_{max} = \phi_s = \tan^{-1} \mu_s = \tan^{-1} 0.3 = 16.7°$ ◀

5-8



$P_{min} = 196 \sin 44° = 136$ N ∠ $\theta = 44°$ ◀

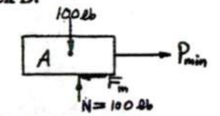
5-9



$P_{min} = 196 \sin 16° = 54.1$ N ∠ $\theta = 16°$ ◀

5-10

The minimum force P required to cause block A to slide on block B:

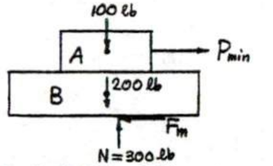


$F_m = \mu_s N = (0.35)(100) = 35$ lb

$\sum F_x = P_{min} - 35 = 0$

$P_{min} = 35$ lb

The minimum force P required to cause the two blocks to slide on the floor:



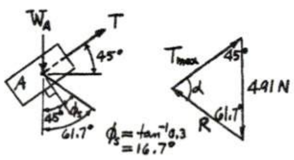
$F_m = \mu_s N = (0.1)(300) = 30$ lb

$\sum F_x = P_{min} - 30 = 0$

$P_{min} = 30$ lb

Hence the minimum force is 30 lb and sliding will occur first between block B and the floor. ◀

5-11

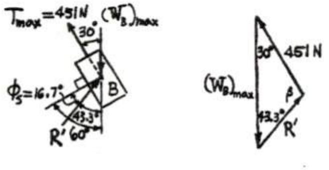


$W_A = 50 \times 9.81 = 491$ N

$\alpha = 180° - (45° + 61.7°) = 73.3°$

$$\frac{T_{max}}{\sin 61.7°} = \frac{491\text{ N}}{\sin 73.3°}$$

$T_{max} = 451$ N

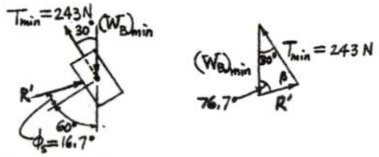


$\beta = 180° - (30° + 43.3°) = 106.7°$

$$\frac{(W_B)_{max}}{\sin 106.7°} = \frac{451\text{ N}}{\sin 43.3°}$$

$(W_B)_{max} = 630$ N

$$(m_B)_{max} = \frac{(W_B)_{max}}{g} = \frac{630}{9.81} = 64.2\text{ kg}$$ ◀



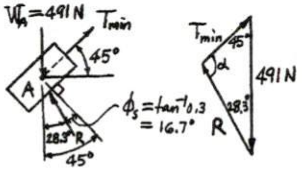
$\beta = 180° - (30° + 76.7°) = 73.3°$

$$\frac{(W_B)_{min}}{\sin 73.3°} = \frac{243\text{ N}}{\sin 76.7°}$$

$(W_B)_{min} = 239$ N

$$(m_B)_{min} = \frac{(W_B)_{min}}{g} = \frac{239}{9.81} = 24.4\text{ kg}$$ ◀

5-12



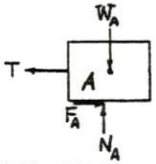
$\alpha = 180° - (45° + 28.3°) = 106.7°$

$$\frac{T_{min}}{\sin 28.3°} = \frac{491\text{ N}}{\sin 106.7°}$$

$T_{min} = 243$ N

5-13

Block A



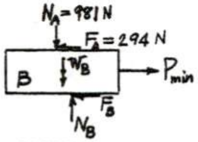
$W_A = 100 \times 9.81 = 981$ N

$\sum F_y = N_A - 981 = 0$

$N_A = 981$ N

When block B is on the verge of sliding, the maximum friction force develops on both contact surfaces. Thus

$F_A = (F_A)_{max} = \mu_s N_A = 0.30(981) = 294$ N



$W_B = 200 \times 9.81 = 1962$ N

$\sum F_y = N_B - 981 - 1962 = 0$

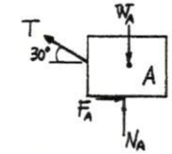
$N_B = 2943$ N

$F_B = (F_B)_{max} = \mu_s N_B = 0.20(2943) = 589$ N

$\sum F_x = P_{min} - 294 - 589 = 0$

$P_{min} = 883$ N ◀

5-14



$W_A = 100 \times 9.81 = 981$ N

$F_A = \mu_s N_A = 0.3\ N_A$

$\sum F_x = 0.3\ N_A - T \cos 30° = 0$ (a)

$\sum F_y = N_A + T \sin 30° - 981 = 0$ (b)

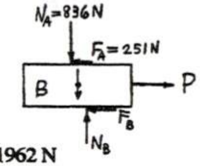
$0.3 \times$ (b) - (a):

$(0.3 \sin 30° + \cos 30°)T - 0.3 \times 981 = 0$

$T = 290$ N

from (b): $N_A = 981 - 290 \sin 30° = 836$ lb

$F_A = 0.3(836) = 251$ N ◀



$W_B = 200 \times 9.81 = 1962$ N

$F_B = \mu_s N_B = 0.2\ N_B$

$\sum F_y = N_B - 836 - 1962 = 0$

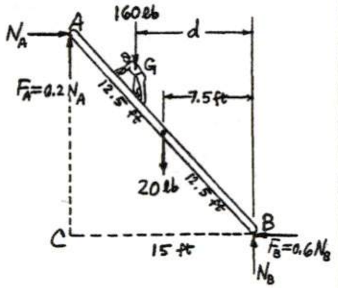
$N_B = 2798$ N

$F_B = 0.20(2798) = 560$ N

$\sum F_x = P - 251 - 560 = 0$

$P = 811$ N ◀

5-15



$AC = \sqrt{(25)^2 - (15)^2} = 20$ ft

$\sum F_x = N_A - 0.6\ N_B = 0$

$N_A = 0.6\ N_B$

$\sum F_y = 0.2\ N_A + N_B - 160 - 20 = 0$

$(0.2)(0.6\ N_B) + N_B = 180$

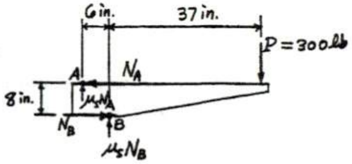
$N_B = 161$ lb

$N_A = 0.6\ (161) = 96.4$ lb

$\sum M_B = (160)(d) + (20)(7.5) - (96.4)(20) - (0.2 \times 96.4)(15) = 0$

$d = 12.9$ ft ◀

5-16



$\sum F_x = N_B - N_A = 0,$

$N_A = N_B$

$\sum M_B = N_A(8) - \mu_s N_A(6) - 300(37) = 0$

$8N_A - 6\mu_s N_A = 11\ 100$ (a)

$\sum F_y = \mu_s N_A - \mu_s N_B - 300 = 0$

$\mu_s N_A = 150$ (b)

⇒ (a): $8N_A - 6(150) = 11\ 100$

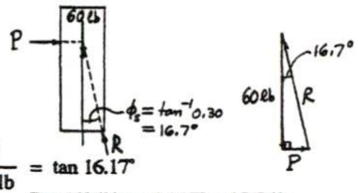
$N_A = 1500$ lb

⇒ (b): $\mu_s = \frac{150}{1500} = 0.10$

$\mu_s \geq 0.10$ ◀

5-17

Minimum horizontal force to cause sliding:



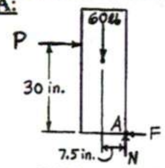
$$\frac{P}{60\text{ lb}} = \tan 16.17°$$

$P = (60\text{ lb})\tan 16.17° = 18.0$ lb

(Cont'd)

5-17 (Cont)

Minimum horizontal force to cause tipping about corner A:



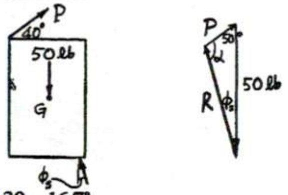
$\sum M_A = -P(30) + 60(7.5) = 0$

$P = 15.0$ lb

The file cabinet will tip first when P = 15.0 lb ◀

5-18

When sliding occurs:



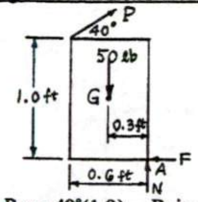
$\phi_s = \tan^{-1} 0.30 = 16.7°$

$\alpha = 180° - (50° + 16.7°) = 113.3°$

$$\frac{P}{\sin 16.7°} = \frac{50 \text{ lb}}{\sin 113.3°}$$

$P = 15.6$ lb

When tipping about corner A occurs:



$\sum M_A = -P\cos 40°(1.0) - P\sin 40°(0.6) + 50(0.3) = 0$

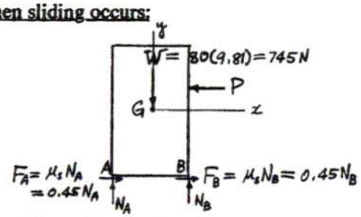
$P(\cos 40° + 0.6 \sin 40°) = 15$

$P = 17.3$ lb

The crate will slide first when P = 15.6 lb. ◀

5-19

(a) When sliding occurs:



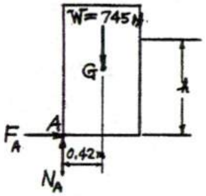
$\sum F_y = N_A + N_B - 745 = 0$

$N_A + N_B = 745$

$\sum F_x = 0.45N_A + 0.45N_B - P = 0$

$P = 0.45(N_A + N_B) = 0.45(745) = 335$ N ◀

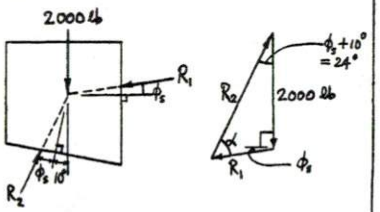
(b) When tipping about corner A occurs:



$\sum M_A = 335\,h - 745(0.42) = 0$

$h = 0.934$ m $= 934$ mm ◀

5-20



$\phi_s = \tan^{-1} 0.25 = 14.0°$

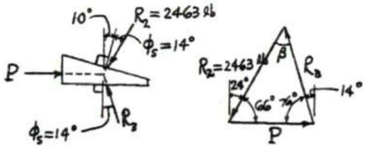
$\alpha = 180° - (24° + 104°) = 52°$

$$\frac{R_2}{\sin 104°} = \frac{2000 \text{ lb}}{\sin 52°}$$

$R_2 = 2463$ lb

(Cont'd)

5-20 (Cont)

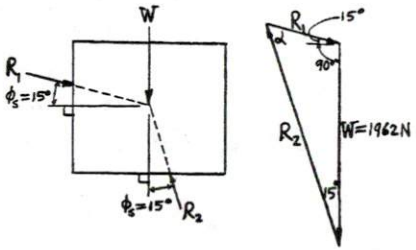


$\beta = 180° - (66° + 76°) = 38°$

$$\frac{P}{\sin 38°} = \frac{2463 \text{ lb}}{\sin 76°}$$

$P = 1563$ lb ◀

5-21

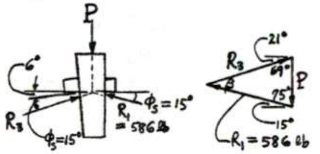


$W = mg = 200 \times 9.81 = 1962$ N

$\alpha = 180° - (105° + 15°) = 60°$

$$\frac{R_1}{\sin 15°} = \frac{1962 \text{ lb}}{\sin 60°}$$

$R_1 = 586$ N

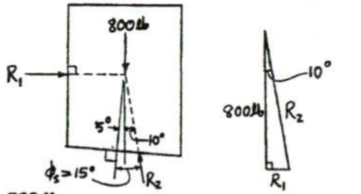


$\beta = 180° - (69° + 75°) = 36°$

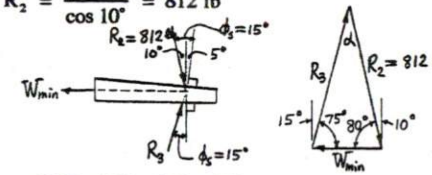
$$\frac{P}{\sin 36°} = \frac{586 \text{ lb}}{\sin 69°}$$

$P = 369$ N ◀

5-22



$$R_2 = \frac{800 \text{ lb}}{\cos 10°} = 812 \text{ lb}$$

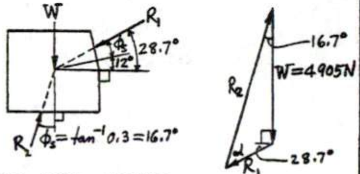


$\alpha = 180° - (75° + 80°) = 25°$

$$\frac{W_{min}}{\sin 25°} = \frac{812 \text{ lb}}{\sin 75°}$$

$W_{min} = 355$ lb ◀

5-23

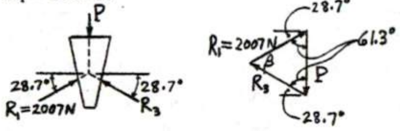


$W = mg = 500 \times 9.81 = 4905$ N

$\alpha = 180° - (16.7° + 118.7°) = 44.6°$

$$\frac{R_1}{\sin 16.7°} = \frac{4905 \text{ N}}{\sin 44.6°}$$

$R_1 = 2007$ N



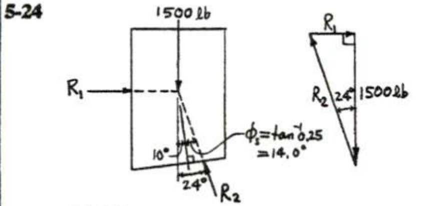
Due to Symmetry, $R_3 = R_1 = 2007$ N

$\beta = 180° - 2(61.3°) = 57.4°$

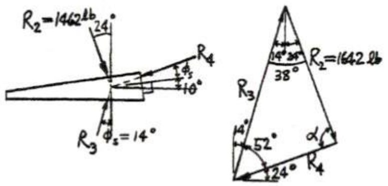
$$\frac{P}{\sin 57.4°} = \frac{2007 \text{ N}}{\sin 61.3°}$$

$P = 1928$ N ◀

5-24

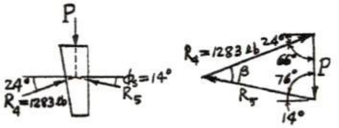


$$R_2 = \frac{1500 \text{ lb}}{\cos 24°} = 1642 \text{ lb}$$



$\alpha = 180° - (38° + 52°) = 90°$

$$R_4 = \frac{1624 \text{ lb}}{\tan 52°} = 1283 \text{ lb}$$



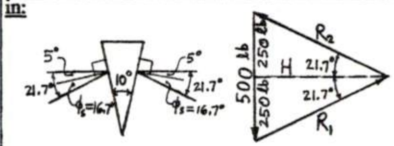
$\beta = 180° - (66° + 76°) = 38°$

$$\frac{P}{\sin 38°} = \frac{1283 \text{ lb}}{\sin 76°}$$

$P = 814$ lb ◀

5-25

Free-body diagram of the wedge as it is being driven in:

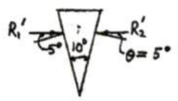


$\phi_s = \tan^{-1} \mu_s = \tan^{-1} 0.30 = 16.7°$

Let H be the horizontal component of R_1 and R_2.

$H = (250 \text{ lb}) / \tan 21.7° = 628$ lb

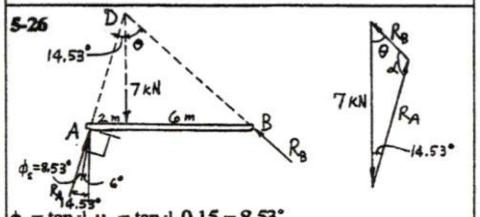
Free-body diagram of the wedge after it is being driven into the log: Without any vertical force (P = 0), the reactions will not have any vertical compontent. The wedge becomes two-force member with $R_1' = R_2' = H$ acting as shown:



$R_1' = R_2' = H = 628$ lb ◀

Note: Since the angle $\theta = 5°$ between R_1' or R_2' and the normals to the surfaces of the wedge is less than $\phi_s = 16.7°$, the wedge will not slip out and is said to be self-locking.

5-26



$\phi_s = \tan^{-1} \mu_s = \tan^{-1} 0.15 = 8.53°$

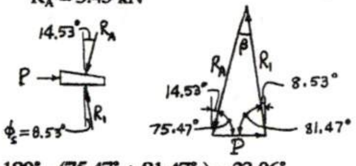
$$CD = \frac{2 \text{ m}}{\tan 14.53°} = 7.72 \text{ m}$$

$$\theta = \tan^{-1} \frac{BC}{CD} = \tan^{-1} \frac{6}{7.72} = 37.9°$$

$\alpha = 180° - (14.53° + 37.9°) = 127.6°$

$$\frac{R_A}{\sin 37.9°} = \frac{7 \text{ kN}}{\sin 127.6°}$$

$R_A = 5.43$ kN

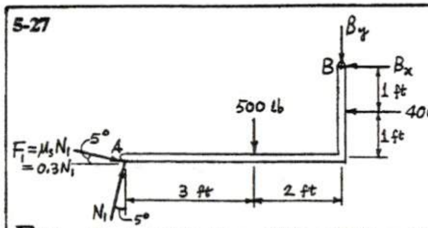


$\beta = 180° - (75.47° + 81.47°) = 23.06°$

$$\frac{P}{\sin 23.06°} = \frac{5.43 \text{ kN}}{\sin 81.47°}$$

$P = 2.15$ kN ◀

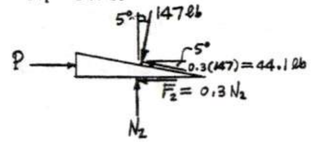
5-27



$$\sum M_B = N_1 \sin 5°(2) - N_1 \cos 5°(5) + 0.3N_1 \cos 5°(2) + 0.3N_1 \sin 5°(5) + 500(2) - 400(1) = 0$$

$-4.078 N_1 + 600 = 0$

$N_1 = 147$ lb



$\sum F_y = N_2 - 147 \cos 5° + 44.1 \sin 5° = 0$

$N_2 = 142.6$ lb

$F_2 = 0.3(142.6) = 42.8$ lb

$\sum F_x = P - 147 \sin 5° - 44.1 \cos 5° - 42.8 = 0$

$P = 99.5$ lb ◀

5-28

For single thread, $n = 1$

$r = 0.025$ m, $p = 0.010$ m,

$W = mg = 2000 \times 9.81 = 19\,620$ N

$$\theta = \tan^{-1} \frac{np}{2\pi r} = \tan^{-1} \frac{1 \times 0.01 \text{ m}}{2\pi(0.025 \text{ m})} = 3.64°$$

$\phi_s = \tan^{-1} \mu_s = \tan^{-1} 0.10 = 5.71°$

Since $\theta < \phi_s$, the jack is self-locking.

The torque required to raise the load is:

$M = Wr \tan(\phi_s + \theta)$

$= (19\,620 \text{ N})(0.025 \text{ m}) \tan(5.71° + 3.64°)$

$= 80.8$ N • m ◀

The torque required to lower the load is:

$M' = Wr \tan(\phi_s + \theta)$

$= (19\,620 \text{ N})(0.025 \text{ m}) \tan(5.71° - 3.64°)$

$= 17.7$ N • m ◀

5-29

For double thread, $n = 2$,

$$r = \frac{d}{2} = \frac{1}{2}\left(\frac{3}{8} \text{ in.}\right) = 0.1875 \text{ in.}$$

$p = 0.1$ in., $\mu_s = 0.30$

Tightening torque = 30 lb • in.

$$\theta = \tan^{-1} \frac{np}{2\pi r} = \tan^{-1} \frac{2(0.1 \text{ in.})}{2\pi(0.1875 \text{ in.})} = 9.63°$$

$\phi_s = \tan^{-1} \mu_s = \tan^{-1}(0.30) = 16.70°$

Since $\theta < \phi_s$, the clamp is self-locking.

(a) The compressive load W on the wood:

$M = Wr \tan(\phi_s + \theta)$

30 lb • in. = W(0.1875 in.) tan (16.70° + 9.63°)

$W = 323$ lb ◀

(b) The torque required to loosen the clamp:

$M' = Wr \tan(\phi_s + \theta)$

$= (323 \text{ lb})(0.1875 \text{ in.}) \tan(16.70° - 9.63°)$

$= 7.51$ lb • in. ◀

5-30

The torque M applied at the sleeve is shared equally by the two rods on each end, hence each rod is subjected to a torque of M/2.

For single thread, $n = 1$

$$W = T = 10 \text{ kips}, \quad r = \frac{1}{2} \text{ in.}, \quad p = \frac{1}{4} \text{ in.}, \quad \phi_s = 8°$$

$$\theta = \tan^{-1} \frac{np}{2\pi r} = \tan^{-1} \frac{1(0.25 \text{ in.})}{2\pi(0.5 \text{ in.})} = 4.55°$$

$$\frac{M}{2} = Wr \tan(\phi_s - \theta)$$

The minimum tightening torque is

$$M = 2(10 \text{ kips})\left(\frac{1}{2} \text{ in.}\right) \tan(8° + 4.55°)$$

$= 2.23$ kip-in. ◀

5-31

Since $\theta < \phi_s$, the turnbuckle is self-locking.

$$\frac{M'}{2} = Wr \tan(\phi_s - \theta)$$

The minimum loosening torque is

$$M' = 2Wr \tan(\phi_s - \theta)$$

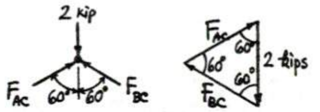
$$= 2(10 \text{ kips})\left(\frac{1}{2} \text{ in.}\right) \tan(8° - 4.55°)$$

$$= 0.603 \text{ kip} \cdot \text{in.} = 603 \text{ lb} \cdot \text{in.}$$ ◄

5-32

Note that AC, BC, AD, BD, and AB are all two-force members.

Joint C



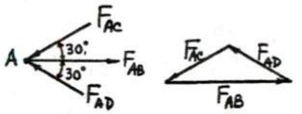
From the force-triangle,

$$F_{AC} = F_{BC} = 2 \text{ kips (C)}$$

Similarly, from the equilibrium of joint D, we have

$$F_{AD} = F_{BD} = 2 \text{ kips (C)}$$

Joint A



$$F_{AB} = F_{AC} \cos 30° + F_{AD} \cos 30°$$
$$= 2(2 \text{ kips}) \cos 30° = 3.46 \text{ kips}$$

The torque M is shared equally by the threads at A and B, each thread is subjected to a torque of M/2.

$$W = F_{AB} = 3.46 \text{ kips}$$

For double threads, n = 2

$$r = \frac{d}{2} = \frac{1.25 \text{ in.}}{2} = 0.625 \text{ in.}, \quad p = \frac{1}{5} = 0.2 \text{ in.}$$

$$\mu_s = 0.15, \quad \phi_s = \tan^{-1}(0.15) = 8.53°$$

$$\theta = \tan^{-1}\frac{np}{2\pi r} = \tan^{-1}\frac{2(0.2 \text{ in.})}{2\pi(0.625 \text{ in.})} = 5.82°$$

$$\frac{M}{2} = Wr \tan(\phi_s + \theta)$$

The torque M for raising load:

$$M = 2Wr \tan(\phi_s + \theta)$$

$$= 2(3.46 \text{ kips})(0.625 \text{ in.}) \tan(8.53° + 5.82°)$$

$$= 1.10 \text{ kip} \cdot \text{in.} = 1100 \text{ lb} \cdot \text{in.}$$ ◄

5-33

$$\frac{M'}{2} = Wr \tan(\phi_s - \theta)$$

The torque M' for lowering load:

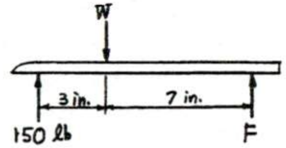
$$M' = Wr \tan(\phi_s - \theta)$$

$$= 2(3.46 \text{ kips})(0.625 \text{ in.}) \tan(8.53° - 5.82°)$$

$$= 0.205 \text{ kip} \cdot \text{in.} = 205 \text{ lb} \cdot \text{in.}$$ ◄

5-34

Free-body of the upper member:



$$\sum M_B = W(7) - 150(10) = 0$$
$$W = 214 \text{ lb}$$

For double threaded screw, n = 2

$$r = \frac{3}{8} \text{ in.}, \quad p = \frac{1}{8} \text{ in.}$$

$$\theta = \tan^{-1}\left[\frac{np}{2\pi r}\right] = \tan^{-1}\left[\frac{2(\frac{1}{8})}{2\pi(\frac{3}{8})}\right] = 6.06°$$

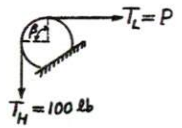
$$\phi_s = \tan^{-1} 0.20 = 11.3°$$

$$M = Wr \tan(\phi_s + \theta)$$

$$= (214 \text{ lb})\left(\frac{3}{8} \text{in.}\right) \tan(11.3° + 6.06°)$$

$$= 25.1 \text{ lb} \cdot \text{in.}$$ ◄

5-35



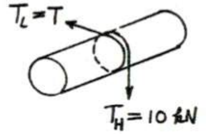
$$\mu_s = 0.30$$

$$\beta = \frac{\pi}{2} \text{ rad.}$$

$$\frac{T_H}{T_L} = \frac{100 \text{ lb}}{P} = e^{\mu_s \beta} = e^{(0.30)\left(\frac{\pi}{2}\right)} = 1.602$$

$$P = \frac{100 \text{ lb}}{1.602} = 62.4 \text{ lb}$$ ◄

5-36

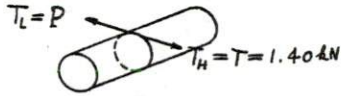


$$\mu_s = 0.25$$

$$\beta = 1\frac{1}{4} \text{ turns} = \frac{5\pi}{2} \text{ rad.}$$

$$\frac{T_H}{T_L} = \frac{10 \text{ kN}}{T} = e^{\mu_s \beta} = e^{(0.25)\left(\frac{5\pi}{2}\right)} = 7.124$$

$$T = \frac{10 \text{ kN}}{7.124} = 1.40 \text{ kN}$$



$$\mu_s = 0.25$$

$$\beta' = 1 \text{ turn} = 2\pi \text{ rad.}$$

$$\frac{T_H}{T_L} = \frac{1.40 \text{ kN}}{P} = e^{\mu_s \beta'} = e^{(0.25)(2\pi)} = 4.81$$

$$P = \frac{1.40 \text{ kN}}{4.81} = 0.292 \text{ kN}$$
$$= 292 \text{ N}$$ ◄

5-37

From the diagram for rod A in Prob. 5-36,

$$\mu_s\left(\frac{5\pi}{2}\right) = \ln \frac{10\ 000 \text{ N}}{T} \qquad \text{(a)}$$

From the diagram for rod B in Prob. 5-36,

$$\mu_s(2\pi) = \ln \frac{T}{400 \text{ N}} \qquad \text{(b)}$$

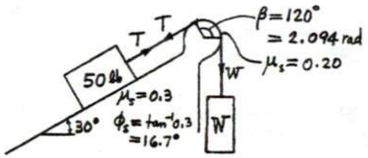
(a) + (b):

$$\mu_s(2.5\pi + 2\pi) = \ln \frac{10\ 000}{T} + \ln \frac{T}{400}$$

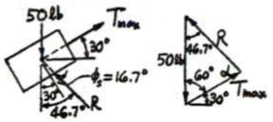
$$= \ln\left(\frac{10\ 000}{T} \times \frac{T}{400}\right) = \ln 25$$

$$\mu_s = \frac{\ln 25}{4.5\pi} = 0.228$$ ◄

5-38



Maximum W:



$$\alpha = 180° - (60° + 46.7°) = 73.3°$$

$$\frac{T_{max}}{\sin 46.7°} = \frac{50 \text{ lb}}{\sin 73.3°}$$

$$T_{max} = 38.0 \text{ lb}$$

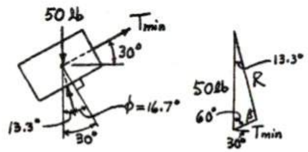
For maximum W_{max}, $W_{max} > T_{max}$

$$\frac{T_H}{T_L} = \frac{W_{max}}{T_{max}} = e^{\mu_s \beta} = e^{(0.20)(2.094)} = 1.52$$

$$W_{max} = 1.52\, T_{max} = 1.52(38.0 \text{ lb})$$
$$= 57.8 \text{ lb}$$ ◄

5-39

Minimum W:



$\beta = 180° - (60° + 13.3°) = 106.7°$

$$\frac{T_{min}}{\sin 13.3°} = \frac{50 \text{ lb}}{\sin 106.7°}$$

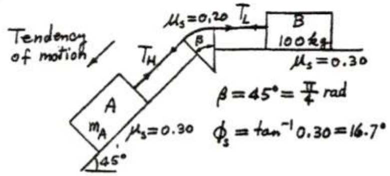
$T_{min} = 12.0$ lb

Since $W_{min} < T_{min}$, we write

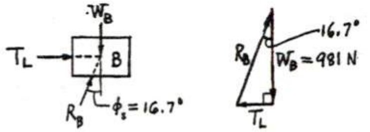
$$\frac{T_H}{T_L} = \frac{T_{min}}{W_{min}} = e^{\mu\beta} = e^{(0.20)(2.094)} = 1.52$$

$$W_{min} = \frac{T_{min}}{1.52} = \frac{12.0 \text{ lb}}{1.52} = 7.90 \text{ lb}$$ ◀

5-40



Block B



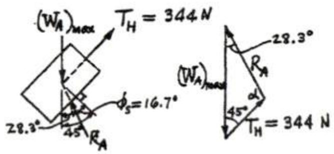
$W_B = mg = 100 \times 9.91 = 981$ N

$T_L = 981 \tan 16.7° = 294.3$ N

$$\frac{T_H}{T_L} = e^{\mu\beta} = e^{(0.20)\left(\frac{\pi}{4}\right)} = 1.17$$

$T_H = 1.17\, T_L = 1.17\,(294.3 \text{ N}) = 344$ N

Block A



$\alpha = 180° - (45° + 28.3) = 106.7°$

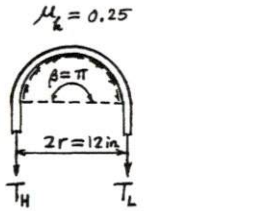
$$\frac{(W_A)_{max}}{\sin 106.7°} = \frac{344 \text{ N}}{\sin 28.3°}$$

$(W_A)_{max} = 695$ N

The largest mass of block A is

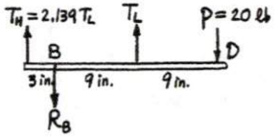
$$(m_A)_{max} = \frac{(W_A)_{max}}{g} = \frac{695}{9.81} = 70.8 \text{ kg}$$ ◀

5-41



$$\frac{T_H}{T_L} = e^{\mu\beta} = e^{(0.25)\pi} = 2.193$$

$T_H = 2.193\, T_L$



$$\sum M_B = T_L(9) - (2.193T_L)(3) - (20)(18) = 0$$

$T_L = 148.7$ lb

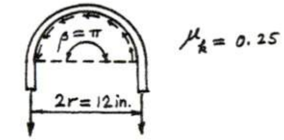
$T_H = 2.193\, T_L = 326$ lb

The torque on the drum is

$M = (T_H - T_L)r = (326 - 148.7)(6)$

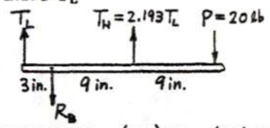
$= 1064$ lb • in. $= 88.7$ lb • ft ◀

5-42



$$\frac{T_H}{T_L} = e^{\mu\beta} = e^{(0.25)\pi} = 2.193$$

$T_H = 2.193\, T_L$



$$\sum M_B = (2.193T_L)(9) - (T_L)(3) - (20)(18) = 0$$

$T_L = 21.51$ lb

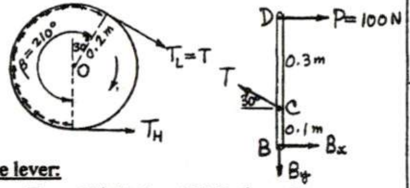
$T_H = 2.193\, T_L = 47.17$ lb

The torque on the drum is

$M = (T_H - T_L)r = (47.17 - 21.51)(6)$

$= 154.0$ lb • in. $= 12.8$ lb • ft ◀

5-43



For the lever:

$$\sum M_B = T \cos 30° \,(0.1) - 100(0.4) = 0$$

$T = 462$ N

For the flywheel:

$$\beta = 210° = 210°\left(\frac{\pi}{180°}\right) = 3.665 \text{ radians}$$

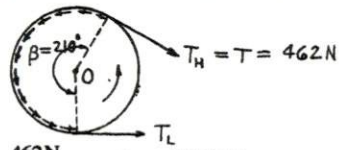
$$\frac{T_H}{T_L} = \frac{T_H}{462 \text{ N}} = e^{\mu\beta} = e^{(0.30)(3.665)} = 3.003$$

$T_H = 3.003(462 \text{ N}) = 1387$ N

$M = (T_H - T_L)r = (1387 - 462)0.2 = 185$ N • m ◀

5-44

From the solution to Prob. 5-43, T = 462 N.

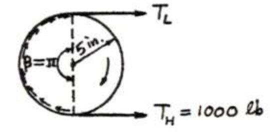


$$\frac{T_H}{T_L} = \frac{462 \text{N}}{T_L} = e^{\mu\beta} = e^{(0.30)(3.665)} = 3.003$$

$$T_L = \frac{462 \text{ N}}{3.003} = 154 \text{ N}$$

$M = (T_H - T_L)r = (462 - 154)0.2 = 61.6$ N • m ◀

5-45

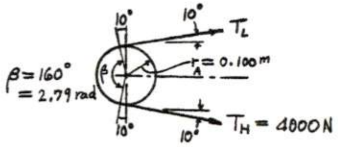


$$\frac{T_H}{T_L} = \frac{1000 \text{ lb}}{T_L} = e^{\mu\beta} = e^{(0.3)\pi} = 2.57$$

$T_L = 390$ lb

$M_{max} = (T_H - T_L)r = (1000 - 390)5 = 3050$ lb • in. ◀

5-46



$$\frac{T_H}{T_L} = \frac{4000 \text{ N}}{T_L} = e^{\mu\beta} = e^{(0.40)(2.79)} = 3.06$$

$T_L = 1307$ N

$M_A = (T_H - T_L)r_A = (4000 - 1307)0.100 = 269$ N • m ◀

5-47

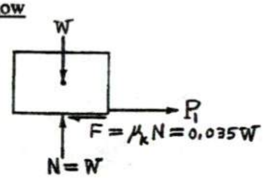
Yes. A car with fully inflated tires would have a better gas mileage because fully inflated tires deform less and have a smaller coefficient of rolling resistance.

5-48

The gas mileage of an automobile travelling on a paved road is better than travelling on a dirt road, because the coefficient of rolling resistance of the tires on a paved road is smaller than that on a dirt road; hence the rolling resistance is smaller and the gas mileage is better on a paved road.

5-49

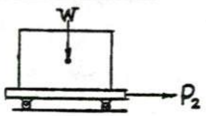
(a) <u>Sled on snow</u>



$$\sum F_x = P_1 - 0.035\,W = 0$$

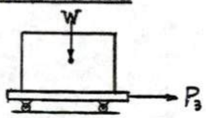
$$P_1 = 0.035\,W$$ ◄

(b) <u>Pneumatic tire on pavement</u>



$$P_2 = \frac{b}{r}W = \frac{0.025\text{ in.}}{10\text{ in.}}W = 0.0025\,W$$ ◄

(c) <u>Steel wheel on steel track</u>



$$P_3 = \frac{b}{r}W = \frac{0.015\text{ in.}}{10\text{ in.}}W = 0.0015\,W$$ ◄

5-50

$$\text{Weight on each wheel} = \frac{50\text{ kips}}{8} = 6.25\text{ kips}$$

Rolling resistance on each wheel

$$= \frac{bW}{r} = \frac{(0.015\text{ in.})(6.25\text{ in.})}{12.5\text{ in.}}$$

$$= 0.0075\text{ kips} = 7.5\text{ lb}$$

The horizontal force required to keep the railroad car to move at a constant speed

$$= 8(7.5\text{ lb}) = 60\text{ lb}$$ ◄

5-51

$$W = 300(9.81) = 2943\text{ N}$$

$$\text{Weight per wheel} = \frac{W}{4} = \frac{2943\text{ N}}{4} = 736\text{ N}$$

Rolling resistance per wheel

$$= \frac{bW}{r} = \frac{(4\text{ mm})(736\text{ N})}{25\text{ mm}} = 118\text{ N}$$

The horizontal force required to overcome the rolling resistance

$$= 4(118\text{ N}) = 471\text{ N}$$ ◄

5-52

Horizontal force required

= rollling resistance

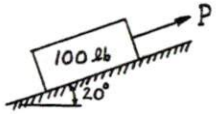
$$= \frac{bW}{r}$$

$$= \frac{(0.05\text{ in})(2200\text{ lb})}{1.075\text{ ft}} = 102.3\text{ lb}$$ ◄

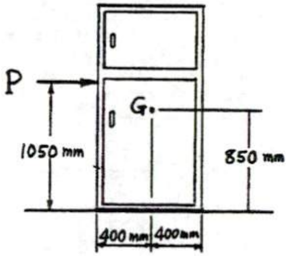
Test Problems for Chapter 5

The following problems may either be given to students for them to practice their problem solving skills or be used as test problems. The answers are provided on the next page.

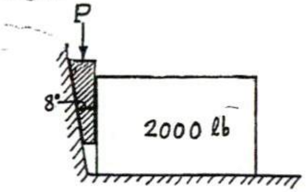
(1) The 100 lb block rests on the incline shown before the force P is applied. Determine the smallest force P required to make the block start to move up the incline. Assume that $\mu_s = 0.40$.



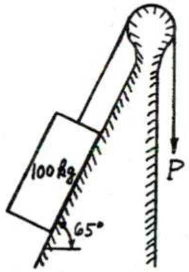
(2) The refrigerator shwon has a mass of 100 kg and a center of gravity at G shown. Knowing that $\mu_s = 0.40$, determint the smallest force P required to cause the refrigerator to slide or tip over.



(3) The 8° wedge of negligible weight is used to adjust the position of the 2000-lb block shown. Know that $\mu_s = 0.25$ at all contact surfaces, determine the smallest force P required to move the weight.

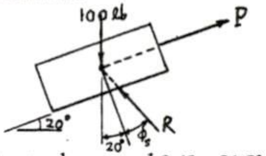


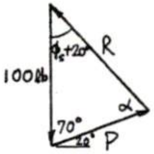
(4) The block shown rests on the incline shown. Determine the smallest force P required to keep the block from sliding down the incline. The static friction coefficients are 0.4 between the block and the incline and 0.3 between the cable and the circular peg.



Solutions to Test Problems for Chapter 5

(1) Before the block can move up the incline, the maximum static friction force must be overcome. The reaction **R** makes an angle ϕ_s with the normal to the incline.





$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.40 = 21.8°$

$\alpha = 180° - 70° - (\phi_s + 20°) = 68.2°$

$$\frac{P}{\sin(\phi_s + 20°)} = \frac{100 \text{ lb}}{\sin\alpha}$$

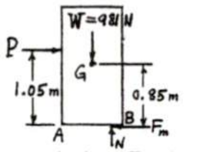
$$P = \frac{(100 \text{ lb})\sin(\phi_s + 20°)}{\sin\alpha} = \frac{(100 \text{ lb})\sin 41.8°}{\sin 68.2°}$$

$P = 71.8$ lb ◀

(2)

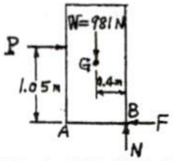
<u>To cause sliding:</u> The smallest force to cause the refrigerator to slide to the right needs to overcome the maximum static friction force.

$W = mg = (100 \text{ kg})(9.81 \text{ m/s}^2)$
$= 981$ N



$\Sigma F_y = N - W = 0$
$N = W = 981$ N
$F_m = \mu_s N$
$= 0.40(981 \text{ N})$
$= 392$ N
$\Sigma F_x = P - 392 \text{ N} = 0$
$P = 392$ N

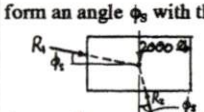
<u>To cause tipping:</u> For the smallest force to cause the refrigerator to tip over about point *B*, the normal and friction forces must act at *B*.

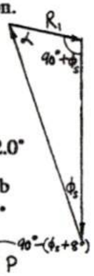


$\Sigma M_B = -P(1.05 \text{ m}) + (981 \text{ N})(0.4 \text{ m}) = 0, \quad P = 374$ N

$P_{min} = 374$ N (cause tipping) ◀

(3) The small force P required must overcome the maximum static friction forces in all the contact surfaces. The reaction at the contact surfaces must form an angle ϕ_s with the normal direction.

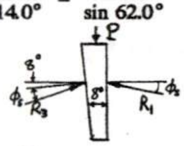


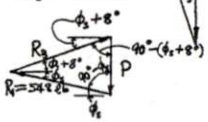


$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.25 = 14.0°$

$\alpha = 180° - \phi_s - (\phi_s + 90°) = 90° - 2\phi_s = 62.0°$

$$\frac{R_1}{\sin 14.0°} = \frac{2000 \text{ lb}}{\sin 62.0°} \qquad R_1 = 548 \text{ lb}$$



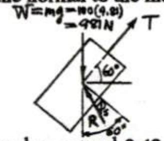


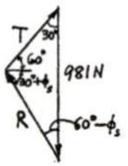
$$\frac{P}{\sin(2\phi_s + 8°)} = \frac{548 \text{ lb}}{\sin[90° - (8° + \phi_s)]}$$

$$P = \frac{(548 \text{ lb})\sin[(2)(14°) + 8°]}{\sin[90° - (8° + 14°)]}$$

$P = 347$ lb ◀

(4)
For the smallest force P, the block tends to move down the incline and the reaction makes an angle ϕ_s with the normal to the incline.



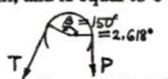


$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.40 = 21.8°$

$$\frac{T}{\sin(60 - \phi_s)} = \frac{981 \text{ N}}{\sin(90° + \phi_s)}$$

$$T = \frac{(981 \text{ N})\sin(60° - 21.8°)}{\sin(90° + 21.8°)} = 653.4 \text{ lb}$$

For the smallest force *P*, the ratio of T to P is a maximum, and is equal to $e^{\mu\beta}$



$$\frac{T}{P} = e^{\mu\beta}$$

$$P = \frac{T}{e^{\mu\beta}} = \frac{653.4 \text{ lb}}{e^{0.3(2.618)}} = 298 \text{ N}$$ ◀

6-1

$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$\cos\theta_z = \sqrt{1 - \cos^2\theta_x - \cos^2\theta_y}$

$= \sqrt{1 - \cos^2 70° - \cos^2 50°}$

$= \pm 0.685$

Since F_z is negative, $\cos\theta_z = -0.685$

$F_x = F\cos\theta_x = 400\cos 70° = +137$ lb ◀

$F_y = F\cos\theta_y = 400\cos 50° = +257$ lb ◀

$F_z = F\cos\theta_z = 400(-0.685) = -274$ lb ◀

6-2

$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$\cos^2 35° + \cos^2 120° + \cos^2\theta_z = 1$

$\cos^2\theta_z = 0.07899$

$\cos\theta_z = -0.2811$ (Negative value is taken because F_z is negative)

$\theta_z = 106.3°$

$F_x = F\cos\theta_x = 400\cos 35° = 328$ lb ◀

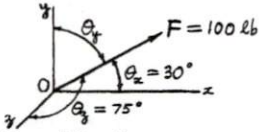
$F_y = F\cos\theta_y = 400\cos 120° = -200$ lb ◀

$F_z = F\cos\theta_z = 400\cos 106.3° = -112$ lb ◀

Check:

$\sqrt{(328)^2 + (200)^2 + (112)^2} = 400$ (Checks)

6-3



$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$\cos^2 30° + \cos^2\theta_y + \cos^2 75° = 1$

$\cos\theta_y = \sqrt{1 - \cos^2 30° - \cos^2 75°} = 0.4278$

$\theta_y = 64.7°$

$F_x = F\cos\theta_x = 100\cos 30° = 86.6$ lb ◀

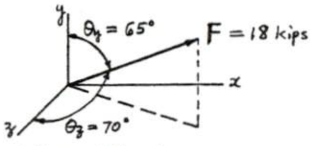
$F_y = F\cos\theta_y = 100\cos 64.7° = 42.81$ lb ◀

$F_z = F\cos\theta_z = 100\cos 75° = 25.9$ lb ◀

Check:

$\sqrt{(86.6)^2 + (42.8)^2 + (25.9)^2} = 100$ (Checks)

6-4



$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$\cos^2\theta_x + \cos^2 65° + \cos^2 70° = 1$

$\cos\theta_x = \sqrt{1 - \cos^2 65° - \cos^2 70°} = 0.839$

$\theta_x = 32.9°$

$F_x = F\cos\theta_x = (18)\cos 32.9° = 15.11$ kips ◀

$F_y = F\cos\theta_y = (18)\cos 65° = 7.61$ kips ◀

$F_z = F\cos\theta_z = (18)\cos 70° = 6.16$ kips ◀

Check:

$\sqrt{(15.11)^2 + (7.61)^2 + (6.16)^2} = 18.0$ (Checks)

6-5

$F_x = +600$ N, $F_y = +800$ N, $F_z = -2400$ N

$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$

$= \sqrt{(+600)^2 + (800)^2 + (-2400)^2}$

$= 2600$ N ◀

$\theta_x = \cos^{-1}\dfrac{F_x}{F} = \cos^{-1}\dfrac{+600}{2600} = 76.7°$ ◀

$\theta_y = \cos^{-1}\dfrac{F_y}{F} = \cos^{-1}\dfrac{+800}{2600} = 72.1°$ ◀

$\theta_z = \cos^{-1}\dfrac{F_z}{F} = \cos^{-1}\dfrac{-2400}{2600} = 157.4°$ ◀

Check:

$\cos^2 76.7° + \cos^2 72.1° + \cos^2 157.4° = 1.00$ (Checks)

6-6

$F_x = 100$ lb
$F_y = 105$ lb
$F_z = -145$ lb

$F = \sqrt{(100)^2 + (105)^2 + (-145)^2} = 205$ lb ◀

$\theta_x = \cos^{-1}\left(\dfrac{100}{205}\right) = 60.8°$ ◀

(Cont'd)

6-6 (Cont)

$$\theta_y = \cos^{-1}\left(\frac{105}{205}\right) = 59.2°$$ ◀

$$\theta_z = \cos^{-1}\left(\frac{-145}{205}\right) = 135.0°$$ ◀

Check:

$\cos^2 60.8° + \cos^2 59.2° + \cos^2 135.0° = 1.00$ (Checks)

6-7

$F_x = -160$ N
$F_y = 300$ N
$F_z = -340$ N

$$F = \sqrt{(-160)^2 + (300)^2 + (-340)^2} = 481 \text{ N}$$ ◀

$$\theta_x = \cos^{-1}\left(\frac{-160}{481}\right) = 109.4°$$ ◀

$$\theta_y = \cos^{-1}\left(\frac{300}{481}\right) = 51.4°$$ ◀

$$\theta_z = \cos^{-1}\left(\frac{-340}{481}\right) = 135.0°$$ ◀

Check:

$\cos^2 109.4° + \cos^2 51.4° + \cos^2 135.0° = 1.00$ (Checks)

6-8

$\theta_y = 40°$
$\theta_z = 75°$
$F_x = -260$ lb

$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

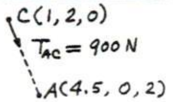
$\cos^2\theta_x + \cos^2 40° + \cos^2 75° = 1$

$$\cos\theta_x = -\sqrt{1 - \cos^2 40° - \cos^2 75°} = -0.588$$

$\theta_x = 126.0°$

$$F = \frac{F_x}{\cos\theta_x} = \frac{-260 \text{ lb}}{\cos 126.0°} = 442 \text{ lb}$$ ◀

6-9



A (4.5, 0, 2)
-) C (1, 2, 0)

$d_x = 3.5$, $d_y = -2$, $d_z = 2$

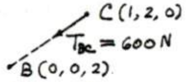
$$d = \sqrt{(3.5)^2 + (2)^2 + (2)^2} = 4.5$$

$$(T_{AC})_x = \frac{3.5}{4.5}(900 \text{ N}) = +700 \text{ N}$$ ◀

$$(T_{AC})_y = \frac{-2}{4.5}(900 \text{ N}) = -400 \text{ N}$$ ◀

$$(T_{AC})_z = \frac{2}{4.5}(900 \text{ N}) = +400 \text{ N}$$ ◀

6-10



B (0, 0, 2)
-) C (1, 2, 0)

$d_x = -1$, $d_y = -2$, $d_z = 2$

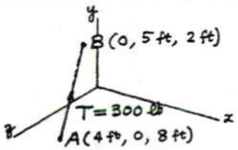
$$d = \sqrt{(1)^2 + (2)^2 + (2)^2} = 3$$

$$(T_{BC})_x = \frac{-1}{3}(600 \text{ N}) = -200 \text{ N}$$ ◀

$$(T_{BC})_y = \frac{-2}{3}(600 \text{ N}) = -400 \text{ N}$$ ◀

$$(T_{BC})_z = \frac{2}{3}(600 \text{ N}) = +400 \text{ N}$$ ◀

6-11



B (0, 5, 2)
-) A (4, 0, 8)

$d_x = -4$, $d_y = 5$, $d_z = -6$

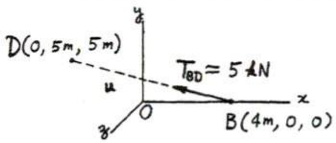
$$d = \sqrt{(4)^2 + (5)^2 + (6)^2} = 8.77 \text{ ft}$$

$$T_x = \frac{d_x}{d}T = \frac{-4}{8.77}(300 \text{ lb}) = -136.8 \text{ lb}$$ ◀

$$T_y = \frac{d_y}{d}T = \frac{5}{8.77}(300 \text{ lb}) = 171.0 \text{ lb}$$ ◀

$$T_z = \frac{d_z}{d}T = \frac{-6}{8.77}(300 \text{ lb}) = -205.2 \text{ lb}$$ ◀

6-12



D (0, 5, 5)
-) B (4, 0, 0)

$d_x = -4$, $d_y = 5$, $d_z = 5$

$$d = \sqrt{(4)^2 + (5)^2 + (5)^2} = 8.12 \text{ m}$$

$$(T_{BD})_x = \frac{-4}{8.12}(5 \text{ kN}) = -2.46 \text{ kN}$$ ◀

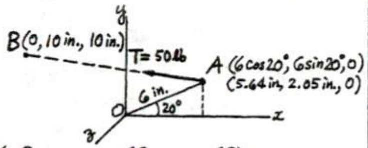
$$(T_{BD})_y = \frac{5}{8.12}(5 \text{ kN}) = 3.08 \text{ kN}$$ ◀

$$(T_{BD})_z = \frac{5}{8.12}(5 \text{ kN}) = 3.08 \text{ kN}$$ ◀

Check:

$$T_{BD} = \sqrt{(2.46)^2 + (3.08)^2 + (3.08)^2} = 5.00$$ (Checks)

6-13



B (0, 10, 10)
-) A (5.64, 2.05, 0)

$d_x = -5.64$, $d_y = 7.95$, $d_z = 10$

$$d = \sqrt{(5.64)^2 + (7.95)^2 + (10)^2} = 13.96 \text{ in.}$$

$$T_x = \frac{-5.64}{13.96}(50 \text{ lb}) = -20.2 \text{ lb}$$ ◀

$$T_y = \frac{7.95}{13.96}(50 \text{ lb}) = 28.5 \text{ lb}$$ ◀

$$T_z = \frac{10}{13.96}(50 \text{ lb}) = 35.8 \text{ lb}$$ ◀

Check:

$$T_{BD} = \sqrt{(20.2)^2 + (28.5)^2 + (35.8)^2} = 50.0$$ (Checks)

6-14

For force Q:

$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$\cos^2 120° + \cos^2 45° + \cos^2\theta_z = 1$

$\cos^2\theta_z = 0.250$

$\cos\theta_z = 0.500$ (P_z is positive)

$\theta_z = 60.0°$

$P_x = (400 \text{ lb}) \cos 120° = -200$ lb
$P_y = (400 \text{ lb}) \cos 45° = +282.8$ lb
$P_z = (400 \text{ lb}) \cos 60° = +200$ lb

Force Q is on the xy-plane:

$Q_x = (500 \text{ lb}) \cos 30° = 433.0$ lb
$Q_y = (500 \text{ lb}) \sin 30° = 250.0$ lb
$Q_z = 0$

$R_x = P_x + Q_x = -200 + 433.0 = 233$ lb ◀

$R_y = P_y + Q_y = 282.8 + 250.0 = 533$ lb ◀

$R_z = P_z + Q_z = 200 + 0 = 200$ lb ◀

6-15

Force P acts from A to B:

B (0, 0.3, 0.6)
-) A (0.6, 0, 0.4)

$d_x = -0.6$, $d_y = 0.3$, $d_z = 0.2$

$$d = \sqrt{(0.6)^2 + (0.3)^2 + (0.2)^2} = 0.7$$

$$P_x = \frac{-0.6}{0.7}(7 \text{ kN}) = -6 \text{ kN}$$

$$P_y = \frac{0.3}{0.7}(7 \text{ kN}) = 3 \text{ kN}$$

$$P_z = \frac{0.2}{0.7}(7 \text{ kN}) = 2 \text{ kN}$$

Force Q acts from A to C:

C (0.6, 0.3, 0)
-) A (0.6, 0, 0.4)

$d_x = 0$, $d_y = 0.3$, $d_z = -0.4$

$$d = \sqrt{(0)^2 + (0.3)^2 + (0.4)^2} = 0.5$$

$Q_x = 0$

$$Q_y = \frac{0.3}{0.5}(5 \text{ kN}) = 3 \text{ kN}$$

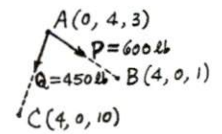
$$Q_z = \frac{-0.4}{0.5}(5 \text{ kN}) = -4 \text{ kN}$$

$R_x = P_x + Q_x = -6 + 0 = -6$ kN ◀

$R_y = P_y + Q_y = 3 + 3 = 6$ kN ◀

$R_z = P_z + Q_z = 2 - 4 = -2$ kN ◀

6-16



B (4,	0,	1)
-) A (0,	4,	3)
$d_x = 4$,	$d_y = -4$,	$d_z = -2$

$$d = \sqrt{(4)^2 + (4)^2 + (2)^2} = 6$$

$$P_x = \frac{4}{6}(600\text{ lb}) = +400\text{ lb}$$

$$P_y = \frac{-4}{6}(600\text{ lb}) = -400\text{ lb}$$

$$P_z = \frac{-2}{6}(600\text{ lb}) = -200\text{ lb}$$

C (4,	0,	10)
-) A (0,	4,	3)
$d_x = 4$,	$d_y = -4$,	$d_z = 7$

$$d = \sqrt{(4)^2 + (4)^2 + (7)^2} = 9$$

$$Q_x = \frac{+4}{9}(450\text{ lb}) = +200\text{ lb}$$

$$Q_y = \frac{-4}{9}(450\text{ lb}) = -200\text{ lb}$$

$$Q_z = \frac{+7}{9}(450\text{ lb}) = +350\text{ lb}$$

$R_x = P_x + Q_x = 400 + 200 = +600\text{ lb}$ ◀

$R_y = P_y + Q_y = -400 - 200 = -600\text{ lb}$ ◀

$R_z = P_z + Q_z = -200 + 350 = +150\text{ lb}$ ◀

6-17

Force P acts from A to B:

B (6,	2,	0)
-) A (0,	-6,	0)
$d_x = 6$,	$d_y = 8$,	$d_z = 0$

$$d = \sqrt{(6)^2 + (8)^2 + (0)^2} = 10$$

$$P_x = \frac{6}{10}(15\text{ kN}) = 9\text{ kN}$$

$$P_y = \frac{8}{10}(15\text{ kN}) = 12\text{ kN}$$

$$P_z = 0$$

Force Q acts from A to C:

C (-3,	0,	2)
-) A (0,	-6,	0)
$d_x = -3$,	$d_y = 6$,	$d_z = 2$

$$d = \sqrt{(3)^2 + (6)^2 + (2)^2} = 7$$

$$Q_x = \frac{-3}{7}(14\text{ kN}) = -6\text{ kN}$$

$$Q_y = \frac{6}{7}(14\text{ kN}) = 12\text{ kN}$$

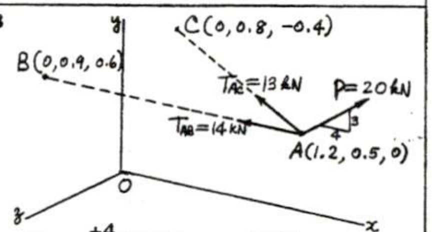
$$Q_z = \frac{2}{7}(14\text{ kN}) = 4\text{ kN}$$

$R_x = P_x + Q_x = 9 - 6 = 3\text{ kN}$ ◀

$R_y = P_y + Q_y = 12 + 12 = 24\text{ kN}$ ◀

$R_z = P_z + Q_z = 0 + 4 = 4\text{ kN}$ ◀

6-18



$$P_x = \frac{+4}{5}(20\text{ kN}) = +16\text{ kN}$$

$$P_y = \frac{+3}{5}(20\text{ kN}) = +12\text{ kN}$$

$$P_z = 0$$

B (0,	0.9,	0.6)
-) A (1.2,	0.5,	0)
$d_x = -1.2$,	$d_y = 0.4$,	$d_z = 0.6$

$$d = \sqrt{(1.2)^2 + (0.2)^2 + (0.6)^2} = 1.4$$

(Cont'd)

6-18 (Cont)

$$(T_{AB})_x = \frac{-1.2}{1.4}(14\text{ kN}) = -12\text{ kN}$$

$$(T_{AB})_y = \frac{0.4}{1.4}(14\text{ kN}) = +4\text{ kN}$$

$$(T_{AB})_z = \frac{0.6}{1.4}(14\text{ kN}) = +6\text{ kN}$$

C (0,	0.8,	-0.4)
-) A (1.2,	0.5,	0)
$d_x = -1.2$,	$d_y = 0.3$,	$d_z = -0.4$

$$d = \sqrt{(1.2)^2 + (0.3)^2 + (0.4)^2} = 1.3$$

$$(T_{AC})_x = \frac{-1.2}{1.3}(13\text{ kN}) = -12\text{ kN}$$

$$(T_{AC})_y = \frac{0.3}{1.3}(13\text{ kN}) = +3\text{ kN}$$

$$(T_{AC})_z = \frac{-0.4}{1.3}(13\text{ kN}) = -4\text{ kN}$$

$R_x = P_x + (T_{AB})_x + (T_{AC})_x = 16 - 12 - 12 = -8\text{ kN}$

$R_y = P_y + (T_{AB})_y + (T_{AC})_y = 12 + 4 + 3 = +19\text{ kN}$

$R_z = P_z + (T_{AB})_z + (T_{AC})_z = 0 + 6 - 4 = +2\text{ kN}$

$$R = \sqrt{(8)^2 + (19)^2 + (2)^2} = 20.7\text{ kN}$$ ◀

$$\theta_x = \cos^{-1}\left(\frac{-8}{20.7}\right) = 112.7^\circ$$ ◀

$$\theta_y = \cos^{-1}\left(\frac{+19}{20.7}\right) = 23.5^\circ$$ ◀

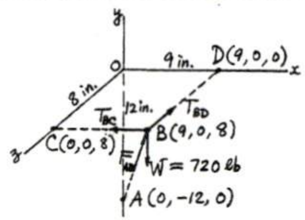
$$\theta_z = \cos^{-1}\left(\frac{+2}{20.7}\right) = 84.5^\circ$$ ◀

Check:

$\cos^2 112.7^\circ + \cos^2 23.5^\circ + \cos^2 84.5^\circ = 0.999$

(Checks)

6-19



B (9,	0,	8)
-) A (0,	-12,	0)
$d_x = 9$,	$d_y = 12$,	$d_z = 8$

$$d = \sqrt{(9)^2 + (12)^2 + (8)^2} = 17$$

$$(F_{AB})_x = \frac{9}{17}F_{AB} = 0.529\,F_{AB}$$

$$(F_{AB})_y = \frac{12}{17}F_{AB} = 0.706\,F_{AB}$$

$$(F_{AB})_z = \frac{8}{17}F_{AB} = 0.471\,F_{AB}$$

$(T_{BC})_x = -T_{BC}$

$(T_{BC})_y = (T_{BC})_z = 0$

$(T_{BD})_x = (T_{BD})_y = 0,$

$(T_{BD})_z = -T_{BD}$

$W_x = 0$

$W_y = -720\text{ lb}$

$W_z = 0$

$\sum F_x = 0.529\,F_{AB} - T_{BC} = 0$ (a)

$\sum F_y = 0.706\,F_{AB} - 720\text{ lb} = 0$ (b)

$\sum F_z = 0.471\,F_{AB} - T_{BD} = 0$ (c)

From Eq. (b)

$F_{AB} = +1020\text{ lb (C)}$ ◀

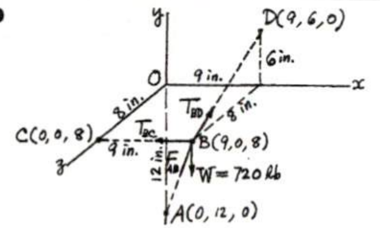
Substitute into Eq. (a)

$T_{BC} = +540\text{ lb (T)}$ ◀

Substitute into Eq. (c)

$T_{BD} = +480\text{ lb (T)}$ ◀

6-20



From Prob. 6-19

$(F_{AB})_x = 0.529\,F_{AB}$

$(F_{AB})_y = 0.706\,F_{AB}$

$(F_{AB})_z = 0.471\,F_{AB}$

(Cont'd)

6-20 (Cont)

$(T_{BC})_x = -T_{BC}$, $(T_{BC})_y = (T_{BC})_z = 0$

$W_x = W_z = 0$, $W_y = -720$ lb

$$\begin{array}{lccc} & D\ (9, & 6, & 0) \\ \text{-)} & B\ (9, & 0, & 8) \\ \hline & d_x = 0, & d_y = 6, & d_z = -8 \end{array}$$

$$d = \sqrt{(0)^2 + (6)^2 + (8)^2} = 10$$

$$(T_{BD})_x = 0$$
$$(T_{BD})_y = \frac{6}{10}T_{BD} = 0.6\,T_{BD}$$
$$(T_{BD})_z = \frac{-8}{10}T_{BD} = -0.8\,T_{BD}$$

$$\sum F_x = 0.529\,F_{AB} - T_{BC} = 0 \quad \text{(a)}$$
$$\sum F_y = 0.706\,F_{AB} + 0.6\,T_{BD} - 720\text{ lb} = 0 \quad \text{(b)}$$
$$\sum F_z = 0.471\,F_{AB} - 0.8\,T_{BD} = 0 \quad \text{(c)}$$

(b)/0.6 + (c)/0.8:

$$(0.706/0.6 + 0.471/0.8)\,F_{AB} - 720/0.6 = 0$$

$F_{AB} = 680$ lb (C) ◀

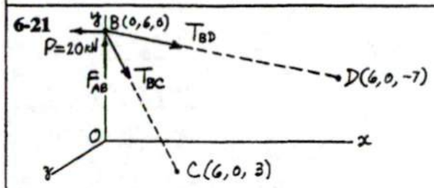
Substitute into (a):

$T_{BC} = 0.529(680) = 360$ lb (T) ◀

Substitute into (c):

$$T_{BD} = \left(\frac{1}{0.8}\right)(0.471)(+680\text{ lb}) = 400\text{ lb (T)}$$ ◀

6-21



$$\begin{array}{lccc} & C\ (6, & 0, & 3) \\ \text{-)} & B\ (0, & 6, & 0) \\ \hline & d_x = 6, & d_y = -6, & d_z = 3 \end{array}$$

$$d = \sqrt{(6)^2 + (6)^2 + (3)^2} = 9$$

$$(T_{BC})_x = \frac{6}{9}T_{BC} = 0.667\,T_{BC}$$
$$(T_{BC})_y = \frac{-6}{9}T_{BC} = -0.667\,T_{BC}$$
$$(T_{BC})_z = \frac{3}{9}T_{BC} = 0.333\,T_{BC}$$

$$\begin{array}{lccc} & D\ (6, & 0, & -7) \\ \text{-)} & B\ (0, & 6, & 0) \\ \hline & d_x = 6, & d_y = -6, & d_z = -7 \end{array}$$

$$d = \sqrt{(6)^2 + (6)^2 + (7)^2} = 11$$

$$(T_{BD})_x = \frac{6}{11}T_{BD} = 0.545\,T_{BD}$$
$$(T_{BD})_y = \frac{-6}{11}T_{BD} = -0.545\,T_{BD}$$
$$(T_{BD})_z = \frac{-7}{11}T_{BD} = -0.636\,T_{BD}$$

$(F_{AB})_x = (F_{AB})_z = 0$
$(F_{AB})_y = F_{AB}$

$P_x = -20$ kN
$P_y = P_z = 0$

$$\sum F_x = 0.667\,T_{BC} + 0.545\,T_{BD} - 20\text{ kN} = 0 \quad \text{(a)}$$
$$\sum F_y = -0.667\,T_{BC} - 0.545\,T_{BD} + F_{AB} = 0 \quad \text{(b)}$$
$$\sum F_z = 0.333\,T_{BC} - 636\,T_{BD} = 0 \quad \text{(c)}$$

(a) + (b): $F_{AB} - 20\text{ kN} = 0$

$F_{AB} = 20$ kN (C) ◀

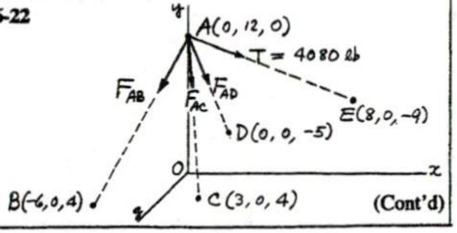
(a) - 2 x (c): $1.817\,T_{BD} - 20\text{ kN} = 0$

$T_{BD} = 11.0$ kN (T) ◀

Substitute into (c):

$T_{BC} = 21.0$ kN (T) ◀

6-22



(Cont'd)

6-22 (Cont)

$$\begin{array}{lccc} & B\ (-6, & 0, & 4) \\ \text{-)} & A\ (0, & 12, & 0) \\ \hline & d_x = -6, & d_y = -12, & d_z = 4 \end{array}$$

$$d = \sqrt{(6)^2 + (12)^2 + (4)^2} = 14$$

$$(F_{AB})_x = \frac{-6}{14}F_{AB} = -0.429\,F_{AB}$$
$$(F_{AB})_y = \frac{-12}{14}F_{AB} = -0.857\,F_{AB}$$
$$(F_{AB})_z = \frac{4}{14}F_{AB} = 0.286\,F_{AB}$$

$$\begin{array}{lccc} & C\ (3, & 0, & 4) \\ \text{-)} & A\ (0, & 12, & 0) \\ \hline & d_x = 3, & d_y = -12, & d_z = 4 \end{array}$$

$$d = \sqrt{(3)^2 + (12)^2 + (4)^2} = 13$$

$$(F_{AC})_x = \frac{3}{13}F_{AC} = 0.231\,F_{AC}$$
$$(F_{AC})_y = -\frac{12}{13}F_{AC} = -0.923\,F_{AC}$$
$$(F_{AC})_z = \frac{4}{13}F_{AC} = 0.308\,F_{AC}$$

$$\begin{array}{lccc} & D\ (0, & 0, & -5) \\ \text{-)} & A\ (0, & 12, & 0) \\ \hline & d_x = 0, & d_y = -12, & d_z = -5 \end{array}$$

$$d = \sqrt{(0)^2 + (12)^2 + (5)^2} = 13$$

$$(F_{AD})_x = 0$$
$$(F_{AD})_y = -\frac{12}{13}F_{AD} = -0.923\,F_{AD}$$
$$(F_{AD})_z = -\frac{5}{13}F_{AD} = -0.385\,F_{AD}$$

$$\begin{array}{lccc} & E\ (8, & 0, & -9) \\ \text{-)} & A\ (0, & 12, & 0) \\ \hline & d_x = 8, & d_y = -12, & d_z = -9 \end{array}$$

$$d = \sqrt{(8)^2 + (12)^2 + (9)^2} = 17$$

$$T_x = \frac{8}{17}(4080\text{ lb}) = +1920\text{ lb}$$
$$T_y = -\frac{12}{17}(4080\text{ lb}) = -2880\text{ lb}$$
$$T_z = -\frac{9}{17}(4080\text{ lb}) = -2160\text{ lb}$$

$$\sum F_x = -0.429\,F_{AB} + 0.231\,F_{AC} + 1920\text{ lb} = 0 \quad \text{(a)}$$
$$\sum F_y = -0.857\,F_{AB} - 0.923\,F_{AC} - 0.923\,F_{AD} - 2880\text{ lb} = 0 \quad \text{(b)}$$
$$\sum F_z = -0.286\,F_{AB} + 0.308\,F_{AC} - 0.385\,F_{AD} - 2160\text{ lb} = 0 \quad \text{(c)}$$

(b)/0.923 - (c)/0.385:

$$-1.671\,F_{AB} - 1.800\,F_{AC} + 2490\text{ lb} \quad \text{(d)}$$

(a)/0.231 + (d)/1.800:

$$-2.785\,F_{AB} + 9695\text{ lb} = 0$$

$F_{AB} = +3480$ lb (T) ◀

Substituting into (a):

$$0.231\,F_{AC} = 0.429(3480) - 1920 = 427$$

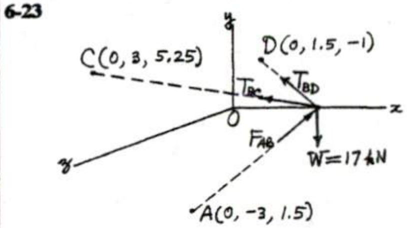
$F_{AC} = -1850$ lb (C) ◀

Substituting into (b):

$$0.923\,F_{AD} = -0.857(3480) - 0.923(-1850) - 2880 = -4155$$

$F_{AD} = -4500$ lb (C) ◀

6-23



$$\begin{array}{lccc} & C\ (0, & 3, & 5.25) \\ \text{-)} & B\ (3, & 0, & 0) \\ \hline & d_x = -3, & d_y = 3, & d_z = 5.25 \end{array}$$

$$d = \sqrt{(3)^2 + (3)^2 + (5.25)^2} = 6.75$$

$$(T_{BC})_x = -\frac{3}{6.75}T_{BC} = -0.444\,T_{BC}$$
$$(T_{BC})_y = \frac{3}{6.75}T_{BC} = 0.444\,T_{BC}$$
$$(T_{BC})_z = \frac{5.25}{6.75}T_{BC} = 0.778\,T_{BC}$$

(Cont'd)

6-23 (Cont)

D (0, 1.5, -1)
-) B (3, 0, 0)
$d_x = -3,\quad d_y = 1.5,\quad d_z = -1$

$$d = \sqrt{(3)^2 + (1.5)^2 + (1)^2} = 3.5$$

$$(T_{BD})_x = \frac{-3}{3.5}T_{BD} = -0.857\,T_{BD}$$
$$(T_{BD})_y = \frac{1.5}{3.5}T_{BD} = 0.429\,T_{BD}$$
$$(T_{BD})_z = \frac{-1}{3.5}T_{BD} = -0.286\,T_{BD}$$

B (3, 0, 0)
-) A (0, -3, 1.5)
$d_x = 3,\quad d_y = 3,\quad d_z = -1.5$

$$d = \sqrt{(3)^2 + (3)^2 + (1.5)^2} = 4.5$$

$$(F_{AB})_x = \frac{3}{4.5}F_{AB} = 0.667\,F_{AB}$$
$$(F_{AB})_y = \frac{3}{4.5}F_{AB} = 0.667\,F_{AB}$$
$$(F_{AB})_z = \frac{-1.5}{4.5}F_{AB} = -0.333\,F_{AB}$$

$W_x = W_z = 0$
$W_y = -17$ kN

$\sum F_x = -0.444\,T_{BC} - 0.857\,T_{BD} + 0.667\,F_{AB} = 0$ (a)
$\sum F_y = 0.444\,T_{BC} + 0.429\,T_{BD} + 0.667\,F_{AB} - 17 = 0$ (b)
$\sum F_z = 0.778\,T_{BC} - 0.286\,T_{BD} - 0.333\,F_{AB} = 0$ (c)

(b)-(a): $0.888\,T_{BC} + 1.286\,T_{BD} - 17 = 0$ (d)

(a) + 2 x (c): $1.112\,T_{BC} - 1.429\,T_{BD} = 0$ (e)

(d)/1.286 + (e)/1.429: $1.469\,T_{BC} - 13.22 = 0$

$T_{BC} = 9.00$ kN (T) ◄

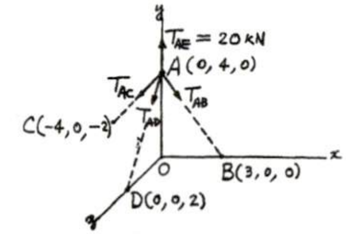
Substituting into (e):

$T_{BD} = 7.00$ kN (T) ◄

Substituting into (a):

$F_{AB} = 15.00$ kN (C) ◄

6-24



B (3, 0, 0)
-) A (0, 4, 0)
$d_x = 3,\quad d_y = -4,\quad d_z = 0$

$$d = \sqrt{(3)^2 + (4)^2 + (0)^2} = 5$$

$$(T_{AB})_x = \frac{3}{5}T_{AB} = 0.6\,T_{AB}$$
$$(T_{AB})_y = -\frac{4}{5}T_{AB} = -0.8\,T_{AB}$$
$$(T_{AB})_z = 0$$

C (-4, 0, -2)
-) A (0, 4, 0)
$d_x = -4,\quad d_y = -4,\quad d_z = -2$

$$d = \sqrt{(4)^2 + (4)^2 + (2)^2} = 6$$

$$(T_{AC})_x = -\frac{4}{6}T_{AC} = -0.667\,T_{AC}$$
$$(T_{AC})_y = -\frac{4}{6}T_{AC} = -0.667\,T_{AC}$$
$$(T_{AC})_z = -\frac{2}{6}T_{AC} = -0.333\,T_{AC}$$

D (0, 0, 2)
-) A (0, 4, 0)
$d_x = 0,\quad d_y = -4,\quad d_z = 2$

$$d = \sqrt{(0)^2 + (4)^2 + (2)^2} = \sqrt{20} = 2\sqrt{5}$$

$$(T_{AD})_x = 0$$
$$(T_{AD})_y = -\frac{4}{2\sqrt{5}}T_{AD} = -0.894\,T_{AD}$$
$$(T_{AD})_z = \frac{2}{2\sqrt{5}}T_{AD} = 0.447\,T_{AD}$$

(Cont'd)

6-24 (Cont)

$(T_{AE})_x = (T_{AE})_z = 0$
$(T_{AE})_y = 20$ kN

$\sum F_x = 0.6\,T_{AB} - 0.667\,T_{AC} = 0$ (a)
$\sum F_y = -0.8\,T_{AB} - 0.667\,T_{AC} - 0.894\,T_{AD} + 20\text{ kN} = 0$ (b)
$\sum F_z = -0.333\,T_{AC} + 0.447\,T_{AD} = 0$ (c)

(b) + 2 x (c): $-0.8\,T_{AB} - 1.333\,T_{AC} + 20\text{ kN} = 0$ (d)

2 x (a) - (d): $2.00\,T_{AB} - 20\text{ kN} = 0$

$T_{AB} = 10$ kN (T) ◄

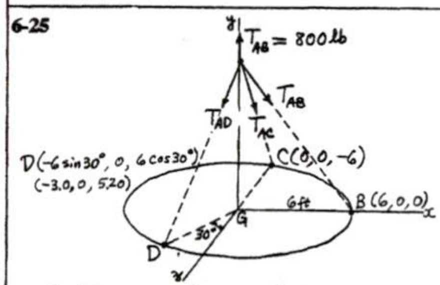
Substituting into (a):

$T_{AC} = 9.00$ kN (T) ◄

Substituting into (c):

$T_{AD} = 6.70$ kN (T) ◄

6-25



B (6, 0, 0)
-) A (0, 8, 0)
$d_x = 6,\quad d_y = -8,\quad d_z = 0$

$$d = \sqrt{(6)^2 + (8)^2 + (0)^2} = 10$$

$$(T_{AB})_x = \frac{6}{10}T_{AB} = 0.6\,T_{AB}$$
$$(T_{AB})_y = \frac{-8}{10}T_{AB} = -0.8\,T_{AB}$$
$$(T_{AB})_z = 0$$

C (0, 0, -6)
-) A (0, 8, 0)
$d_x = 0,\quad d_y = -8,\quad d_z = -6$

$$d = \sqrt{(0)^2 + (8)^2 + (6)^2} = 10$$

$$(T_{AC})_x = 0$$
$$(T_{AC})_y = \frac{-8}{10}T_{AC} = -0.8\,T_{AC}$$
$$(T_{AC})_z = \frac{-6}{10}T_{AC} = -0.6\,T_{AC}$$

D (-3, 0, 5.20)
-) A (0, 8, 0)
$d_x = -3,\quad d_y = -8,\quad d_z = 5.20$

$$d = \sqrt{(3)^2 + (8)^2 + (5.20)^2} = 10.00$$

$$(T_{AD})_x = -0.3\,T_{AD}$$
$$(T_{AD})_y = -0.8\,T_{AD}$$
$$(T_{AD})_z = 0.52\,T_{AD}$$

$(T_{AE})_x = (T_{AE})_z = 0$
$(T_{AE})_y = 800$ lb

$\sum F_x = 0.6\,T_{AB} - 0.3\,T_{AD} = 0$ (a)
$\sum F_y = -0.8\,T_{AB} - 0.8\,T_{AC} - 0.8\,T_{AD} + 800 = 0$ (b)
$\sum F_z = -0.6\,T_{AC} + 0.52\,T_{AD} = 0$ (c)

0.8 x (a) + 0.6 x (b):
$-0.48\,T_{AC} - 0.72\,T_{AD} + 480 = 0$ (d)

(c)/0.6 - (d)/0.48:
$2.367\,T_{AD} - 1000 = 0$

$T_{AD} = 422.5$ lb (T) ◄

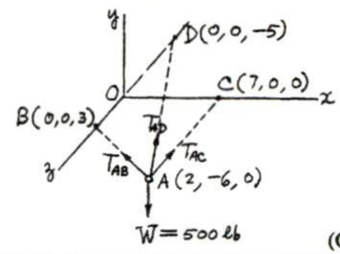
Substituting into (c):

$T_{AC} = 366.2$ lb (T) ◄

Substituting into (a):

$T_{AB} = 211.3$ lb (T) ◄

6-26



(Cont'd)

6-26 (Cont)

$$\begin{array}{llll} & B & (0, & 0, & 3) \\ -) & A & (2, & -6, & 0) \\ \hline & & d_x=-2, & d_y=6, & d_z=3 \end{array}$$

$$d = \sqrt{(2)^2 + (6)^2 + (3)^2} = 7$$

$$(T_{AB})_x = \frac{-2}{7}T_{AB} = -0.286\,T_{AB}$$

$$(T_{AB})_y = \frac{6}{7}T_{AB} = 0.857\,T_{AB}$$

$$(T_{AB})_z = \frac{3}{7}T_{AB} = 0.429\,T_{AB}$$

$$\begin{array}{llll} & C & (7, & 0, & 0) \\ -) & A & (2, & -6, & 0) \\ \hline & & d_x=5, & d_y=6, & d_z=0 \end{array}$$

$$d = \sqrt{(5)^2 + (6)^2 + (0)^2} = 7.81$$

$$(T_{AC})_x = \frac{5}{7.81}T_{AC} = 0.640\,T_{AC}$$

$$(T_{AC})_y = \frac{6}{7.81}T_{AC} = 0.768\,T_{AC}$$

$$(T_{AC})_z = 0$$

$$\begin{array}{llll} & D & (0, & 0, & -5) \\ -) & A & (2, & -6, & 0) \\ \hline & & d_x=-2, & d_y=6, & d_z=-5 \end{array}$$

$$d = \sqrt{(2)^2 + (6)^2 + (5)^2} = 8.06$$

$$(T_{AD})_x = \frac{-2}{8.06}T_{AD} = -0.248\,T_{AD}$$

$$(T_{AD})_y = \frac{6}{8.06}T_{AD} = 0.744\,T_{AD}$$

$$(T_{AD})_z = \frac{-5}{8.06}T_{AD} = -0.620\,T_{AD}$$

$W_x = W_z = 0$
$W_y = -500$ lb

$$\sum F_x = -0.286\,T_{AB} + 0.640\,T_{AC} - 0.248\,T_{AD} = 0 \quad (a)$$
$$\sum F_y = 0.857\,T_{AB} + 0.786\,T_{AC} + 0.744\,T_{AD} - 500 = 0 \quad (b)$$
$$\sum F_z = 0.429\,T_{AB} - 0.620\,T_{AD} = 0 \quad (c)$$

(a)/0.640 - (b)/0.768:
$$-1.563\,T_{AB} - 1.356\,T_{AD} + 651\text{ lb} = 0 \quad (d)$$

(c)/0.620 - (d)/1.356:
$$1.845\,T_{AB} - 480.1\text{ lb} = 0$$

$$T_{AB} = 260.2\text{ lb (T)}$$ ◀

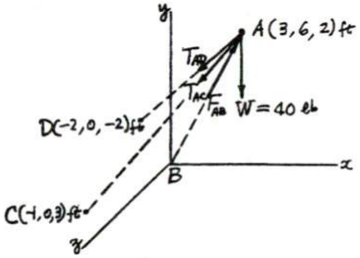
Substituting into (c):

$$T_{AD} = 180.0\text{ lb (T)}$$ ◀

Substituting into (a):

$$T_{AC} = 186.0\text{ lb (T)}$$ ◀

6-27



$$\begin{array}{llll} & A & (3, & 6, & 2) \\ -) & B & (0, & 0, & 0) \\ \hline & & d_x=3, & d_y=6, & d_z=2 \end{array}$$

$$d = \sqrt{(3)^2 + (6)^2 + (2)^2} = 7$$

$$(F_{AB})_x = \frac{3}{7}F_{AB} = 0.429\,F_{AB}$$

$$(F_{AB})_y = \frac{6}{7}F_{AB} = 0.857\,F_{AB}$$

$$(F_{AB})_z = \frac{2}{7}F_{AB} = 0.286\,F_{AB}$$

$$\begin{array}{llll} & C & (-1, & 0, & 3) \\ -) & A & (3, & 6, & 2) \\ \hline & & d_x=-4, & d_y=-6, & d_z=1 \end{array}$$

$$d = \sqrt{(4)^2 + (6)^2 + (1)^2} = 7.28$$

$$(T_{AC})_x = \frac{-4}{7.28}T_{AC} = -0.549\,T_{AC}$$

$$(T_{AC})_y = \frac{-6}{7.28}T_{AC} = -0.824\,T_{AC}$$

$$(T_{AC})_z = \frac{1}{7.28}T_{AC} = 0.137\,T_{AC}$$

(Cont'd)

6-27 (Cont)

$$\begin{array}{llll} & D & (-2, & 0, & -2) \\ -) & A & (3, & 6, & 2) \\ \hline & & d_x=-5, & d_y=-6, & d_z=-4 \end{array}$$

$$d = \sqrt{(5)^2 + (6)^2 + (4)^2} = 8.77$$

$$(T_{AD})_x = \frac{-5}{8.77}T_{AD} = -0.570\,T_{AD}$$

$$(T_{AD})_y = \frac{-6}{8.77}T_{AD} = -0.684\,T_{AD}$$

$$(T_{AD})_z = \frac{-4}{8.77}T_{AD} = -0.456\,T_{AD}$$

$W_x = W_z = 0$
$W_y = -40$ lb

$$\sum F_x = 0.429\,F_{AB} - 0.549\,T_{AC} - 0.570\,T_{AD} = 0 \quad (a)$$
$$\sum F_y = 0.857\,F_{AB} - 0.824\,T_{AC} - 0.684\,T_{AD} - 40 = 0 \quad (b)$$
$$\sum F_z = 0.286\,F_{AB} + 0.137\,T_{AC} - 0.456\,T_{AD} = 0 \quad (c)$$

(a)/0.570 - (b)/0.684:
$$-0.500\,F_{AB} + 0.242\,T_{AC} + 58.5 = 0 \quad (d)$$

(a)/0.570 - (c)/0.456:
$$0.125\,F_{AB} - 1.264\,T_{AC} = 0 \quad (e)$$

(d)/0.242 + (e)/1.264:
$$-1.967\,F_{AB} + 242 = 0$$

$$F_{AB} = 123\text{ lb (C)}$$ ◀

Substituting into (e):

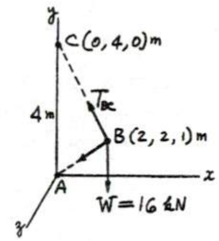
$$T_{AC} = 12.2\text{ lb (T)}$$ ◀

Substituting into (a):

$$T_{AD} = 80.8\text{ lb (T)}$$ ◀

6-28

Joint B



$W_x = W_z = 0$
$W_y = -16$ kN

$$\begin{array}{llll} & B & (2, & 2, & 1) \\ -) & A & (0, & 0, & 0) \\ \hline & & d_x=2, & d_y=2, & d_z=1 \end{array}$$

$$d = \sqrt{(2)^2 + (2)^2 + (1)^2} = 3$$

$$(F_{AB})_x = \frac{2}{3}F_{AB} = 0.667\,F_{AB}$$

$$(F_{AB})_y = \frac{2}{3}F_{AB} = 0.667\,F_{AB}$$

$$(F_{AB})_z = \frac{1}{3}F_{AB} = 0.333\,F_{AB}$$

$$\begin{array}{llll} & C & (0, & 4, & 0) \\ -) & B & (2, & 2, & 1) \\ \hline & & d_x=-2, & d_y=2, & d_z=-1 \end{array}$$

$$d = \sqrt{(2)^2 + (2)^2 + (1)^2} = 3$$

$$(T_{BC})_x = -\frac{2}{3}T_{BC} = -0.667\,T_{BC}$$

$$(T_{BC})_y = \frac{2}{3}T_{BC} = 0.667\,T_{BC}$$

$$(T_{BC})_z = -\frac{1}{3}T_{BC} = -0.333\,T_{BC}$$

$$\sum F_x = 0.667\,F_{AB} - 0.667\,T_{BC} = 0 \quad (a)$$
$$\sum F_y = 0.667\,F_{AB} + 0.667\,T_{BC} - 16 = 0 \quad (b)$$
$$\sum F_z = 0.333\,F_{AB} - 0.333\,T_{BC} = 0 \quad (c)$$

Equations (a) and (c) are independent, since (a) = 2(c). But we have only two unknowns, so two independent equations are enough to solve for two unknowns.

(a) + (b): $1.333\,F_{AB} - 16 = 0$

$$F_{AB} = 12.0\text{ kN (C)}$$ ◀

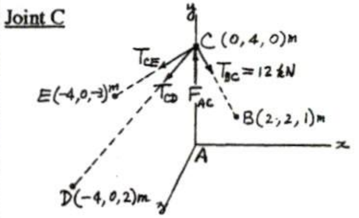
Substituting into (a):

$$T_{BC} = 12.0\text{ kN (T)}$$ ◀

(Cont'd)

6-28 (Cont)

Joint C



$(F_{AC})_x = (F_{AC})_z = 0,$
$(F_{AC})_y = F_{AC}$

	D (-4,	0,	2)
-)	C (0,	4,	0)
	$d_x = -4$,	$d_y = -4$,	$d_z = 2$

$$d = \sqrt{(4)^2 + (4)^2 + (2)^2} = 6$$

$$(T_{CD})_x = -\frac{4}{6} T_{CD} = -0.667\, T_{CD}$$

$$(T_{CD})_y = -\frac{4}{6} T_{CD} = -0.667\, T_{CD}$$

$$(T_{CD})_z = \frac{2}{6} T_{CD} = 0.333\, T_{CD}$$

	E (-4,	0,	-3)
-)	C (0,	4,	0)
	$d_x = -4$,	$d_y = -4$,	$d_z = -3$

$$d = \sqrt{(4)^2 + (4)^2 + (3)^2} = \sqrt{41}$$

$$(T_{CE})_x = -\frac{4}{\sqrt{41}} T_{CE} = -0.625\, T_{CE}$$

$$(T_{CE})_y = -\frac{4}{\sqrt{41}} T_{CE} = -0.625\, T_{CE}$$

$$(T_{CE})_z = -\frac{3}{\sqrt{41}} T_{CE} = -0.469\, T_{CE}$$

	B (2,	2,	1)
-)	C (0,	4,	0)
	$d_x = 2$,	$d_y = -2$,	$d_z = 1$

$$d = \sqrt{(2)^2 + (2)^2 + (1)^2} = 3$$

$$(T_{BC})_x = \frac{2}{3} T_{BC} = \frac{2}{3}(12 \text{ kN}) = 8 \text{ kN}$$

$$(T_{BC})_y = -\frac{2}{3} T_{BC} = -\frac{2}{3}(12 \text{ kN}) = -8 \text{ kN}$$

$$(T_{BC})_z = \frac{1}{3} T_{BC} = \frac{1}{3}(12 \text{ kN}) = 4 \text{ kN}$$

$$\sum F_x = -0.667\, T_{CD} - 0.625\, T_{CE} + 8 = 0 \qquad \text{(a)}$$

$$\sum F_y = -0.667\, T_{CD} - 0.625\, T_{CE} + F_{AC} - 8 = 0 \qquad \text{(b)}$$

$$\sum F_z = 0.333\, T_{CD} - 0.469\, T_{CE} + 4 = 0 \qquad \text{(c)}$$

(a) + 2(c): $-1.563\, T_{CE} + 16 = 0$

$T_{CE} = 10.24$ kN (T) ◀

Substitute in (a):

$T_{CD} = 2.40$ kN (T) ◀

Substitute in (b):

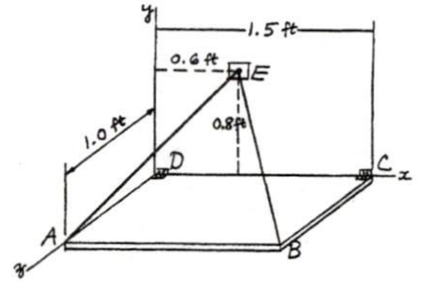
$F_{AC} = 16.0$ kN (C) ◀

Test Problems for Chapter 6

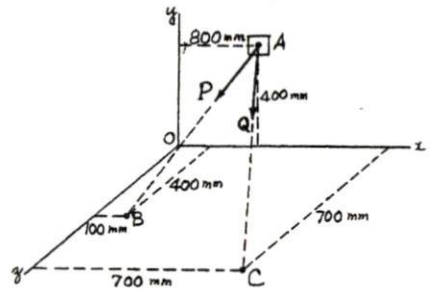
The following problems may either be given to students for them to practice their problem solving skills or be used as test problems. The answers are provided on the next page.

(1) A force acts at the origin in a direction defined by the angles $\theta_x = 125°$ and $\theta_y = 50°$. Knowing that the y component of the force is $F_z = +500$ N, determine (a) the value of θ_y, (b) the components F_x and F_y.

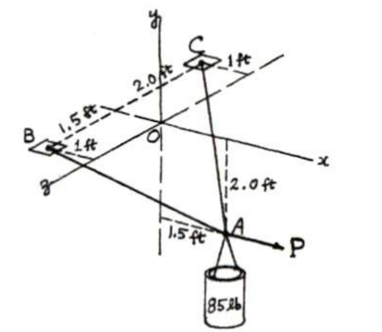
(2) Knowing that the tension in cable BE is 313 lb, determine the components of the force exerted on the platform at B.



(3) Given that P is 600 N and Q is 900 N, determine the magnitude and the direction angle of the resultant of the two forces exerted on point A.



(4) A container of weight 85 lb is supported by cables AB and AC as shown. Determine the magnitude of the force P along the x-axis that must be applied at A to maintain the container in the position shown.



Solutions to Test Problems for Chapter 6

(1)

(a)

$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$\cos^2 125° + \cos^2 50° + \cos^2 \theta_z = 1$

$\cos^2\theta_z = 0.258$

$\cos\theta_z = 0.508$ (neg. value dropped because F_z is +)

$\cos\theta_z = 59.5°$ ◄

(b)

$F_z = +500\text{ N} = F\cos 59.5°$

$F = \dfrac{500\text{ N}}{\cos 59.5°} = 985\text{ N}$

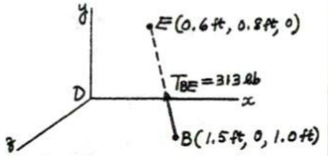
$F_x = F\cos\theta_x = (985\text{ N})\cos 125°$

$= -565\text{ N}$ ◄

$F_y = F\cos\theta_y = (985\text{ N})\cos 50°$

$= +633\text{ N}$ ◄

(2)



$d_x = 0.6\text{ ft} - 1.5\text{ ft} = -0.9\text{ ft}$
$d_y = 0.8\text{ ft} - 0 = 0.8\text{ ft}$
$d_z = 0 - 1.0\text{ ft} = -1.0\text{ ft}$

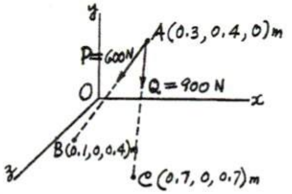
$d = \sqrt{(0.9\text{ ft})^2 + (0.8\text{ ft})^2 + (1.0\text{ ft})^2} = 1.565\text{ ft}$

$(T_{BE})_x = \dfrac{d_x}{d}T_{BE} = \dfrac{-0.9\text{ ft}}{1.565\text{ ft}}(313\text{ ft}) = -180\text{ ft}$ ◄

$(T_{BE})_y = \dfrac{d_y}{d}T_{BE} = \dfrac{0.8\text{ ft}}{1.565\text{ ft}}(313\text{ ft}) = +160\text{ ft}$ ◄

$(T_{BE})_z = \dfrac{d_z}{d}T_{BE} = \dfrac{-1.0\text{ ft}}{1.565\text{ ft}}(313\text{ ft}) = -200\text{ ft}$ ◄

(3)



$(d_{AB})_x = -0.2\text{ m}$
$(d_{AB})_y = -0.4\text{ m}$
$(d_{AB})_z = +0.4\text{ m}$

$d_{AB} = \sqrt{(0.2\text{ m})^2 + (0.4\text{ m})^2 + (0.4\text{ m})^2} = 0.6\text{ m}$

$P_x = \dfrac{(d_{AB})_x}{d_{AB}}P = \dfrac{-0.2\text{ m}}{0.6\text{ m}}(600\text{ N}) = -200\text{ N}$

$P_y = \dfrac{(d_{AB})_y}{d_{AB}}P = \dfrac{-0.4\text{ m}}{0.6\text{ m}}(600\text{ N}) = -400\text{ N}$

$P_z = \dfrac{(d_{AB})_z}{d_{AB}}P = \dfrac{0.4\text{ m}}{0.6\text{ m}}(600\text{ N}) = +400\text{ N}$

$(d_{AC})_x = +0.4\text{ m}$
$(d_{AC})_y = -0.4\text{ m}$
$(d_{AC})_z = +0.7\text{ m}$

$d_{AC} = \sqrt{(0.4\text{ m})^2 + (0.4\text{ m})^2 + (0.7\text{ m})^2} = 0.9\text{ m}$

$Q_x = \dfrac{(d_{AC})_x}{d_{AC}}Q = \dfrac{+0.4\text{ m}}{0.9\text{ m}}(900\text{ N}) = +400\text{ N}$

$Q_y = \dfrac{(d_{AC})_y}{d_{AC}}Q = \dfrac{-0.4\text{ m}}{0.9\text{ m}}(900\text{ N}) = -400\text{ N}$

$Q_z = \dfrac{(d_{AC})_z}{d_{AC}}Q = \dfrac{+0.7\text{ m}}{0.9\text{ m}}(900\text{ N}) = +700\text{ N}$

$R_x = P_x + Q_x = +200\text{ N}$
$R_y = P_y + Q_y = -800\text{ N}$
$R_z = P_z + Q_z = +1100\text{ N}$

$R = \sqrt{(200\text{ N})^2 + (800\text{ N})^2 + (1100\text{ N})^2}$

$= 1375\text{ N}$ ◄

(Cont'd)

Solutions to Test Problems for Chapter 6 (Cont'd)

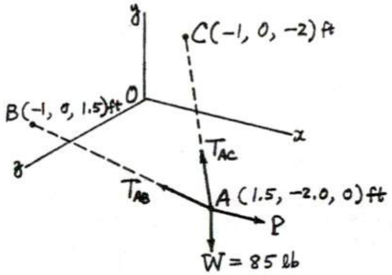
(3) (Cont)

$\theta_x = \cos^{-1}\dfrac{R_x}{R} = \cos^{-1}\dfrac{200}{1375} = 81.6°$ ◄

$\theta_y = \cos^{-1}\dfrac{R_y}{R} = \cos^{-1}\dfrac{-800}{1375} = 125.6°$ ◄

$\theta_z = \cos^{-1}\dfrac{R_z}{R} = \cos^{-1}\dfrac{1100}{1375} = 36.9°$ ◄

(4)



$(d_{AB})_x = -2.5\text{ ft}$
$(d_{AB})_y = 2.0\text{ ft}$
$(d_{AB})_z = 1.5\text{ ft}$

$d_{AB} = \sqrt{(2.5\text{ ft})^2 + (2.0\text{ ft})^2 + (1.5\text{ ft})^2} = 3.54\text{ ft}$

$(T_{AB})_x = \dfrac{(d_{AB})_x}{d_{AB}}T_{AB} = \dfrac{-2.5\text{ ft}}{3.54\text{ ft}}(T_{AB}) = -7.06\,T_{AB}$

$(T_{AB})_y = \dfrac{(d_{AB})_y}{d_{AB}}T_{AB} = \dfrac{2.0\text{ ft}}{3.54\text{ ft}}(T_{AB}) = 0.565\,T_{AB}$

$(T_{AB})_z = \dfrac{(d_{AB})_z}{d_{AB}}T_{AB} = \dfrac{1.5\text{ ft}}{3.54\text{ ft}}(T_{AB}) = 0.424\,T_{AB}$

$(d_{AC})_x = -2.5\text{ ft}$
$(d_{AC})_y = 2.0\text{ ft}$
$(d_{AC})_z = -2.0\text{ ft}$

$d_{AC} = \sqrt{(2.5\text{ ft})^2 + (2.0\text{ ft})^2 + (2\text{ ft})^2} = 3.77\text{ ft}$

$(T_{AC})_x = \dfrac{(d_{AC})_x}{d_{AC}}T_{AC} = \dfrac{-2.5\text{ ft}}{3.77\text{ ft}}(T_{AC}) = -0.663\,T_{AC}$

$(T_{AC})_y = \dfrac{(d_{AC})_y}{d_{AC}}T_{AC} = \dfrac{2.0\text{ ft}}{3.77\text{ ft}}(T_{AC}) = 0.531\,T_{AC}$

$(T_{AC})_z = \dfrac{(d_{AC})_z}{d_{AC}}T_{AC} = \dfrac{-2.0\text{ ft}}{3.77\text{ ft}}(T_{AC}) = -0.531\,T_{AC}$

$P_x = P$
$P_y = 0$
$P_z = 0$

$W_x = 0$
$W_y = -85\text{ lb}$
$W_z = 0$

$\Sigma F_x = -0.706\,T_{AB} - 0.663\,T_{AC} + P = 0$ (a)

$\Sigma F_y = -0.565\,T_{AB} + 0.531\,T_{AC} - 85 = 0$ (b)

$\Sigma F_z = 0.424\,T_{AB} - 0.531\,T_{AC} = 0$ (c)

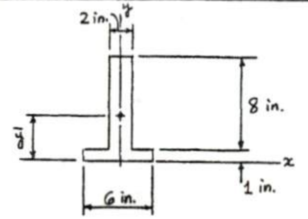
(b) + (c): $0.989\,T_{AB} - 85 = 0$
$T_{AB} = 85.9\text{ lb}$

Substitute into (c):
$T_{AC} = 0.424(85.9)/0.531 = 68.6\text{ lb}$

Substitute into (a):
$P = 0.706(85.9\text{ lb}) + 0.663(68.6\text{ lb})$

$= 106\text{ lb}$ ◄

7-1



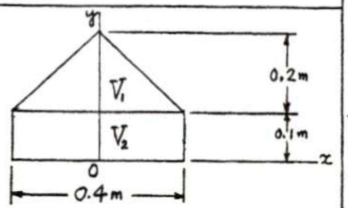
$V_1 = \pi(1)^2(8) = 25.1 \text{ in.}^3$

$V_2 = \pi(3)^2(1) = 28.3 \text{ in.}^3$

$$\bar{y} = \frac{V_1 y_1 + V_2 y_2}{V_1 + V_2} = \frac{(25.1)(5) + (28.3)(0.5)}{25.1 + 28.3}$$

$= 2.62$ in. ◀

7-2



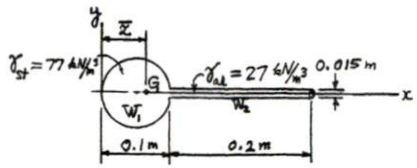
$$V_1 = \frac{1}{3}[\pi(0.2)^2](0.2) = 0.00838 \text{ m}^3$$

$V_2 = \pi(0.2)^2(0.1) = 0.01257 \text{ m}^3$

$$\bar{y} = \frac{V_1 y_1 + V_2 y_2}{V_1 + V_2} = \frac{(0.00838)\left[0.1 + \frac{1}{4}(0.2)\right] + (0.01257)(0.05)}{0.00838 + 0.01257}$$

$= 0.09$ m ◀

7-3



$$W_1 = \frac{4}{3}\pi r^3 \gamma_{st} = \frac{4}{3}\pi(0.05 \text{ m})^3(77 \text{ kN/m}^3)$$

$= 0.0403$ kN

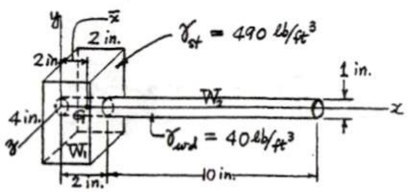
$$W_2 = \pi r^2 \gamma_{al} = \pi(0.0075 \text{ m})^2(0.2 \text{ m})(27 \text{ kN/m}^3)$$

$= 0.000954$ kN

$$\bar{x} = \frac{W_1 x_1 + W_2 x_2}{W_1 + W_2} = \frac{(0.0403)(0.05) + (0.000954)(0.2)}{0.0403 + 0.000954}$$

$= 0.0535 \text{ m} = 53.5$ mm ◀

7-4



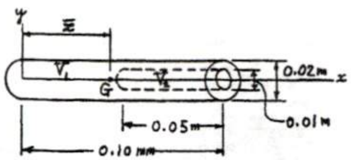
$$W_1 = \left[\frac{(2)(2)(4) - \pi(0.5)^2(2)}{12^3}\right](490) = 4.09 \text{ lb}$$

$$W_2 = \frac{\pi(0.5)^2(12)}{12^3}(40) = 0.218 \text{ lb}$$

$$\bar{x} = \frac{W_1 x_1 + W_2 x_2}{W_1 + W_2} = \frac{(4.09)(1) + (0.218)(6)}{4.09 + 0.218}$$

$= 1.25$ in. ◀

7-5



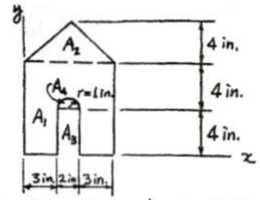
$$V_1 = \frac{\pi}{4}(0.02)^2(0.1) = 3.14 \times 10^{-5} \text{ m}^3$$

$$V_2 = -\frac{\pi}{4}(0.01)^2(0.05) = -3.93 \times 10^{-6} \text{ m}^3$$

$$\bar{x} = \frac{V_1 x_1 + V_2 x_2}{V_1 + V_2} = \frac{(3.14 \times 10^{-5})(0.05) + (-3.93 \times 10^{-6})(0.075)}{3.14 \times 10^{-5} + (-3.93 \times 10^{-6})}$$

$= 0.0464 \text{ m} = 46.4$ m ◀

7-6



Area	A (in.²)	y (in.)	Ay (in.³)
A_1	64.0	4.0	256.0
A_2	16.0	9.33	149.3
A_3	-8.0	2.0	-16.0
A_4	-1.57	4.42	-6.94
Σ	70.43		382.4

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{382.34}{70.43} = 5.43 \text{ in.}$$

The area is symmetrical with respect to the vertical line x = 4 in., hence

$\bar{x} = 4$ in.

Since the thickness is uniform, the center of gravity must be located at the mid-thickness, hence

$\bar{z} = 0.5$ in.

Therefore the center of gravity is located at the point

G (4, 5.43, 0.5) in. ◀

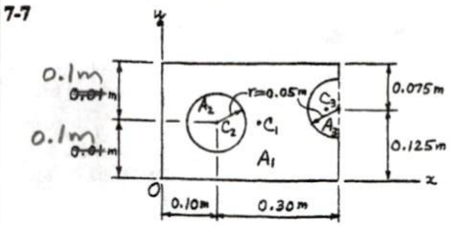
The weight of the block is

$W = \gamma V = \gamma(\Sigma A)t$

$$= (55 \text{ lb/ft}^3)(70.43 \text{ in.} \times 1 \text{ in.})\left(\frac{1 \text{ ft}^3}{1728 \text{ in.}^3}\right)$$

$= 2.24$ lb ◀

7-7



Area	A (in.²)	x (in.)	Ax (in.³)	y (in.)	Ay (in.³)
A_1	0.08	0.2	0.016	0.1	0.008
A_2	-0.00785	0.1	-0.000785	0.1	-0.000785
A_3	-0.00393	0.379	-0.00149	0.125	-0.000491
Σ	0.0682		0.0137		0.00672

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{0.0137}{0.0682} = 0.201 \text{ m}$$

$= 201$ m ◀

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{0.00672}{0.0682} = 0.0985 \text{ m}$$

$= 98.5$ mm ◀

For uniform thickness:

$\bar{z} = t/2 = 20$ mm ◀

7-8

$W = 2040 + 1980 = 4020$ lb

$$\bar{x} = \frac{W_B}{W}b = \frac{2040}{4020}(12) = 6.09 \text{ ft}$$ ◀

$$\bar{y} = \frac{b}{W \tan\theta}(W_B - W_B') = \frac{12}{4020 \tan 30^\circ}(2040 - 1560)$$

$= 2.40$ ft ◀

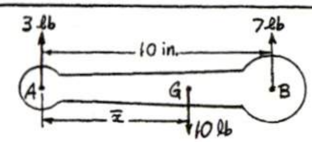
7-9

$$\bar{x} = \frac{W_B}{W}b = \frac{6.67}{13.3}(3) = 1.50 \text{ m}$$ ◀

$$\bar{y} = \frac{b}{W \tan\theta}(W_B - W_B) = \frac{3}{13.3 \tan 30^\circ}(6.67 - 5.34)$$

$= 0.520$ m ◀

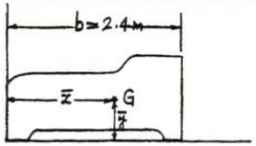
7-10



$$\sum M_A = -10\bar{x} + 7(10) = 0$$

$\bar{x} = 7$ in. ◀

7-11



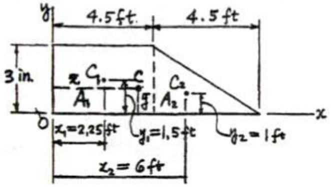
$$\bar{x} = \frac{W_B}{W}b = \frac{17.5}{25}(2.4) = 1.68\text{ m} \blacktriangleleft$$

$$\bar{y} = \frac{b}{W\tan\theta}(W_B - W_B')$$

$$= \frac{2.4}{25\tan 20^\circ}(17.5 - 15)$$

$$= 0.659\text{ m} \blacktriangleleft$$

7-12



$A_1 = 4.5 \times 3 = 13.5\text{ ft}^2$

$A_2 = \frac{1}{2}(4.5 \times 3) = 6.75\text{ ft}^2$

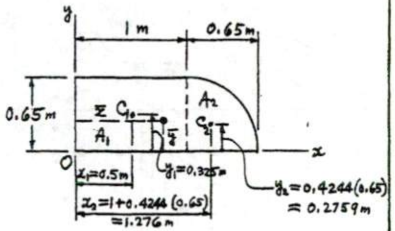
$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{(13.5)(2.25) + (6.75)(6)}{13.5 + 6.75}$$

$$= 3.5\text{ ft} \blacktriangleleft$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{(13.5)(1.5) + (6.75)(1)}{13.5 + 6.75}$$

$$= 1.33\text{ ft} \blacktriangleleft$$

7-13



$A_1 = 1 \times 0.65 = 0.65\text{ m}^2$

$A_2 = \frac{\pi}{4}(0.65)^2 = 0.3318\text{ m}^2$

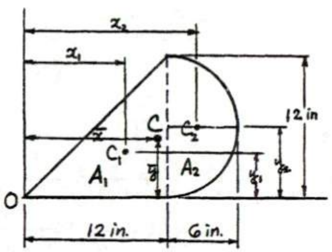
$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{(0.65)(0.5) + (0.3318)(1.276)}{0.65 + 0.3318}$$

$$= 0.762\text{ m} = 762\text{ mm} \blacktriangleleft$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{(0.65)(0.325) + (0.3318)(0.2759)}{0.65 + 0.3318}$$

$$= 0.308\text{ m} = 308\text{ mm} \blacktriangleleft$$

7-14



$A_1 = \frac{1}{2}(12 \times 12) = 72\text{ in.}^2$

$A_2 = \frac{\pi}{2}(6)^2 = 56.55\text{ in.}^2$

$x_1 = \frac{2}{3}(12\text{ in.}) = 8\text{ in.}$

$x_2 = 12\text{ in.} + 0.4244(6\text{ in.}) = 14.55\text{ in.}$

$y_1 = \frac{1}{3}(12\text{ in.}) = 4\text{ in.}$

$y_2 = 6\text{ in.}$

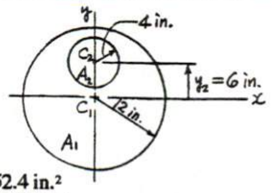
$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{(72)(8) + (56.55)(14.55)}{72 + 56.55}$$

$$= 10.88\text{ in.} \blacktriangleleft$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{(72)(4) + (56.55)(6)}{72 + 56.55}$$

$$= 4.88\text{ in.} \blacktriangleleft$$

7-15



$A_1 = \pi(12)^2 = 452.4\text{ in.}^2$

$A_2 = -\pi(4)^2 = -50.27\text{ in.}^2$

The area is symmetrical about the y-axis, hence,

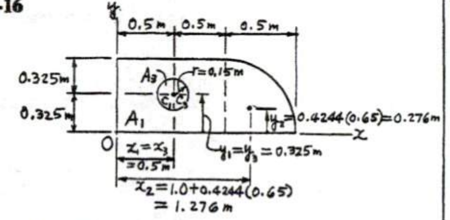
$$\bar{x} = 0 \blacktriangleleft$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2}$$

$$= \frac{(452.4)(0) + (-50.27)(6)}{452.4 + (-50.27)}$$

$$= -0.750\text{ in.} \blacktriangleleft$$

7-16



$A_1 = 1 \times 0.65 = 0.65\text{ m}^2$

$A_2 = \frac{\pi}{4}(0.65)^2 = 0.3318\text{ m}^2$

$A_3 = -\pi(0.15)^2 = -0.0707\text{ m}^2$

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3}$$

$$= \frac{(0.65)(0.5) + (0.3318)(1.276) + (-0.0707)(0.5)}{0.65 + 0.3318 + (-0.0707)}$$

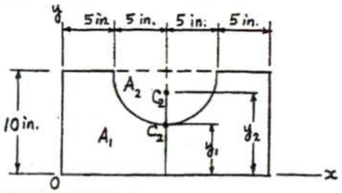
$$= 0.783\text{ m} = 783\text{ mm} \blacktriangleleft$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3}$$

$$= \frac{(0.65)(0.325) + (0.3318)(0.276) + (-0.0707)(0.325)}{0.65 + 0.3318 + (-0.0707)}$$

$$= 0.307\text{ m} = 307\text{ mm} \blacktriangleleft$$

7-17



$A_1 = (20)(10) = 200\text{ in.}^2$

$A_2 = -\frac{\pi}{2}(5)^2 = -39.3\text{ in.}^2$

$y_1 = 5\text{ in.}$

$y_2 = 10 - 0.4244(5) = 7.88\text{ in.}$

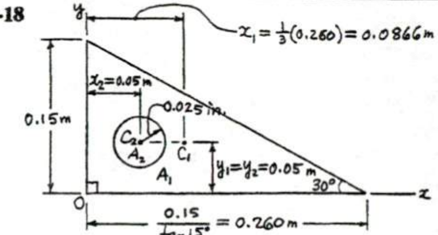
Due to Symmetry:

$$\bar{x} = 0 \blacktriangleleft$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{(200)(5) + (-39.3)(7.88)}{200 + (-39.3)}$$

$$= 4.30\text{ in.} \blacktriangleleft$$

7-18



$A_1 = \frac{1}{2}(0.260)(0.15) = 0.0195\text{ m}^2$

$\underline{A_2 = -\pi(0.025)^2 = -0.00196\text{ m}^2}$

$A_1 + A_2 = 0.01754\text{ m}^2$

$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2}$$

$$= \frac{(0.0195)(0.0866) + (-0.00196)(0.05)}{0.01754}$$

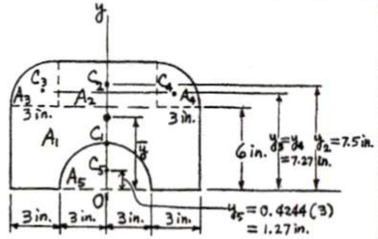
$$= 0.0907\text{ m} = 90.7\text{ mm} \blacktriangleleft$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2}$$

$$= \frac{(0.0195)(0.05) + (-0.00196)(0.05)}{0.01754}$$

$$= 0.05\text{ m} = 50\text{ mm} \blacktriangleleft$$

7-19



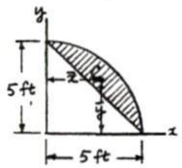
Area	A (in.²)	y (in.)	Ay (in.³)
A_1	72	3	216.0
A_2	18	7.5	135.0
A_3	7.07	7.27	51.4
A_4	7.07	7.27	51.4
A_5	-14.14	1.27	-18.0
Σ	90.0		435.8

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{435.8}{90.0} = 4.84 \text{ in.}$$ ◀

Due to Symmetry:

$$\bar{x} = 0$$ ◀

7-20



A_1 = quarter circle $= \frac{\pi}{4} r^2 = \frac{\pi}{4}(5)^2 = 19.6$

A_2 = triangle $= -\frac{1}{2} bh = -\frac{1}{2}(5)(5) = -12.5$

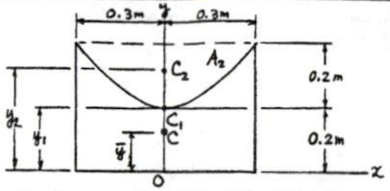
$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

$$= \frac{(19.6)[0.4244(5)] + (-12.5)\left[\frac{1}{3}(5)\right]}{19.6 + (-12.5)}$$

$= 2.92$ ft ◀

$\bar{y} = \bar{x} = 2.92$ ft ◀

7-21



Rectangular Area: $A_1 = 0.6 \times 0.4 = 0.24 \text{ m}^2$

Parabolic Area: $A_2 = -\frac{2}{3}(0.6)(0.2) = -0.08 \text{ m}^2$

$y_1 = 0.2$ m

$y_2 = 0.2 + \frac{3}{5}(0.2) = 0.32$ m

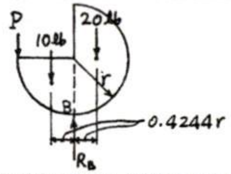
$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(0.24)(0.2) + (-0.08)(0.32)}{0.24 + (-0.08)}$$

$= 0.14$ m $= 140$ mm ◀

Due to Symmetry:

$$\bar{x} = 0$$ ◀

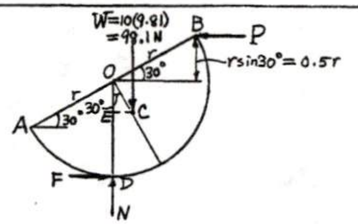
7-22



$$\sum M_B = (P)(r) + (10)(0.4244r) - (20)(0.4244r) = 0$$

$P = 4.24$ lb ◀

7-23



OC = 0.4244 r

CE = OC sin 30° = (0.4244 r) sin 30° = 0.2122 r

(a) $\sum M_D = (P)(r + 0.5r) - (98.1)(0.2122\, r) = 0$

$P = 13.88$ N ◀

(Cont'd)

7-23 (Cont)

(b) $\sum F_y = N - 98.1 = 0$

$N = 98.1$ N

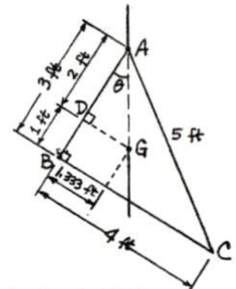
$\sum F_x = F - 13.88 = 0$

$F = 13.88 \text{ N} \le F_m = \mu_s N = \mu_s (98.1 \text{ N})$

$$(\mu_s)_{min} = \frac{13.88 \text{ N}}{98.1 \text{ N}} = 0.141$$ ◀

7-24

When the plate is in equilibrium, the center of gravity G must be vertically below A.



In the right triangle ADG, we have

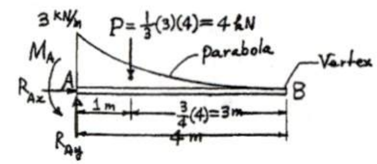
$AD = \frac{2}{3} AB = 2$ ft

$DG = \frac{1}{3} BC = 1.33\dot{3}$ ft

$$\tan\theta = \frac{DG}{AD} = \frac{1.333}{2}$$

$$\theta = \tan^{-1}\frac{1.333}{2} = 33.7°$$ ◀

7-25



$\sum F_x = R_{AX} + 0 = 0$

$R_{AX} = 0$ ◀

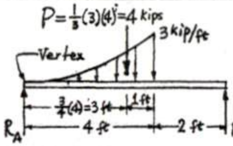
$\sum F_y = R_{Ay} - 4 = 0$

$R_{Ay} = 4 \text{ kN} \uparrow$ ◀

$\sum M_A = M_A - (4)(1) = 0$

$M_A = 4 \text{ kN} \cdot \text{m}$ ↺ ◀

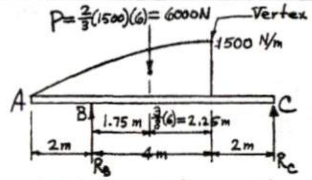
7-26



From the free-body diagram, we see that the resultant load acts at the midspan. Thus,

$$R_A = R_B = \frac{1}{2} P = 2 \text{ kips}$$ ◀

7-27



$\sum M_C = -R_B(6) + (6000)(2 + 2.25) = 0$

$R_B = 4250 \text{ N} \uparrow$ ◀

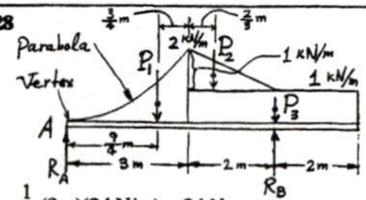
$\sum M_B = R_C(6) - (6000)(1.75) = 0$

$R_C = 1750 \text{ N} \uparrow$ ◀

Check:

$\sum F_y = 4250 + 1750 - 6000 = 0$ (Checks)

7-28



$P_1 = \frac{1}{3}$ (3m)(2 kN/m) = 2 kN

$P_2 = \frac{1}{2}$ (2 m)(1 kN/m) = 1 kN

P_3 = (4 m)(1 kN/m) = 4 kN

(Cont'd)

7-28 (Cont)

$$\sum M_B = -R_A(5) + 2\left(2+\frac{3}{4}\right) + 1\left(2-\frac{2}{3}\right) + 4(0) = 0$$

$R_A = 1.37$ kN ↑ ◀

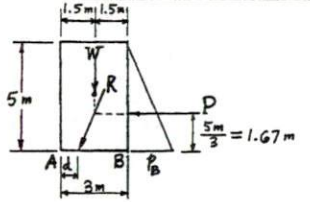
$$\sum M_A = R_B(5) - 2\left(\frac{9}{4}\right) - 1\left(3+\frac{2}{3}\right) - 4(5) = 0$$

$R_B = 5.63$ kN ↑ ◀

Check:

$\sum F_y = 1.37 + 5.63 - 2 - 1 - 4 = 0$ (Checks)

7-29



$W = \gamma_c V = (23.6\ \text{kN/m}^3)(3 \times 5 \times 1\ \text{m}^3) = 354$ kN

$p_B = \gamma_W h_B = (9.80\ \text{kN/m}^3)(5\ \text{m}) = 49\ \text{kN/m}^2$

$P = \left(\frac{1}{2} p_B h_B\right)t = \left(\frac{1}{2} \times 49 \times 5\right)(1) = 122.5$ kN

$R_x = P = 122.5$ kN ←

$R_y = W = 354$ kN ↓

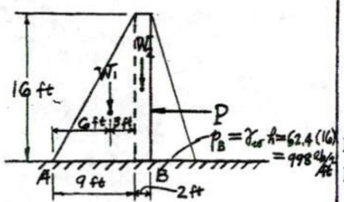
$M_A = +122.5(1.67) - 354(1.5) = -326$ kN • m ↺

$R_y d = 354\, d = 326$

$$d = \frac{326}{354} = 0.921\ \text{m}$$ ◀

Which is less than AB/3 = 1 m. The resultant does not act through the middle-third of the base and so the dam is not safe.

7-30



$W_1 = \left[\frac{1}{2}(9)(16)(1)\right](150) = 10\,800$ lb

$W_2 = [2(16)(1)](150) = 4\,800$ lb

$P = \frac{1}{2}(998)(16)(1) = 7\,980$ lb

$R_x = P = 7\,980$ lb ←

$R_y = W_1 + W_2 = 15\,600$ lb ↓

$\sum M_A = 7980\left[\frac{1}{3}(16)\right] - 10\,800(6) - 4800(10)$

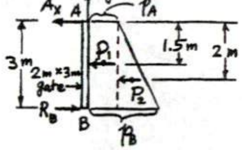
$= -70\,240$ lb • ft

$R_y d = 15\,600\, d = 70\,240$

$d = 4.50\ \text{ft} < \frac{1}{3}AB = 3.67$ ft ◀

The resultant does not act through the middle-third of the base and so the dam is not safe. ◀

7-31



$p_A = \gamma_W h_A = (9.80\ \text{kN/m}^3)(2\ \text{m}) = 19.6\ \text{kN/m}^2$

$p_B = \gamma_W h_B = (9.80\ \text{kN/m}^3)(5\ \text{m}) = 49.0\ \text{kN/m}^2$

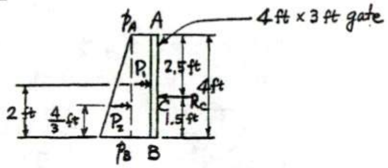
$P_1 = (19.6)(2 \times 3) = 117.6$ kN

$P_2 = \frac{1}{2}(49.0 - 19.6)(2)(3) = 88.2$ kN

$$\sum M_A = (R_B)(3) - (117.6)(1.5) - (88.2)(2) = 0$$

$R_B = 117.6$ kN ◀

7-32



$p_A = \gamma(h - 4)$

$p_B = \gamma h$

(Cont'd)

7-32 (Cont)

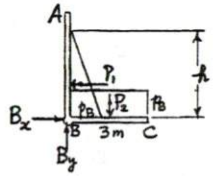
$P_1 = p_A(4 \times 3) = 12\,\gamma(h - 4)$

$P_2 = \frac{1}{2}(p_B - p_A)(4 \times 3) = 6[\gamma h - \gamma(h - 4)] = 24\,\gamma$

$$\sum M_C = -12\gamma(h-4)(2-1.5) + 24\gamma\left(1.5 - \frac{4}{3}\right) = 0$$

$h = 4.67$ ft ◀

7-33



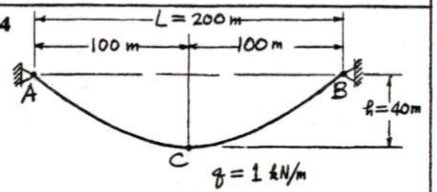
$P_1 = \frac{1}{2} p_B h w = \frac{1}{2}\gamma h^2 w$

$P_2 = p_B(3\,w) = 3\,\gamma h w$

$$\sum M_B = \left(\frac{1}{2}\gamma h^2 w\right)\left(\frac{h}{3}\right) - (3\gamma h w)(1.5) = 0$$

$h = \sqrt{(6)(4.5)} = 5.20$ m ◀

7-34



(a)

$$T_{min} = T_o = \frac{qL^2}{8h} = \frac{(1\ \text{kN/m})(200\ \text{m})^2}{8(40\ \text{m})} = 125\ \text{kN}$$ ◀

(b)

$T_{max} = T_B = \sqrt{T_o^{\,2} + q^2 x_A^{\,2}}$

$= \sqrt{(125\ \text{kN})^2 + (1\ \text{kN/m})^2(100\ \text{m})^2}$

$= 160$ kN ◀

$$\tan\theta_B = \frac{q\,x_B}{T_o} = \frac{(1\ \text{kN/m})(100\ \text{m})}{125\ \text{kN}} = 0.800$$

$\theta_B = 38.7°$ ◀

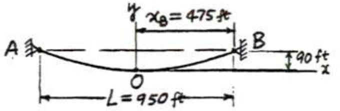
(c)

$$s_{AB} = 2\,s_{BC} = 2\,x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_B}\right)^4\right]$$

$$= 2(100)\left[1 + \frac{2}{3}\left(\frac{40}{100}\right)^2 - \frac{2}{5}\left(\frac{40}{100}\right)^4\right]$$

$= 219.2$ m ◀

7-35



The load is carried by two cables. Hence

$q = \frac{1}{2}(8\ \text{kip/ft}) = 4$ kip/ft

$$T_{min} = T_O = \frac{qL^2}{8h} = \frac{(4\ \text{kip/ft})(950\ \text{ft})^2}{(8)(90\ \text{ft})}$$

$= 5014$ kips ◀

$T_{max} = T_B = \sqrt{(T_O)^2 + q^2 x_B^2}$

$= \sqrt{(5014)^2 + (4)^2(475)^2}$

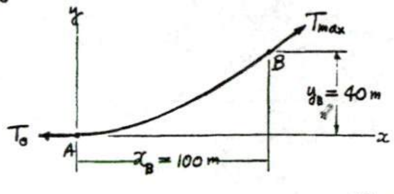
$= 5362$ kips ◀

$$\text{Length} = 2\,x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_B}\right)^4\right]$$

$$= 2(475)\left[1 + \frac{2}{3}\left(\frac{90}{475}\right)^2 - \frac{2}{5}\left(\frac{90}{475}\right)^4\right]$$

$= 972$ ft ◀

7-36



(Cont'd)

7-36 (Cont)

$q = 3$ kN/m

$$T_O = \frac{q\,x_B^2}{2\,y_B} = \frac{(3\text{ kN/m})(100\text{ m})^2}{2(40\text{ m})}$$

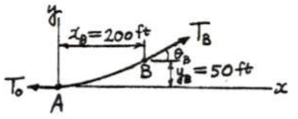
$$= 375\text{ kN}$$

$$T_{max} = T_B = \sqrt{(T_O)^2 + q^2 x_B^2}$$

$$= \sqrt{(375\text{ kN})^2 + (3\text{ kN/m})^2(100\text{ m})^2}$$

$$= 480\text{ kN}$$ ◀

7-37



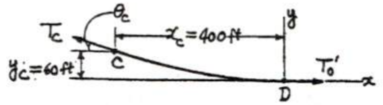
$q = 3$ kip/ft

$$T_O = \frac{q\,x_B^2}{2\,y_B} = \frac{(2\text{ kip/ft})(200\text{ ft})^2}{2(50\text{ ft})} = 800\text{ kips}$$

$$T_B = \sqrt{(T_O)^2 + q^2 x_B^2}$$

$$= \sqrt{(800\text{ kips})^2 + (2\text{ kip/ft})^2(200\text{ ft})^2} = 894\text{ kips}$$

$$\theta_B = \tan^{-1}\frac{q\,x_B}{T_O} = \tan^{-1}\frac{2(200)}{800} = 26.6°$$



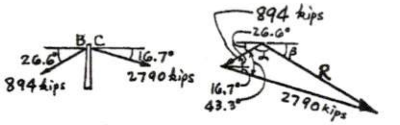
$$T_0' = \frac{q\,x_C^2}{2\,y_C} = \frac{(2\text{ kip/ft})(-400\text{ ft})^2}{(2)(60\text{ ft})} = 2670\text{ kips}$$

$$T_C = \sqrt{(T_0')^2 + q^2 x_C^2}$$

$$= \sqrt{(2670\text{ kips})^2 + (2\text{ kip/ft})^2(-400\text{ ft})^2}$$

$$= 2790\text{ kips}$$

$$\theta_C = \tan^{-1}\left|\frac{q\,x_C}{T_0'}\right| = \tan^{-1}\left|\frac{2(-400)}{2670}\right| = 16.7°$$



$$R = \sqrt{(894)^2 + (2790)^2 - (2)(894)(2790)\cos 43.3°}$$

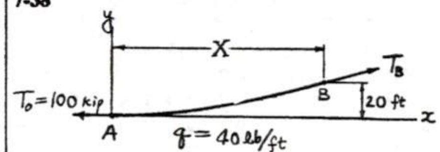
$$= 2230\text{ kips}$$

$$\alpha = \cos^{-1}\frac{(894)^2 + (2230)^2 - (2790)^2}{(2)(894)(2230)} = 120.3°$$

$$\beta = 180° - (26.6° + 120.3°) = 33.1°$$

R = 2230 kips ∡ 33.1° ◀

7-38



(a)

$$T_O = 100\,000\text{ lb} = \frac{q\,x_B^2}{2\,y_B} = \frac{(40\text{ lb/ft})X^2}{(2)(20\text{ ft})}$$

$$X = \sqrt{100\,000\text{ ft}^2} = 316.2\text{ ft}$$ ◀

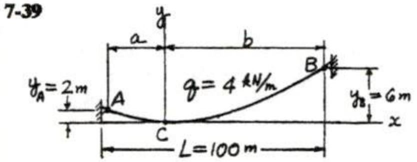
(b)

$$L = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_B}\right)^4\right]$$

$$= (316.2)\left[1 + \frac{2}{3}\left(\frac{20}{316.2}\right)^2 - \frac{2}{5}\left(\frac{20}{316.2}\right)^4\right]$$

$$= 317.0\text{ ft}$$ ◀

7-39



$$T_o = \frac{qL^2}{2(\sqrt{y_A} + \sqrt{y_B})^2} = \frac{(4)(100)^2}{2(\sqrt{2} + \sqrt{6})^2} = 1340\text{ kN}$$

(Cont'd)

7-39 (Cont)

$$a = \sqrt{\frac{2T_o y_A}{q}} = \sqrt{\frac{2(1340)(2)}{4}} = 36.6\text{ m}$$

$$b = \sqrt{\frac{2T_o y_B}{q}} = \sqrt{\frac{2(1340)(6)}{4}} = 63.4\text{ m}$$

Check:
a + b = 36.6 + 63.4 = 100 (Checks)

(a)

$$T_{max} = T_B = \sqrt{(T_O)^2 + q^2 x_B^2}$$

$$= \sqrt{(1340)^2 + (4)^2(63.4)^2}$$

$$= 1364\text{ kN}$$ ◀

(b)

$$s_{AC} = |x_A|\left[1 + \frac{2}{3}\left(\frac{y_A}{x_A}\right)^2 - \frac{2}{5}\left(\frac{y_A}{x_A}\right)^4\right]$$

$$= (36.6)\left[1 + \frac{2}{3}\left(\frac{2}{36.6}\right)^2 - \frac{2}{5}\left(\frac{2}{36.6}\right)^4\right]$$

$$= 36.7\text{ m}$$

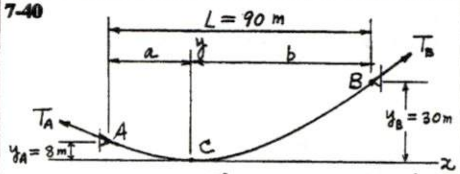
$$s_{BC} = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_B}\right)^4\right]$$

$$= (63.4)\left[1 + \frac{2}{3}\left(\frac{6}{63.4}\right)^2 - \frac{2}{5}\left(\frac{6}{63.4}\right)^4\right]$$

$$= 63.8\text{ m}$$

$$s_{AB} = 36.7\text{ m} + 63.8\text{ m} = 100.5\text{ m}$$ ◀

7-40



$$T_{min} = T_o = \frac{qL^2}{2(\sqrt{y_A} + \sqrt{y_B})^2} = \frac{(0.5)(90)^2}{2(\sqrt{8} + \sqrt{30})^2}$$

$$= 29.35\text{ kN}$$ ◀

$$a = \sqrt{\frac{2T_o y_A}{q}} = \sqrt{\frac{2(29.35)(8)}{0.5}} = 30.6\text{ m}$$

$$b = \sqrt{\frac{2T_o y_B}{q}} = \sqrt{\frac{2(29.35)(30)}{0.5}} = 59.4\text{ m}$$

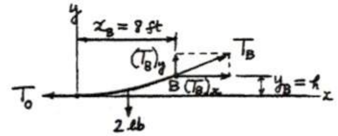
Check:
a + b = 30.6 + 59.4 = 90 (Checks)

$$T_{max} = T_B = \sqrt{(T_O)^2 + q^2 x_B^2}$$

$$= \sqrt{(29.35)^2 + (0.5)^2(59.4)^2}$$

$$= 41.8\text{ kN}$$ ◀

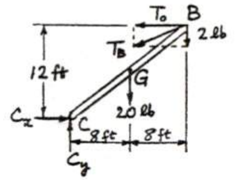
7-41



$$\sum F_x = (T_B)_x - T_o = 0$$
$$(T_B)_x = T_O$$

$$\sum F_y = (T_B)_y - 2 = 0$$
$$(T_B)_y = 2\text{ lb}$$

$$q = 4\text{ lb} / 16\text{ ft} = 0.25\text{ lb/ft}$$



$$\sum M_C = (T_O)(12) - (2)(16) - (20)(8) = 0$$
$$T_o = 16\text{ lb}$$

At B: $x_B = 8$ ft

$$h = y_B = \frac{q\,x_B^2}{2\,T_O}$$

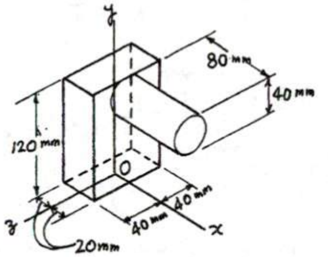
$$= \frac{(0.25\text{ lb/ft})(8\text{ ft})^2}{2\,(16\text{ lb})}$$

$$= 0.5\text{ ft}$$ ◀

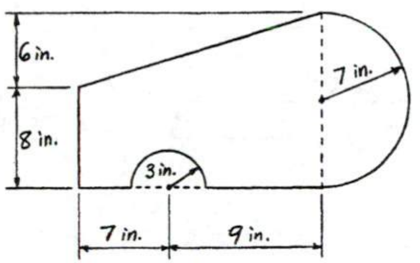
Test Problems for Chapter 7

The following problems may either be given to students for them to practice their problem solving skills or be used as test problems. The answers are provided on the next page.

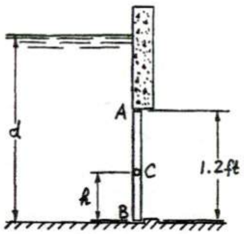
(1) A machine part consists of a rectangular solid made of steel ($\gamma_{st} = 77$ kN/m^3) and a circular cylinder made of aluminum (γ_{al} = 27 kN/m^3). Determine the center of gravity of the body.



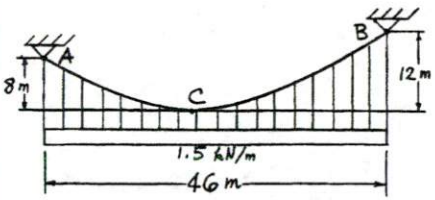
(2) Determine the centroid of the plane area shown.



(3) An automatic valve consists of a 0.8 ft x 1.2 ft gate which is pivoted about a horizontal axis through C as shown. When the water level rises to a certain level, the gate will open automatically. If the gate opens when the depth of water reaches $d = 2.0$ ft, determine the value of h for the location of the pivot axis from the bottom of the gate.



(4) Cable AB supports a load distributed uniformly along the 45 m horizontal length as shown. The sags at the support from the lowest point are indicated in the figure. Determine the minimum and maximum tensions in the cable.



Solutions to Test Problems for Chapter 7

(1)

The body is symmetric with respect to the xy-plane. The centroid must be located on this plane. Hence

$\bar{z} = 0$ ◀

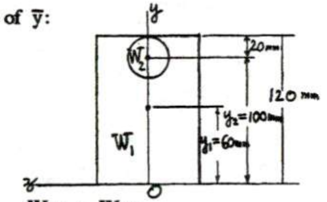
The weight of rectangular solid is

$W_1 = \gamma_{st} V_1 = (77\ \text{kN/m}^3)(0.04\ \text{m})(0.08\ \text{m})(0.12\ \text{m}) = 0.0296\ \text{kN} = 29.6\ \text{N}$

The weight of the circular cylinder is

$W_2 = \gamma_{al} V_2 = (27\ \text{kN/m}^3)\left[\frac{\pi}{4}(0.040\ \text{m})^2(0.080\ \text{m})\right] = 0.00271\ \text{kN} = 2.71\ \text{N}$

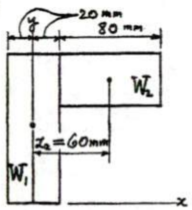
The value of $\bar{y}$:



$$\bar{y} = \frac{W_1 y_1 + W_2 y_2}{W_1 + W_2} = \frac{(29.6\ \text{N})(60\ \text{mm}) + (2.71\ \text{N})(100\ \text{mm})}{29.6\ \text{N} + 2.71\ \text{N}}$$

$= 63.4$ mm ◀

The value of $\bar{x}$:

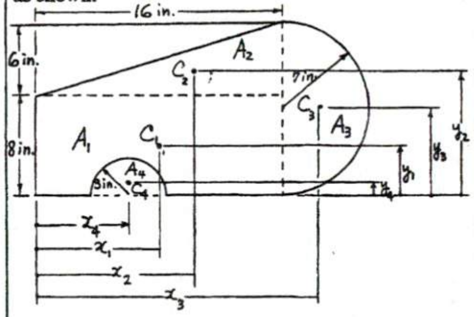


$$\bar{x} = \frac{W_1 x_1 + W_2 x_2}{W_1 + W_2} = \frac{(29.6\ \text{N})(0) + (2.71\ \text{N})(60\ \text{mm})}{29.6\ \text{N} + 2.71\ \text{N}}$$

$= 5.03$ mm ◀

(2)

The given area is divided into four component areas as shown.



The area and the centroid of each component area is:

$A_1 = (16)(8) = 128$ in.^{2}

$x_1 = 8$ in.

$y_1 = 4$ in.

$A_2 = \frac{1}{2}(16)(6) = 48$ in.^{2}

$x_2 = \frac{2}{3}(16) = 10.67$ in.

$y_2 = 8 + \frac{1}{3}(6) = 10$ in.

$A_3 = \frac{\pi}{2}(7)^2 = 77.0$ in.^{2}

$x_3 = 16 + \frac{4(7)}{3\pi} = 18.97$ in.

$y_3 = 7$ in.

$A_4 = \frac{\pi}{2}(3)^2 = -14.13$ in.^{2}

$x_4 = 7$ in.,

$y_4 = \frac{4(3)}{3\pi} = 1.27$ in.

$$\bar{x} = \frac{\Sigma A_x}{\Sigma A} = \frac{128(8) + 48(10.67) + 77.0(18.97) + (-14.14)(7)}{128 + 48 + 77.0 - 14.14}$$

$= 12.13$ in. ◀

(Cont'd)

Solutions to Test Problems for Chapter 7 (Cont)

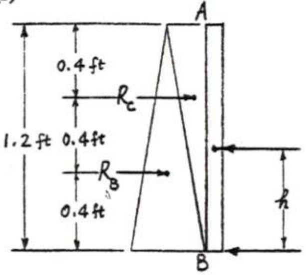
(2) (Cont)

$$\bar{y} = \frac{\Sigma A_y}{\Sigma A}$$

$$= \frac{128(4) + 48(10) + 77.0(7) + (-14.14)(1.27)}{128 + 48 + 77.0 - 14.14}$$

$= 6.33$ in. ◀

(3)



$p_C = \gamma_w(d - 1.2 \text{ ft})$

$= (62.4 \text{ lb/ft}^3)(2.0 \text{ ft} - 1.2 \text{ ft})$

$= 49.92 \text{ lb/ft}^2$

$p_B = \gamma_w d$

$= (62.4 \text{ lb/ft}^3)(2.0 \text{ ft})$

$= 124.8 \text{ lb/ft}^2$

$P_1 = \frac{1}{2}(49.92 \text{ lb/ft}^2)(1.2 \text{ ft})(0.8 \text{ ft}) = 24.0 \text{ lb}$

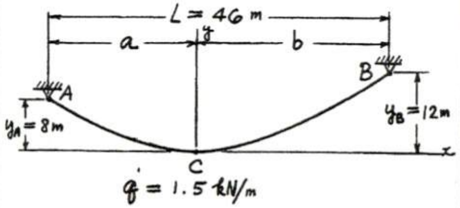
$P_2 = \frac{1}{2}(124.8 \text{ lb/ft}^2)(1.2 \text{ ft})(0.8 \text{ ft}) = 59.9 \text{ lb}$

$\Sigma M_C = (24.0)(0.8 - h) - (59.9)(h - 0.4) = 0$

$19.2 - 24.0h - 59.9h + 24.0 = 0$

$h = 0.514$ ft ◀

(4)



From Eq. 7-16, the minimum tension is

$$T_O = \frac{qL^2}{2\left(\sqrt{y_A} + \sqrt{y_B}\right)^2} = \frac{(1.5 \text{ kN/m})(46 \text{ m})^2}{2\left(\sqrt{8 \text{ m}} + \sqrt{12 \text{ m}}\right)^2}$$

$= 40.0$ kN ◀

From Eq. 7-15, the distance b is

$$b = \sqrt{\frac{2T_o y_B}{q}} = \sqrt{\frac{2(40 \text{ kN})(12 \text{ m})}{1.5 \text{ kN/m}}} = 25.3 \text{ m}$$

From Eq. 7-10, the maximum tension at B is

$$T_{max} = \sqrt{T_o^2 + q^2 x_B^2}$$

$$= \sqrt{(40.0 \text{ kN})^2 + (1.5 \text{ kN/m})^2 (25.3 \text{ m})^2}$$

$= 47.4$ kN ◀

8-1

$$\bar{r}_x = \sqrt{\frac{\bar{I}_x}{A}} = \sqrt{\frac{bh^3/12}{bh}} = \frac{h}{\sqrt{12}}$$ ◀

$$\bar{r}_y = \sqrt{\frac{\bar{I}_y}{A}} = \sqrt{\frac{hb^3/12}{bh}} = \frac{b}{\sqrt{12}}$$ ◀

8-2

$$A = \frac{\pi}{4}d^2$$

$$\bar{I} = \frac{\pi}{64}d^4$$

$$\bar{r} = \sqrt{\frac{\bar{I}}{A}} = \sqrt{\frac{\frac{\pi}{64}d^4}{\frac{\pi}{4}d^2}} = \frac{d}{4}$$ ◀

8-3

$$I_x = \bar{I}_x + Ad^2$$

$$= \frac{\pi}{4}r^4 + (\pi r^2)d^2$$

$$= \frac{\pi}{4}(5 \text{ in.})^4 + \pi(5 \text{ in.})^2(10 \text{ in.})^2$$

$= 8340 \text{ in.}^4$ ◀

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{8340 \text{ in.}^4}{\pi(5 \text{ in.})^2}}$$

$= 10.3$ in. ◀

8-4

$I_y = Ar_y^2 = (100 \text{ in.})(12.4 \text{ in.})^2 = 15\,376 \text{ in.}^4$

From $I_y = \bar{I}_y + Ad^2$ we write

$\bar{I}_y = I_y - Ad^2 = 15376 - 100(12)^2 = 976 \text{ in.}^4$ ◀

$$\bar{r}_y = \sqrt{\frac{\bar{I}_y}{A}} = \sqrt{\frac{976 \text{ in.}^4}{100 \text{ in.}^2}} = 3.12 \text{ in.}$$ ◀

8-5

$b = 100 \text{ mm} = 0.1 \text{ m}$

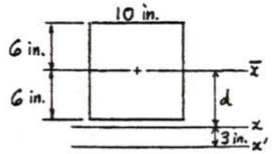
$h = 200 \text{ mm} = 0.2 \text{ m}$

$$\bar{J} = \frac{bh}{12}(b^2 + h^2)$$

$$= \frac{(0.1 \text{ m})(0.2 \text{ m})}{12}\left[(0.1 \text{ m})^2 + (0.2 \text{ m})^2\right]$$

$= 8.33 \times 10^{-5} \text{ m}^4$ ◀

8-6



$I_x = \bar{I}_x + Ad^2$

$$7320 \text{ in.}^4 = \frac{(10 \text{ in.})(12 \text{ in.})^3}{12} + (10 \times 12 \text{ in.}^2) = d^2$$

From which we get

$d = 7$ in.

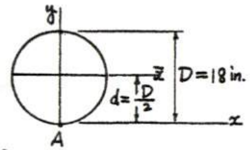
$I_{x'} = \bar{I}_x + A(d + 3)^2$

$$= \frac{(10 \text{ in.})(12 \text{ in.})^3}{12}$$

$+ (120 \text{ in.}^2)(10 \text{ in.})^2$

$= 13\,440 \text{ in.}^4$ ◀

8-7



$$I_x = \bar{I}_x + Ad^2$$

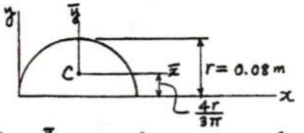
$$= \frac{\pi D^4}{64} + \left(\frac{\pi D^2}{4}\right)\left(\frac{D}{2}\right)^2 = \frac{5\pi}{64}D^4$$

$$J_A = I_x + I_y$$

$$= \frac{5\pi}{64}D^4 + \frac{\pi}{64}D^4 = \frac{6\pi}{64}D^4$$

$$= \frac{6\pi}{64}(18 \text{ in.})^4 = 30\,920 \text{ in.}^4$$ ◀

8-8



$$A = \frac{\pi}{2}r^2 = \frac{\pi}{2}(0.08\ \text{m})^2 = 0.01005\ \text{m}^2$$

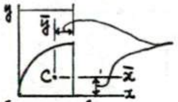
$$I_x = \frac{\pi r^4}{8} = \frac{\pi(0.08\ \text{m})^4}{8} = 1.608 \times 10^{-5}\ \text{m}^4$$

$$I_y = \bar{I}_y + Ad^2 = \frac{\pi r^4}{8} + A(r)^2 = \frac{\pi(0.08\ \text{m})^4}{8} + (0.01005\ \text{m}^2)(0.08\ \text{m})^2 = 8.04 \times 10^{-5}\ \text{m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{1.608 \times 10^{-5}\ \text{m}^4}{0.01005\ \text{m}^2}} = 0.040\ \text{m} = 40\ \text{mm}$$ ◄

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{8.04 \times 10^{-5}\ \text{m}^4}{0.01005\ \text{m}^2}} = 0.0894\ \text{m} = 89.4\ \text{mm}$$ ◄

8-9



$$\frac{4r}{3\pi} = \frac{4(6)}{3\pi} = 2.55\ \text{in.}$$

$$A = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(6\ \text{in.})^2 = 28.3\ \text{in.}^2$$

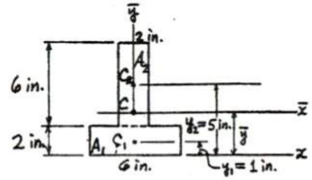
$$I_x = \bar{I}_x + Ad^2 = 0.0549r^4 + Ad^2 = 0.0549(6\ \text{in.})^4 + (28.3\ \text{in.})(2.55\ \text{in.})^2 = 2.55\ \text{in.}^4$$

$$I_y = \bar{I}_y + Ad^2 = 0.0549r^4 + Ad^2 = 0.0549(6\ \text{in.})^4 + (28.3\ \text{in.})(6\ \text{in.} - 2.55\ \text{in.})^2 = 408\ \text{in.}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{255\ \text{in.}^4}{28.3\ \text{in.}^2}} = 3.00\ \text{in.}$$ ◄

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{408\ \text{in.}^4}{28.3\ \text{in.}^2}} = 3.80\ \text{in.}$$ ◄

8-10



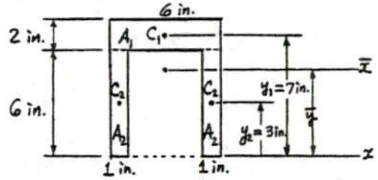
$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{(6 \times 2)(1) + (2 \times 6)(5)}{6 \times 2 + 2 \times 6} = 3\ \text{in.}$$

$$\bar{I}_x = \left[\bar{I}_1 + A_1(\bar{y} - y_1)^2\right] + \left[\bar{I}_2 + A_2(\bar{y} - y_2)^2\right]$$

$$= \left[\frac{6(2)^3}{12} + (6 \times 2)(3-1)^2\right] + \left[\frac{2(6)^3}{12} + (2 \times 6)(3-5)^2\right]$$

$$= 4 + 48 + 36 + 48 = 136\ \text{in.}^4$$ ◄

8-11



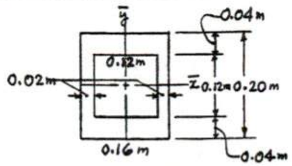
$$\bar{y} = \frac{A_1y_1 + 2A_2y_2}{A_1 + 2A_2} = \frac{(6 \times 2)(7) + 2(1 \times 6)(3)}{6 \times 2 + 2(1 \times 6)} = 5\ \text{in.}$$

$$\bar{I}_x = \left[\bar{I}_1 + A_1(\bar{y} - y_1)^2\right] + 2\left[\bar{I}_2 + A_2(\bar{y} - y_2)^2\right]$$

$$= \left[\frac{6(2)^3}{12} + (12)(5-7)^2\right] + 2\left[\frac{1(6)^3}{12} + (6)(5-3)^2\right]$$

$$= 136\ \text{in}^4$$ ◄

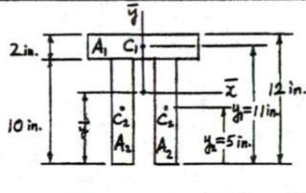
8-12



Treating the section as a 0.16 m x 0.2 m rectangle subtract a 0.12 m x 0.12 m square, we have

$$\bar{I}_x = (\bar{I}_x)_1 - (\bar{I}_x)_2 = \frac{(0.16)(0.2)^3}{12} - \frac{(0.12)(0.12)^3}{12} = 8.94 \times 10^{-5}\ \text{m}^4$$ ◄

8-13



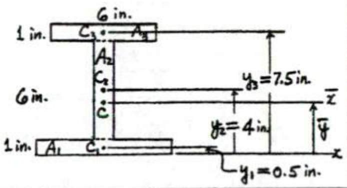
$$\bar{y} = \frac{A_1y_1 + 2A_2y_2}{A_1 + 2A_2} = \frac{(10 \times 2)(11) + 2(2 \times 10)(5)}{10 \times 2 + 2(2 \times 10)} = 7\ \text{in.}$$

$$\bar{I}_x = \left[\bar{I}_1 + A_1(\bar{y} - y_1)^2\right] + 2\left[\bar{I}_2 + A_2(\bar{y} - y_2)^2\right]$$

$$= \left[\frac{10(2)^3}{12} + (10 \times 2)(7-11)^2\right] + 2\left[\frac{2(10)^3}{12} + (2 \times 10)(7-5)^2\right]$$

$$= 820\ \text{in.}^4$$ ◄

8-14



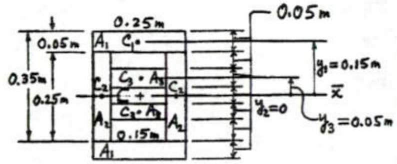
$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3} = \frac{8(0.5) + 6(4) + 6(7.5)}{8 + 6 + 6} = 3.65\ \text{in.}$$

$$\bar{I}_x = \sum\left[\bar{I}_x + A(\bar{y} - \tilde{y})^2\right]$$

$$= \left[\frac{8(1)^3}{12} + 8(3.65-0.5)^2\right] + \left[\frac{1(6)^3}{12} + 6(3.65-4)^2\right] + \left[\frac{6(1)^3}{12} + 6(3.65-7.5)^2\right]$$

$$= 188.2\ \text{in.}^4$$ ◄

8-15



$$\bar{I}_x = 2\left[(\bar{I}_x)_1 + A_1y_1^2\right] + 2(\bar{I}_x)_2 + 2\left[(\bar{I}_x)_3 + A_3y_3^2\right]$$

$$= 2\left[\frac{0.25(0.05)^3}{12} + (0.25 \times 0.05)(0.15)^2\right] + 2\left[\frac{0.05(0.25)^3}{12}\right] + 2\left[\frac{0.15(0.05)^3}{12} + (0.15 \times 0.05)(0.05)^2\right]$$

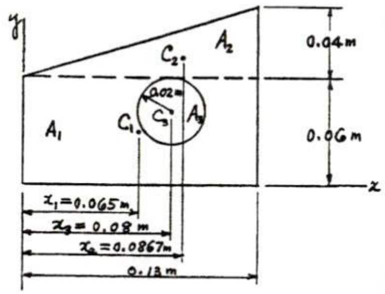
$$= 7.39 \times 10^{-4}\ \text{m}^4$$ ◄

8-24 (Cont)

$$I_x = \sum\left[\bar{I}_x + Ay^2\right]$$
$$= [2.34 \times 10^{-6} + 0.0078(0.03)^2]$$
$$+ [2.31 \times 10^{-7} + 0.0026(0.0733)^2]$$
$$+ [(-1.26 \times 10^{-7}) + (-0.00126)(0.04)^2]$$
$$= 2.142 \times 10^{-5}\ m^4$$ ◄

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{2.142 \times 10^{-5}}{0.00914}} = 0.0484\ m$$
$$= 48.4\ mm$$ ◄

8-25

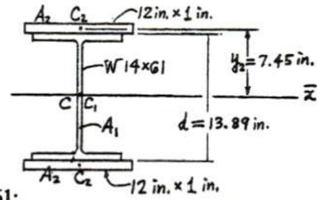


$$A_1 = (0.13)(0.06) = 0.0078\ m^2$$
$$A_2 = \frac{1}{2}(0.13)(0.04) = 0.0026\ m^2$$
$$A_3 = -\pi(0.02)^2 = -0.00126\ m^2$$

$$(\bar{I}_y)_1 = \frac{(0.06)(0.13)^3}{12} = 1.099 \times 10^{-5}\ m^4$$
$$(\bar{I}_y)_2 = \frac{(0.04)(0.13)^3}{36} = 2.441 \times 10^{-5}\ m^4$$
$$(\bar{I}_y)_3 = -\frac{\pi(0.02)^4}{4} = -1.26 \times 10^{-7}\ m^4$$

$$I_y = \sum\left[\bar{I}_y + Ax^2\right]$$
$$= [1.099 \times 10^{-5} + 0.0078(0.065)^2]$$
$$+ [2.441 \times 10^{-5} + 0.0026(0.0867)^2]$$
$$+ [(-1.26 \times 10^{-7}) + (-0.00126)(0.08)^2]$$
$$= 7.971 \times 10^{-5}\ m^4$$ ◄

8-26

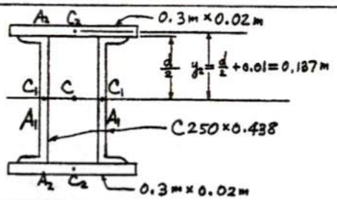


For W 14 x 61:
$A_1 = 17.9\ in.^2$, $d = 13.89\ in.$, $(\bar{I}_x)_1 = 640\ in.^4$
$A = A_1 + 2A_2 = 17.9\ in.^2 + 2(12 \times 1\ in.^2) = 41.9\ in.^2$

$$\bar{I}_x = (\bar{I}_x)_1 + 2\left[(\bar{I}_x)_2 + A_2y_2^2\right]$$
$$= 640 + 2\left[\frac{12(1)^3}{12} + (12 \times 1)(7.45)^2\right]$$
$$= 1974\ in.^4$$ ◄

$$\bar{r}_x = \sqrt{\frac{\bar{I}_x}{A}} = \sqrt{\frac{1974\ in.^4}{41.9\ in.^2}} = 6.36\ in.$$ ◄

8-27

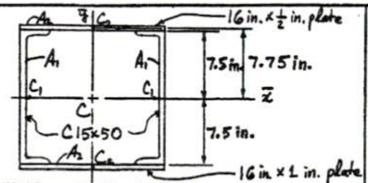


For C 250 x 0.438:
$d = 0.254\ m$
$A_1 = 5.69 \times 10^{-3}\ m^2$
$(\bar{I}_x)_1 = 42.9 \times 10^{-6}\ m^4$
$A = 2A_1 + 2A_2 = 2(5.69 \times 10^{-3}) + 2(0.3 \times 0.02)$
$= 0.0234\ m^2$

$$\bar{I}_x = 2(\bar{I}_x)_1 + 2\left[(\bar{I}_x)_2 + A_2y_2^2\right]$$
$$= 2(42.9 \times 10^{-6})$$
$$+ 2\left[\frac{0.3(0.02)^3}{12} + (0.3 \times 0.02)(0.137)^2\right]$$
$$= 3.11 \times 10^{-4}\ m^4$$ ◄

$$\bar{r}_x = \sqrt{\frac{\bar{I}_x}{A}} = \sqrt{\frac{3.11 \times 10^{-4}}{0.0234}}$$
$$= 0.1153\ m = 115.3\ mm$$ ◄

8-28

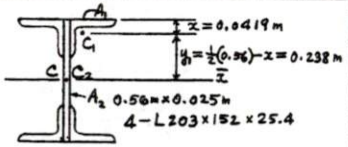


For C15 x 50 [from Table A-3(a)]:
$d = 15\ in.$
$A_1 = 14.7\ in.^2$
$(\bar{I}_x)_1 = 404\ in.^4$

$$A = 2A_1 + 2A_2$$
$$= 2(14.7\ in.^2) + 2\left(16 \times \frac{1}{2}\ in.^2\right) = 45.4\ in.^2$$
$$\bar{I}_x = 2(\bar{I}_x)_1 + 2\left[(\bar{I}_x)_2 + A_2y_2^2\right]$$
$$= 2(404) + 2\left[\frac{16(1/2)^3}{12} + \left(16 \times \frac{1}{2}\right)(7.75)^2\right]$$
$$= 1770\ in.^4$$ ◄

$$\bar{r}_x = \sqrt{\frac{\bar{I}_x}{A}} = \sqrt{\frac{1770\ in.^4}{45.4\ in.^2}} = 6.24\ in.$$ ◄

8-29

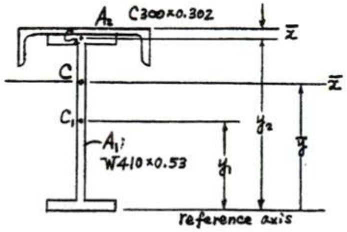


For L 203 x 152 x 25.4 [from table A-4(b)]:
$A_1 = 8.39 \times 10^{-3}\ m^2$
$(\bar{I}_x)_1 = I_y$ in the table $= 16.1 \times 10^{-6}\ m^4$
$x = 0.0419\ m$
$A = 4A_1 + A_2 = 4[(8.39 \times 10^{-3}) + 0.56 \times 0.025$
$= 0.0476\ m^2$

$$\bar{I}_x = 4\left[(\bar{I}_x)_1 + A_1y_1^2\right] + (\bar{I}_x)_2$$
$$= 4[16.1 \times 10^{-6} + (8.39 \times 10^{-3})(0.238)^2] + \frac{0.025(0.56)^3}{12}$$
$$= 2.33 \times 10^{-3}\ m^4$$ ◄

$$\bar{r}_x = \sqrt{\frac{\bar{I}_x}{A}} = \sqrt{\frac{2.33 \times 10^{-3}}{0.0476}}$$
$$= 0.221\ m = 221\ mm$$ ◄

8-30



For W410 x 0.53 [from Table A-1(b)]:
$A_1 = 6.84 \times 10^{-3}\ m^2$
$d = 0.403\ m$
$(I_x)_1 = 186 \times 10^{-6}\ m^4$

For C300 x 0.302 [from Table A-3(b)]:
$A_2 = 3.93 \times 10^{-3}\ m^2$
$(I_x)_2 = I_y$ in the table $= 1.61 \times 10^{-6}\ m^4$
$\bar{x} = 0.0177\ m$
$t_w = 0.00716\ m$

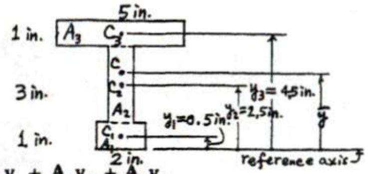
$A = A_1 + A_2 = 6.86 \times 10^{-3} + 3.93 \times 10^{-3} = 0.01079\ m^2$
$y_1 = \frac{d}{2} = 0.2015\ m$
$y_2 = d + t_w - \bar{x} = 0.403 + 0.00716 - 0.0177 = 0.3925\ m$

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2}$$
$$= \frac{(6.84 \times 10^{-3})(0.2015) + (3.93 \times 10^{-3})(0.3925)}{0.01079}$$
$$= 0.271\ m$$

$$\bar{I}_x = \sum\left[I_x + A(\bar{y} - y)^2\right]$$
$$= [186 \times 10^{-6} + (6.84 \times 10^{-3})(0.271 - 0.2015)^2]$$
$$+ [1.61 \times 10^{-6} + (3.93 \times 10^{-3})(0.271 - 0.3925)^2]$$
$$= 2.79 \times 10^{-4}\ m^4$$ ◄

$$\bar{r}_x = \sqrt{\frac{\bar{I}_x}{A}} = \sqrt{\frac{2.79 \times 10^{-4}}{0.01079}}$$
$$= 0.161\ m = 161\ mm$$ ◄

8-16



$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3}$$

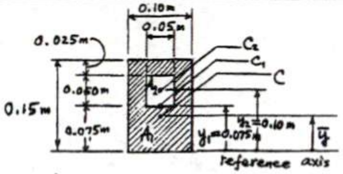
$$= \frac{2(0.5) + 3(2.5) + 5(4.5)}{2 + 3 + 5} = 3.1 \text{ in.}$$

$$\bar{I}_x = \sum\left[\bar{I}_x + A(\bar{y} - y)^2\right]$$

$$= \left[\frac{2(1)^3}{12} + 2(3.1-0.5)^2\right] + \left[\frac{1(3)^3}{12} + 3(3.1-2.5)^2\right] + \left[\frac{5(1)^3}{12} + 5(3.1-4.5)^2\right]$$

$$= 27.2 \text{ in.}^4$$ ◀

8-17



$$\bar{y} = \frac{A_1y_1 - A_2y_2}{A_1 - A_2}$$

$$= \frac{(0.10 \times 0.15)(0.075) - (0.05 \times 0.05)(0.10)}{0.10 \times 0.15 - 0.05 \times 0.05}$$

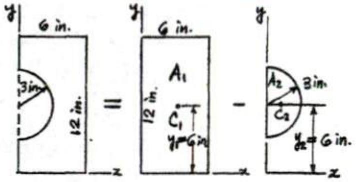
$= 0.070$ m

$$\bar{I}_x = \left[\bar{I}_1 + A_1(\bar{y} - y_1)^2\right] - \left[\bar{I}_2 + A_2(\bar{y} - y_2)^2\right]$$

$$= \left[\frac{0.10(0.15)^3}{12} + (0.10 \times 0.15)(0.070 - 0.075)^2\right] - \left[\frac{0.05(0.05)^3}{12} + (0.05 \times 0.05)(0.070 - 0.10)^2\right]$$

$$= 2.57 \times 10^{-5} \text{ m}^4$$ ◀

8-18



$$I_x = \left[(\bar{I}_x)_1 + A_1y_1^2\right] - \left[(\bar{I}_x)_2 + A_2y_2^2\right]$$

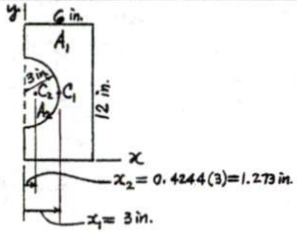
$$= \left[\frac{6(12)^3}{12} + (6 \times 12)(6)^2\right] - \left[\frac{\pi(3)^3}{8} + \frac{\pi}{2}(3)^2(6)^2\right]$$

$$= 2936 \text{ in.}^4$$ ◀

$$A = A_1 - A_2 = 6 \times 12 - \frac{\pi}{2}(3)^2 = 57.9 \text{ in.}^2$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{2936 \text{ in.}^4}{57.9 \text{ in.}^2}} = 7.12 \text{ in.}$$ ◀

8-19



$$I_y = \left[(\bar{I}_y)_1 + A_1x_1^2\right] - \left[(\bar{I}_y)_2 + A_2x_2^2\right]$$

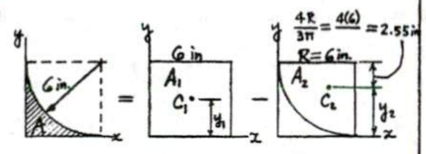
$$= \left[\frac{12(6)^3}{12} + (12 \times 6)(3)^2\right] - \left[0.1098(3)^4 + \frac{\pi}{2}(3)^2(1.273)^2\right]$$

$$= 832 \text{ in.}^4$$ ◀

$$A = A_1 - A_2 = 6 \times 12 - \frac{\pi}{2}(3)^2 = 57.9 \text{ in.}^2$$

$$r_y = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{832 \text{ in.}^4}{57.9 \text{ in.}^2}} = 3.79 \text{ in.}$$ ◀

8-20



$$A = A_1 - A_2 = 6^2 - \frac{\pi}{4}(6)^2 = 7.73 \text{ in.}^2$$

$$I_x = \left[(\bar{I}_x)_1 + A_1y_1^2\right] - \left[(\bar{I}_x)_2 + A_2y_2^2\right]$$

$$= \left[\frac{6(6)^3}{12} + (6^2)(3)^2\right] - \left[0.0549(6)^4 + \frac{\pi}{4}(6)^2(6 - 2.55)^2\right]$$

$$= 24.3 \text{ in.}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{24.3 \text{ in.}^4}{7.73 \text{ in.}^2}} = 1.77 \text{ in.}$$ ◀

8-21

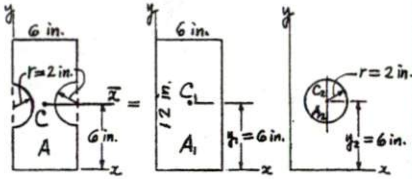
J_o = one-half of J_o of a circular ring

$$= \frac{1}{2}\left[\frac{\pi}{64}(d_o^4 - d_i^4)\right]$$

$$= \frac{\pi}{128}\left[(0.6 \text{ m})^4 - (0.4 \text{ m})^4\right]$$

$$= 5.11 \times 10^{-3} \text{ m}^4$$ ◀

8-22

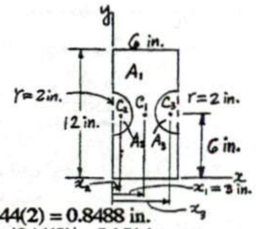


$$I_x = \left[(\bar{I}_x)_1 + A_1y_1^2\right] - \left[(\bar{I}_x)_2 + A_2y_2^2\right]$$

$$= \left[\frac{6(12)^3}{12} + (6 \times 12)(6)^2\right] - \left[\frac{\pi}{4}(2)^4 + \pi(2^2)(6)^2\right]$$

$$= 2990 \text{ in.}^4$$ ◀

8-23



$x_2 = 0.4244(2) = 0.8488$ in.

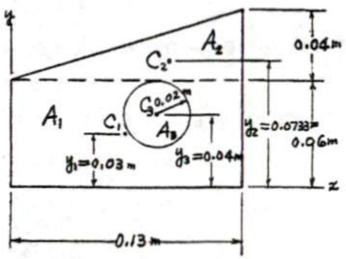
$x_3 = 6 - 0.4244(2) = 5.151$ in.

$$I_y = \left[(\bar{I}_y)_1 + A_1x_1^2\right] - (\bar{I}_y)_2 - \left[(\bar{I}_y)_3 + A_3x_3^2\right]$$

$$= \left[\frac{12(6)^3}{12} + (12 \times 6)(3)^2\right] - \left[\frac{\pi(2)^4}{8}\right] - \left[0.1098(2)^4 + \frac{\pi(2)^2}{2}(5.151)^2\right]$$

$$= 689 \text{ in.}^4$$ ◀

8-24



$A_1 = (0.13)(0.06)$	=	0.0078 m^2
$A_2 = \frac{1}{2}(0.13)(0.04)$	=	0.0026 m^2
$A_3 = -\pi(0.02)^2$	=	-0.00126 m^2
$A = A_1 + A_2 + A_3$	=	0.00914 m^2

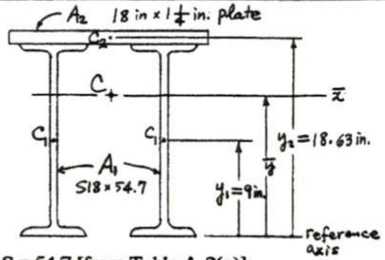
$$(\bar{I}_x)_1 = \frac{(0.13)(0.06)^3}{12} = 2.34 \times 10^{-6} \text{ m}^4$$

$$(\bar{I}_x)_2 = \frac{(0.13)(0.04)^3}{36} = 2.31 \times 10^{-7} \text{ m}^4$$

$$(\bar{I}_x)_3 = -\frac{\pi(0.02)^4}{4} = -1.26 \times 10^{-7} \text{ m}^4$$

(Cont'd)

8-31



For S18 x 54.7 [from Table A-2(a)]:
$A_1 = 16.1 \text{ in.}^2$
$d = 18.00 \text{ in.}$
$(\bar{I}_x)_1 = 804 \text{ in.}^4$

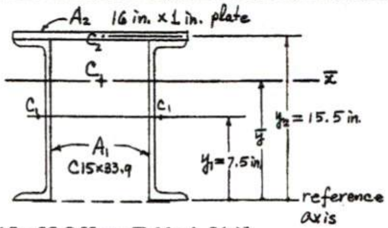
$A = 2A_1 + A_2 = 2(16.1) + 18 \times 1.25 = 54.7 \text{ in.}^2$

$$\bar{y} = \frac{2A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{2(16.1)(9.0) + (18 \times 1.25)(18.63)}{54.7} = 12.96 \text{ in.}$$

$$\bar{I}_x = 2\left[(\bar{I}_x)_1 + A_1(\bar{y} - y_1)^2\right] + \left[(\bar{I}_x)_2 + A_2(\bar{y} - y_2)^2\right]$$
$$= 2\left[804 + 16.1(12.96 - 9)^2\right] + \left[\frac{18(1.25)^3}{12} + (18 \times 1.25)(12.96 - 18.63)^2\right]$$
$$= 2840 \text{ in.}^4$$ ◄

$$\bar{r}_x = \sqrt{\frac{\bar{I}_x}{A}} = \sqrt{\frac{2840 \text{ in.}^4}{54.7 \text{ in.}^2}} = 7.21 \text{ in.}$$ ◄

8-32



For C 15 x 33.9 [from Table A-3(a)]:
$A_1 = 9.96 \text{ in.}^2$
$d = 15.00 \text{ in.}$
$(\bar{I}_x)_1 = 315 \text{ in.}^4$

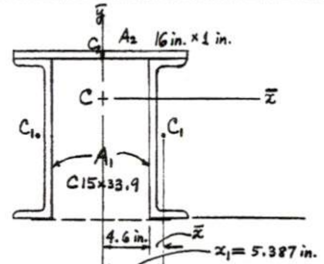
$A = 2A_1 + A_2 = 2(9.96) + 16 \times 1 = 35.9 \text{ in.}^2$

$$\bar{y} = \frac{2A_1y_1 + A_2y_2}{A} = \frac{2(9.96)(7.5) + (16 \times 1)(15.5)}{35.9} = 11.07 \text{ in.}$$

$$\bar{I}_x = 2\left[(\bar{I}_x)_1 + A_1(\bar{y} - y_1)^2\right] + \left[(\bar{I}_x)_2 + A_2(\bar{y} - y_2)^2\right]$$
$$= 2\left[315 + 9.96(11.07 - 7.5)^2\right] + \left[\frac{16(1)^3}{12} + (16 \times 1)(11.07 - 15.5)^2\right]$$
$$= 1199 \text{ in.}^4$$

$$\bar{r}_x = \sqrt{\frac{\bar{I}_x}{A}} = \sqrt{\frac{1199 \text{ in.}^4}{35.9 \text{ in.}^2}} = 5.78 \text{ in.}$$ ◄

8-33



For C 15 x 33.9 [from Table A-3(a)]:
$A_1 = 9.96 \text{ in.}^2$
$(\bar{I}_y)_1 = 8.13 \text{ in.}^4$
$\bar{x} = 0.787 \text{ in.}$

$A = 2A_1 + A_2 = 2(9.96) + 16 \times 1 = 35.9 \text{ in.}^2$

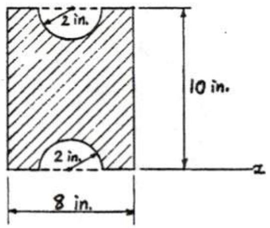
$$\bar{I}_y = 2\left[(\bar{I}_y)_1 + A_1x_1^2\right] + (\bar{I}_y)_2 = 2[8.13 + (9.96)(5.387)^2] + \frac{1(16)^3}{12} = 936 \text{ in.}^4$$

$$\bar{r}_y = \sqrt{\frac{\bar{I}_y}{A}} = \sqrt{\frac{936 \text{ in.}^4}{35.9 \text{ in.}^2}} = 5.11 \text{ in.}$$ ◄

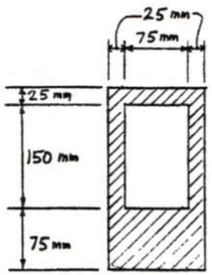
Test Problems for Chapter 8

The following problems may either be given to students for them to practice their problem solving skills or be used as test problems. The answers are provided on the next page.

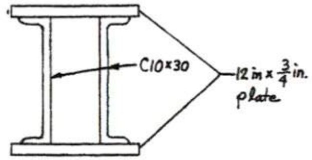
(1) Find the moment of inertia and the radius of gyration of the shaded area about the *x*-axis.



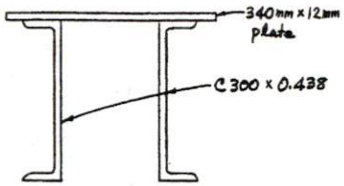
(2) Abeam has a box section as shown. Find the location of the horizontal centroidal axis and determine the moment of inertia of the section about this axis.



(3) Determine the moment of inertia and the radius of gyration of the built-up steel section shown about its horizontal centroidal axis.

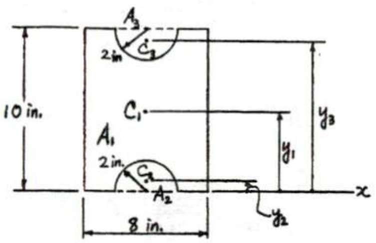


(4) For the built-up steel section shown, find the location of the horizontal centroidal axis and determine its moment of inertia about this axis.



Solutions to Test Problems for Chapter 8

(1)



$A_1 = (8\text{ in.})(10\text{ in.}) = 80\text{ in.}^2$

$A_2 = A_3 = -\pi(2\text{ in.})^2 = -12.57\text{ in.}^2$

$y_1 = 5\text{ in.}$

$y_2 = 0.4244\,r = 0.4244\,(2\text{ in.}) = 0.8488\text{ in.}$

$y_3 = 10\text{ in.} - 0.8488\text{ in.} = 9.15\text{ in.}$

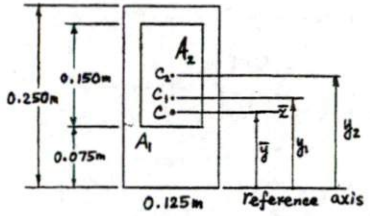
$$I_1 = \frac{bh^3}{12} = \frac{(8\text{ in.})(10\text{ in.})^3}{12} = 666.7\text{ in.}^4$$

$I_2 = I_3 = -0.1098\,r^4 = -0.1098(2\text{ in.})^4 = -1.76\text{ in.}^4$

$I_x = \Sigma\,[\,I + Ay^2]$

$= [666.7 + 80(5)^2]$
$+ [-1.76 + (-12.57)(0.8488)^2]$
$+ [-1.76 + (-12.57)(9.15)^2]$
$= 1063\text{ in.}^4$ ◀

(2)



$A_1 = (0.125\text{ m})(0.250\text{ m}) = 0.03125\text{ m}^2$

$A_2 = -(0.075\text{ m})(0.150\text{ m}) = -0.01125\text{ m}^2$

$$y_1 = \frac{0.250\text{ m}}{2} = 0.125\text{ m}$$

$$y_2 = 0.075\text{ m} + \frac{0.150\text{ m}}{2} = 0.150\text{ m}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2}$$

$$= \frac{(0.03125\text{m}^2)(0.125\text{m}) + (-0.01125\text{m}^2)(0.150\text{m})}{0.03125\text{ m}^2 - 0.01125\text{ m}^2}$$

$= 0.1109\text{ m} = 110.9\text{ mm}$ ◀

$$I_1 = \frac{bh^3}{12} = \frac{(0.125\text{ m})(0.250\text{ m})^3}{12} = 1.628 \times 10^{-4}\text{ m}^4$$

$$I_2 = \frac{bh^3}{12} = -\frac{(0.075\text{ m})(0.150\text{ m})^3}{12} = -2.109\times10^{-5}\text{ m}^4$$

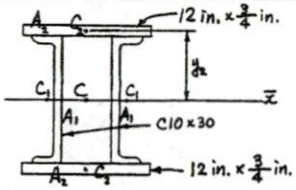
$\bar{I}_x = [I_1 + A_1(\bar{y} - y_1)^2] + [I_2 + A_2(\bar{y} - y_2)^2]$

$= [1.628 \times 10^{-4} + (0.03125)(0.1109 - 0.125)^2]$

$+ [-2.109 \times 10^{-5} + (-0.01125)(0.1109 - 0.150)^2]$

$= -1.31 \times 10^{-4}\text{ m}^4$ ◀

(3)



$A_1 = 8.82\text{ in.}^2$
$d = 10\text{ in.}$
$y_1 = 0$
$I_1 = 103\text{ in.}^4$

$$A_2 = (12\text{ in})\left(\frac{3}{4}\text{ in.}\right) = 9\text{ in.}^2$$

(Cont'd)

Solutions to Test Problems for Chapter 8 (Cont)

(3) (Cont)

$$y_2 = \frac{1}{2}(10.0\text{ in.}) + \frac{1}{2}\left(\frac{3}{4}\text{ in.}\right) = 5.375\text{ in.}$$

$$I_2 = \frac{bh^3}{12} = \frac{(12\text{ in.})\left(\frac{3}{4}\text{ in.}\right)^3}{12} = 0.42\text{ in.}^4$$

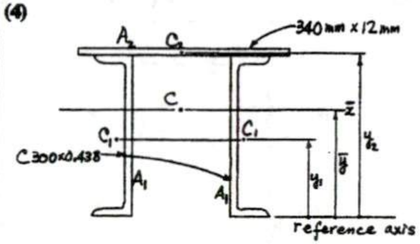
$A = \Sigma A = 2A_1 + 2A_2$
$= 2(8.82\text{ in.}^2) + 2(9\text{ in.}^2) = 35.64\text{ in.}^2$

$\bar{I}_x = 2I_1 + 2[I_2 + A_2y_2^2]$

$= 2(103\text{ in.}^4) + 2[0.42\text{ in.}^4 + (9\text{ in.}^2)(5.375\text{ in.})^2]$
$= 727\text{ in.}^4$ ◀

$$\bar{r}_x = \sqrt{\frac{\bar{I}_x}{A}} = \sqrt{\frac{727\text{ in.}^4}{35.64\text{ in.}^2}} = 4.52\text{ in.}$$ ◀

(4)



$A_1 = 5.69 \times 10^{-3}\text{ m}^2$
$d = 0.3048\text{ m.},$

$$y_1 = \frac{d}{2} = 0.1524\text{ m}$$

$I_1 = 67.4 \times 10^{-6}\text{ m}^4$

$A_2 = (0.340\text{ m})(0.012\text{ m}) = 4.08 \times 10^{-3}\text{ m}^2$

$$y_2 = d + \frac{t}{2} = 0.3048\text{ m} + \frac{1}{2}(0.012\text{ m}) = 0.3108\text{ m}$$

$$I_2 = \frac{bh^3}{12} = \frac{(0.340\text{ m})(0.012\text{ m})^3}{12} = 4.90 \times 10^{-8}\text{ m}^4$$

$$\bar{y} = \frac{2A_1y_1 + A_2y_2}{2A_1 + A_2}$$

$$= \frac{2(5.69\times10^{-3})(0.1524) + (4.08\times10^{-3})(0.3108)}{2(5.69 \times 10^{-3}) + (4.08 \times 10^{-3})}$$

$= 0.1942\text{ m}$

$\bar{I}_x = 2[I_1 + A_1(\bar{y} - y_1)^2] + [I_2 + A_2(\bar{y} - y_2)^2]$

$= 2[67.4 \times 10^{-6} + (5.69 \times 10^{-3})(0.1942 - 0.1524)^2]$

$+ [4.90 \times 10^{-8} + (4.08 \times 10^{-3})(0.1942 - 0.3108)^2]$

$= 2.10 \times 10^{-4}\text{ m}^4$ ◀

9-1

$P = \frac{1}{3}(1.5)(200\ \text{lb}) = 1000\ \text{lb}$

$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.25\ \text{in.})^2 = 0.0491\ \text{in.}^2$

$\sigma = \frac{P}{A} = \frac{1000\ \text{lb}}{0.0491\ \text{in.}^2} = 20\,370\ \text{psi}$ ◀

9-2

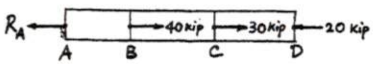
$P = \frac{1}{3}(80\ \text{kN}) = 26.67\ \text{kN}$

$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.015\ \text{m})^2 = 1.767 \times 10^{-4}\ \text{m}^2$

$\sigma = \frac{P}{A} = \frac{26.67\ \text{kN}}{1.767 \times 10^{-4}\ \text{m}^2}$

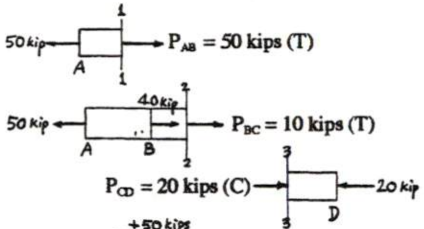
$= 151 \times 10^3\ \text{kPa} = 151\ \text{MPa}$ ◀

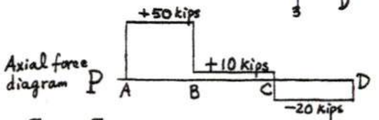
9-3



$\sum F_x = -R_A + 40 + 30 - 20 = 0$

$R_A = +50\ \text{kips} \leftarrow$





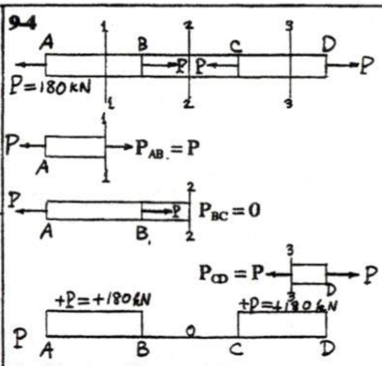
$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(2\ \text{in.})^2 = 3.14\ \text{in.}^2$

$\sigma_{AB} = \frac{P_{AB}}{A} = \frac{+50\ \text{kips}}{3.14\ \text{in.}^2} = +15.92\ \text{ksi (T)}$ ◀

$\sigma_{BC} = \frac{P_{BC}}{A} = \frac{+10\ \text{kips}}{3.14\ \text{in.}^2} = +3.18\ \text{ksi (T)}$ ◀

$\sigma_{CD} = \frac{P_{CD}}{A} = \frac{-20\ \text{kips}}{3.14\ \text{in.}^2} = -6.37\ \text{ksi(C)}$ ◀

9-4



For 70 mm x 50 mm section,

$A = (0.070\ \text{m})(0.050\ \text{m}) = 0.0035\ \text{m}^2$

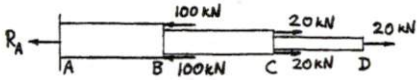
$\sigma_{AB} = \sigma_{CD} = \frac{P}{A} = \frac{+180\ \text{kN}}{0.0035\ \text{m}^2}$

$= 51\,400\ \text{kN/m}^2$

$= 51.4\ \text{MPa (T)}$ ◀

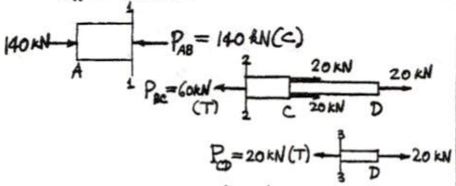
$\sigma_{BC} = 0$ ◀

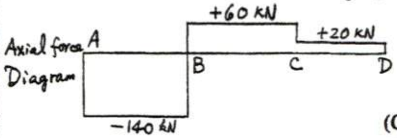
9-5



$\sum F_x = -R_A - (100)(2) + (20)(2) + 20 = 0$

$R_A = -140\ \text{kN} \rightarrow$





(Cont'd)

9-5 (Cont)

$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{-140\ \text{kN}}{\frac{\pi}{4}(0.06\ \text{m})^2} = -49\,500\ \text{kN/m}^2$

$= 49.5\ \text{MPa (C)}$ ◀

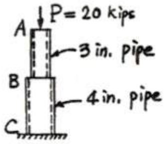
$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{+60\ \text{kN}}{\frac{\pi}{4}(0.04\ \text{m})^2} = +47\,700\ \text{kN/m}^2$

$= 47.7\ \text{MPa (T)}$ ◀

$\sigma_{CD} = \frac{P_{CD}}{A_{CD}} = \frac{+20\ \text{kN}}{\frac{\pi}{4}(0.02\ \text{m})^2} = +63\,700\ \text{kN/m}^2$

$= 63.7\ \text{MPa (T)}$ ◀

9-6



From the appendix, Table A-5(a):
For 3" standard weight steel pipe: $A = 2.23\ \text{in.}^2$
For 4" standard weight steel pipe: $A = 3.17\ \text{in.}^2$

$\sigma_{AB} = \frac{P}{A} = \frac{-20\ \text{kips}}{2.23\ \text{in.}^2} = -8.97\ \text{ksi (C)}$ ◀

$\sigma_{BC} = \frac{P}{A} = \frac{-20\ \text{kips}}{3.17\ \text{in.}^2} = -6.31\ \text{ksi (C)}$ ◀

9-7

$P = 40\ \text{kips (T)}$

$\sigma_{allow} = 22\ \text{ksi}$

$A = \frac{P}{\sigma_{allow}} = \frac{40\ \text{kips}}{22\ \text{kips/in.}^2} = 1.818\ \text{in.}^2$

$A = \frac{\pi}{4}d^2 = 0.7854\,d^2 = 1.818\ \text{in.}^2$

$d = \sqrt{\frac{1.818\ \text{in.}^2}{0.7854}} = 1.52\ \text{in.}$

Use $d = 1\frac{9}{16}$ in. (1.563 in.) ◀

9-8

$P = 200\ \text{kN}$

$\sigma_{allow} = 150\ \text{MPa} = 150\,000\ \text{kN/m}^2$

$A = \frac{P}{\sigma_{allow}} = \frac{200\ \text{kN}}{150\,000\ \text{kN/m}^2} = 0.001333\ \text{m}^2$

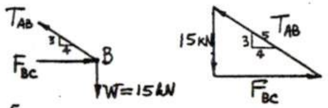
$A = \frac{\pi}{4}d^2 = 0.7854\,d^2 = 0.001333\ \text{m}^2$

$d = \sqrt{\frac{0.001333\ \text{m}^2}{0.7854}} = 0.0412\ \text{m} = 41.2\ \text{mm}$

Use d = 42 mm ◀

9-9

Joint B



$\frac{T_{AB}}{W} = \frac{5}{3}$

$T_{AB} = \frac{5}{3}(W) = \frac{5}{3}(15) = 25\ \text{kN (T)}$

$\sigma_{AB} = \frac{T_{AB}}{A_{AB}} = \frac{25\ \text{kN}}{\frac{\pi}{4}(0.01\ \text{m})^2} = 318\,000\ \text{kN/m}^2$

$= 318\ \text{MPa (T)}$ ◀

9-10

From the solution to Prob 9-9 we find

$T_{AB} = \frac{5}{3}W = \frac{5}{3}(30\ \text{kN}) = 50\ \text{kN}$

$\sigma_{allow} = 150\ \text{MPa} = 150\,000\ \text{kN/m}^2$

$A = \frac{T_{AB}}{\sigma_{allow}} = \frac{50\ \text{kN}}{150\,000\ \text{kN/m}^2} = 3.33 \times 10^{-4}\ \text{m}^2$

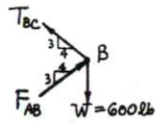
$A = \frac{\pi}{4}d^2 = 0.7854\,d^2 = 3.33 \times 10^{-4}\ \text{m}^2$

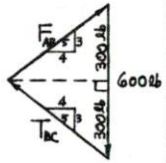
$d = \sqrt{\frac{3.33 \times 10^{-4}\ \text{m}^2}{0.7854}} = 0.0206\ \text{m} = 20.6\ \text{mm}$

Use d = 21 mm ◀

9-11

Joint B





From the force-triangle, by inspection,

$$F_{AB} = T_{BC} = 500 \text{ lb}$$

$$\sigma_{AB} = \frac{500 \text{ lb}}{0.5 \text{ in.}^2} = 1000 \text{ psi (C)} \blacktriangleleft$$

$$\sigma_{BC} = \frac{500 \text{ lb}}{0.025 \text{ in.}^2} = 20\ 000 \text{ psi (T)} \blacktriangleleft$$

9-12

From the solution to Prob 9-11 we find

$$F_{AB} = 2\left(\frac{5}{6}W\right) = \frac{10}{6}(1000 \text{ lb}) = 1667 \text{ lb}$$

$$\sigma_{allow} = 1200 \text{ psi}$$

$$A = \frac{F_{AB}}{\sigma_{allow}} = \frac{1667 \text{ lb}}{1200 \text{ lb / in.}^2} = 1.389 \text{ in.}^2$$

$$A = \frac{\pi}{4}d^2 = 0.7854\, d^2 = 1.389 \text{ in.}^2$$

$$d = \sqrt{\frac{1.389 \text{ in.}^2}{0.7854}} = 1.33 \text{ in.}$$

Use $d = 1\frac{3}{8}$ in. (1.375 in) ◀

9-13

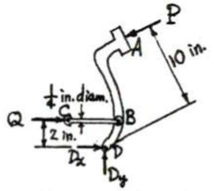
$$A = \frac{\pi}{4}d^2 = 0.7854\,(0.080 \text{ m})^2 = 0.00503 \text{ m}^2$$

$$\sigma_{allow} = 200 \text{ MPa} = 200\ 000 \text{ kN/m}^2$$

$$P = 2A\sigma_{allow} = 2(0.00503 \text{ m}^2)(200\ 000 \text{ kN/m}^2)$$

$$= 2010 \text{ kN} \blacktriangleleft$$

9-14

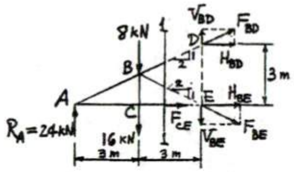


$$\sum M_D = -Q(2) + 20(10) = 0$$

$$Q = 100 \text{ lb}$$

$$\sigma_{BC} = \frac{Q}{A} = \frac{100 \text{ lb}}{\frac{\pi}{4}(0.25 \text{ in.})^2} = 2040 \text{ psi (C)} \blacktriangleleft$$

9-15



$$\sum M_E = -H_{BD}(3) - 24(6) + (8+16)(3) = 0$$

$$H_{BD} = -24 \text{ kN}$$

$$V_{BD} = \frac{1}{2}H_{BD} = -12 \text{ kN}$$

$$F_{BD} = \sqrt{(24)^2 + (12)^2} = 26.8 \text{ kN (C)}$$

$$\sum M_A = -V_{BE}(6) - (8+16)(3) = 0$$

$$V_{BE} = -12 \text{ kN}$$

$$H_{BE} = 2V_{BE} = -24 \text{ kN}$$

$$F_{BE} = \sqrt{(24)^2 + (12)^2} = 26.8 \text{ kN (C)}$$

$$\sum M_B = F_{CE}(1.5) - (24)(3) = 0$$

$$F_{CE} = +48 \text{ kN (T)}$$

Check:

$$\sum F_x = H_{BD} + H_{BE} + F_{CE} = 0$$

$$= -24 + (-24) + 48 = 0 \qquad \text{(Checks)}$$

$$\sum F_y = V_{BD} + V_{BE} + 24 - 8 - 16 = 0$$

$$= -12 - (-12) + 24 - 8 - 16 = 0 \qquad \text{(Checks)}$$

$$\sigma_{allow}^{(T)} = 140 \text{ MPa} = 140\ 000 \text{ kN/m}^2$$

$$\sigma_{allow}^{(C)} = 70 \text{ MPa} = 70\ 000 \text{ kN/m}^2$$

(Cont'd)

9-15 (Cont)

$$(A_{BD})_{req} = \frac{F_{BD}}{\sigma_{allow}^{(C)}} = \frac{26.8 \text{ kN}}{70\ 000 \text{ kN / m}^2}$$

$$= 383 \times 10^{-6} \text{ m}^2 = 383 \text{ mm}^2 \blacktriangleleft$$

$$(A_{BE})_{req} = \frac{F_{BE}}{\sigma_{allow}^{(C)}} = \frac{26.8 \text{ kN}}{70\ 000 \text{ kN / m}^2}$$

$$= 383 \times 10^{-6} \text{ m}^2 = 383 \text{ mm}^2 \blacktriangleleft$$

$$(A_{CE})_{req} = \frac{F_{CE}}{\sigma_{allow}^{(T)}} = \frac{48 \text{ kN}}{140\ 000 \text{ kN / m}^2}$$

$$= 343 \times 10^{-6} \text{ m}^2 = 343 \text{ mm}^2 \blacktriangleleft$$

9-16

$$\tau_u = \frac{P}{A} = \frac{8000 \text{ lb}}{(2 \text{ in.})(4 \text{ in.})} = 1000 \text{ psi} \blacktriangleleft$$

9-17

(a)

$$\tau = \frac{P}{4A_s} = \frac{P}{4\left(\frac{\pi}{4}d^2\right)} = \frac{120 \text{ kN}}{\pi\,(0.02 \text{ m})^2}$$

(4 rivets)

$$= 95\ 500 \text{ kN/m}^2 = 95.5 \text{ MPa} \blacktriangleleft$$

(b)

$$\sigma_b = \frac{P}{4td} = \frac{120 \text{ kN}}{(4)(0.012 \text{ m})(0.02 \text{ m})}$$

$$= 125\ 000 \text{ kN/m}^2 = 125 \text{ MPa} \blacktriangleleft$$

9-18

The shear stress (double shear) in the pin:

$$\tau = \frac{P}{2A} = \frac{10 \text{ kips}}{2\left(\frac{\pi}{4}\right)\left(\frac{3}{4} \text{ in.}\right)} = 11.3 \text{ ksi} \blacktriangleleft$$

The bearing stress between the pin and the plates:

$$\sigma_b = \frac{P}{(2t)d} = \frac{10 \text{ kips}}{\left(\frac{1}{2} \text{ in.}\right)\left(\frac{3}{4} \text{ in.}\right)} = 26.7 \text{ ksi} \blacktriangleleft$$

9-19

$$P = \frac{M}{r} = \frac{950 \text{ N} \cdot \text{m}}{\frac{1}{2}(0.035 \text{ m})} = 54.3 \times 10^3 \text{ N} = 54.3 \text{ kN}$$

(a)

$$A_s = bL = (0.008 \text{ m})(0.050 \text{ m}) = 4.00 \times 10^{-4} \text{ m}^2$$

$$\tau = \frac{P}{A_s} = \frac{54.3 \text{ kN}}{4.00 \times 10^{-4} \text{ m}^2} = 135.7 \times 10^3 \text{ kN/m}^2$$

$$= 135.7 \text{ MPa} \blacktriangleleft$$

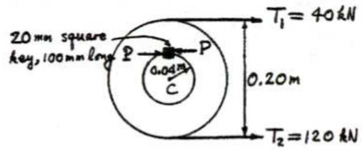
(b)

$$A_b = \frac{h}{2}L = \frac{0.008\text{m}}{2}(0.050 \text{ m}) = 2.00 \times 10^{-4} \text{ m}^2$$

$$\sigma_b = \frac{P}{A_b} = \frac{54.3 \text{ kN}}{2.00 \times 10^{-4} \text{ m}^2} = 272 \times 10^3 \text{ kN/m}^2$$

$$= 272 \text{ MPa} \blacktriangleleft$$

9-20



$$M = (T_2 - T_1)\,r_{pulley} = (120 \text{ kN} - 40 \text{ kN})(0.10 \text{ m})$$

$$= 8.00 \text{ kN} \cdot \text{m}$$

$$P = \frac{M}{r} = \frac{8.00 \text{ kN} \cdot \text{m}}{0.04 \text{ m}} = 200 \text{ kN}$$

(a)

$$\tau = \frac{P}{A_s} = \frac{200 \text{ kN}}{(0.02 \text{ m})(0.10 \text{ m})} = 100\ 000 \text{ kN / m}^2$$

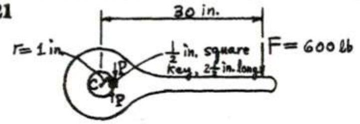
$$= 100 \text{ MPa} \blacktriangleleft$$

(b)

$$\sigma_b = \frac{P}{A_b} = \frac{200 \text{ kN}}{\left(\frac{0.02 \text{ m}}{2}\right)(0.1 \text{ m})} = 200\ 000 \text{ kN / m}^2$$

$$= 200 \text{ MPa} \blacktriangleleft$$

9-21



$M = (600\text{ lb})(30\text{ in.}) = 18\,000\text{ lb}\cdot\text{in.}$

$$P = \frac{M}{r} = \frac{18\,000\text{ lb}\cdot\text{in.}}{1\text{ in.}} = 18\,000\text{ lb}$$

(a)

$$\tau = \frac{P}{A_s} = \frac{18\,000\text{ lb}}{\left(\frac{1}{2}\text{ in.}\right)\left(2\frac{1}{2}\text{ in.}\right)} = 14\,400\text{ psi}$$

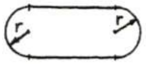
$= 14.4$ ksi ◄

(b)

$$\sigma_b = \frac{P}{A_b} = \frac{18\,000\text{ lb}}{\left(\frac{1}{4}\text{ in.}\right)\left(2\frac{1}{2}\text{ in.}\right)} = 28\,800\text{ psi}$$

$= 28.8$ ksi ◄

9-22



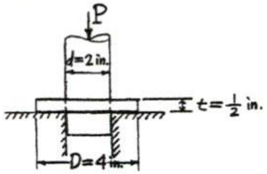
radius $r = 0.04$ m
thickness $t = 0.004$ m

$A_s = (2\pi r + 2L)t$
$= [2\pi\,(0.04\text{ m}) + 2\,(0.12\text{ m})]\,(0.004\text{ m})$
$= 0.001\,965\text{ m}^2$

$P_{min} = A_s\,\tau_u = (0.001\,965\text{ m}^2)(300\,000\text{ kN/m}^2)$

$= 590$ kN ◄

9-23



(a)

$A_b = \frac{\pi}{4}[(4\text{ in.})^2 - (2\text{ in.})^2] = 9.42\text{ in.}^2$

$P = \sigma_b A_b = (4000\text{ lb/in}^2)(9.42\text{ in.}^2)$

$= 37\,700$ lb

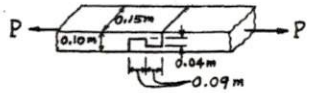
$= 37.7$ kips ◄

(b)

$$\tau = \frac{P}{A_s} = \frac{P}{(\pi d)(t)}$$

$$= \frac{37.7\text{ kips}}{\pi(2\text{ in.})\left(\frac{1}{2}\text{ in.}\right)} = 12.0\text{ ksi}$$ ◄

9-24



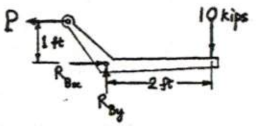
$$\tau = \frac{P}{A_s} = \frac{50\text{ kN}}{(0.09\text{ m})(0.15\text{ m})} = 3700\text{ kN/m}^2$$

$= 3.70$ MPa ◄

$$\sigma_b = \frac{P}{A_b} = \frac{50\text{ kN}}{(0.04\text{ m})(0.15\text{ m})} = 8330\text{ kN/m}^2$$

$= 8.33$ MPa ◄

9-25



$\sum M_B = P(1) - (10)(2) = 0$
$P = 20$ kips
$\sum F_x = R_{Bx} - 20 = 0$
$R_{Bx} = 20$ kips →
$\sum F_y = R_{By} - 10 = 0$
$R_{By} = 10$ kips ↑

(Cont'd)

9-25 (Cont)

$$R_B = \sqrt{(20)^2 + (10)^2} = 22.4\text{ kips}$$

The pin at B is in double shear. Thus each area carries one-half of the load.

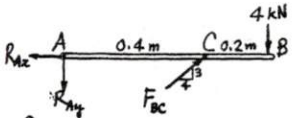
$$A_s = \frac{R_B/2}{\tau_{allow}} = \frac{22.4\text{ kips}/2}{15\text{ kip/in.}^2} = 0.747\text{ in.}^2$$

$A = \frac{\pi}{4}d^2 = 0.7854\,d^2 = 0.747\text{ in.}^2$

$$d = \sqrt{\frac{0.747\text{ in.}^2}{0.7854}} = 0.975\text{ in.}$$

Use $d = 1$ in. ◄

9-26



$\sum M_A = \frac{3}{5}F_{BC}(0.4) - (4)(0.6) = 0$
$F_{BC} = 10$ kN (C)
$\sum F_x = -R_{Ax} - \left(\frac{4}{5}\right)(10) = 0$
$R_{Ax} = 8$ kN ←
$\sum F_y = -R_{Ay} + \left(\frac{3}{5}\right)(10) - 4 = 0$
$R_{Ay} = 2$ kN ↓

$$R_A = \sqrt{(8)^2 + (2)^2} = 8.25\text{ kN}$$

The bolt at A is in double shear. Thus each area carries one-half of the load.

$$A_s = \frac{R_A/2}{\tau_{allow}} = \frac{8.25\text{ kN}/2}{120\,000\text{ kN/m}^2} = 3.44\times10^{-5}\text{ m}^2$$

$A = \frac{\pi}{4}d_A^2 = 0.7854\,d_A^2 = 3.44\times10^{-5}\text{ m}^2$

$$d_A = \sqrt{\frac{3.44\times10^{-5}\text{ m}^2}{0.7854}}$$

$= 6.62\times10^{-3}$ m $= 6.62$ mm

Use $d_A = 7$ mm ◄

$R_B = F_{BC} = 10$ kN

The bolt at B is in double shear. Thus each area carries one-half of the load.

$$A_s = \frac{R_B/2}{\tau_{allow}} = \frac{10\text{ kN}/2}{120\,000\text{ kN/m}^2} = 4.17\times10^{-5}\text{ m}^2$$

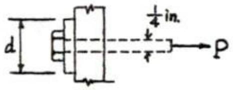
$A = \frac{\pi}{4}d_A^2 = 0.7854\,d_A^2 = 4.17\times10^{-5}\text{m}^2$

$$d_B = \sqrt{\frac{4.17\times10^{-5}\text{ m}^2}{0.7854}}$$

$= 7.28\times10^{-3}$ m $= 7.28$ mm

Use $d_B = 8$ mm ◄

9-27



$P = \sigma A = (20\text{ kip/in.}^2)\left(\frac{\pi}{4}\right)(0.25\text{ in.})^2$
$= 0.982$ kips $= 982$ lb

$$A_b = \frac{P}{(\sigma_b)_{allow}} = \frac{982\text{ lb}}{300\text{ lb/in.}^2} = 3.27\text{ in.}^2$$

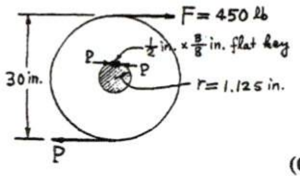
$A = \frac{\pi}{4}(d^2 - 0.25^2)$
$= 0.7854\,(d^2 - 0.0625) = 3.27\text{ in.}^2$

$$d^2 - 0.0625 = \frac{3.27}{0.7854} = 4.163$$

$d = \sqrt{4.163 + 0.0625} = 2.06$ in.

Use $d = 2\frac{1}{16}$ in. (2.0625 in.) ◄

9-28



(Cont'd)

9-28 (Cont)

$M = (450\text{ lb})(30\text{ in.}) = 13\ 500\text{ lb}\cdot\text{in.}$

$$P = \frac{M}{r} = \frac{13\ 500\text{ lb}\cdot\text{in.}}{1.125\text{ in.}} = 12\ 000\text{ lb}$$

$$A_s = \frac{P}{\tau_{allow}} = \frac{12\ 000\text{ lb}}{8000\text{ lb/in.}^2} = 1.5\text{ in.}^2$$

$$A_s = bL = (0.5\text{ in.})L = 1.5\text{ in.}^2$$

$$L = 3.0\text{ in.}$$

$$A_b = \frac{P}{(\sigma_b)_{allow}} = \frac{12\ 000\text{ lb}}{20\ 000\text{ lb/in.}^2} = 0.6\text{ in.}^2$$

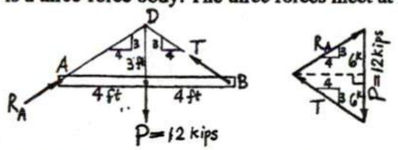
$$A_b = \left(\frac{h}{2}\right)L = \left(\frac{3/8}{2}\right)L = (0.1875)L = 0.6\text{ in.}^2$$

$$L = 3.2\text{ in.}$$

$$\text{Use } L = 3\frac{1}{4}\text{ in.} \quad ◄$$

9-29

AB is a three-force body. The three forces meet at D.



From the force triangle, by inspection, we have

$$T = R_A = 10\text{ kips}$$

(a) Size of eye bar:

$$A = \frac{T}{\sigma_{allow}} = \frac{10\text{ kips}}{20\text{ kip/in.}^2} = 0.5\text{ in.}^2$$

$$A = \frac{\pi}{4}d_{bar}^2\ d^2 = 0.7854\ d_{bar}^2 = 0.5\text{ in.}^2$$

$$d_{bar} = \sqrt{\frac{0.5\text{ in.}^2}{0.7854}} = 0.798\text{ in.}$$

$$\text{Use } d_{bar} = \frac{13}{16}\text{ in.} \quad ◄$$

(b) Size of the pin at A:

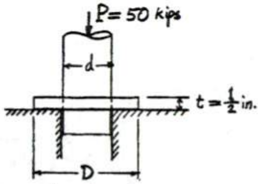
$$A_S = \frac{R_A/2}{\tau_{allow}} = \frac{10\text{ kips}/2}{12\text{ kip/in.}^2} = 0.417\text{ in.}^2$$

$$A_S = \frac{\pi}{4}d_{pin}^2 = (0.7854)\,d_{pin}^2 = 0.417\text{ in.}^2$$

$$d_{pin} = \sqrt{\frac{0.417\text{ in.}^2}{0.7854}} = 0.729\text{ in.}$$

$$\text{Use } d_{pin} = \frac{3}{4}\text{ in.} \quad ◄$$

9-30



Compressive stress in column:

$$A = \frac{P}{\sigma_{allow}} = \frac{50\text{ kips}}{20\text{ kip/in.}^2} = 2.5\text{ in.}^2$$

$$A = \frac{\pi}{4}d^2 = 0.7854\ d^2 = 2.5\text{ in.}^2$$

$$d = \sqrt{\frac{2.5\text{ in.}^2}{0.7854}} = 1.78\text{ in.}$$

Shear stress in collar:

$$A_s = \frac{P}{\tau_{allow}} = \frac{50\text{ kips}}{15\text{ kip/in.}^2} = 3.33\text{ in.}^2$$

$$A_s = \pi\,d\,t = \pi\,d\left(\frac{1}{2}\text{ in.}\right) = 1.571\ d = 3.33\text{ in.}^2$$

$$d = 2.12\text{ in.}$$

$$\text{Use } d = 2\frac{1}{8}\text{ in.}\quad (2.125\text{ in.}) \quad ◄$$

Bearing stress between the collar and the support:

$$A_b = \frac{P}{(\sigma_b)_{allow}} = \frac{50\text{ kips}}{5\text{ kip/in.}^2} = 10\text{ in.}^2$$

$$A = \frac{\pi}{4}(D^2 - 2.125^2) = 0.7854\,(D^2 - 4.52) = 10\text{ in.}^2$$

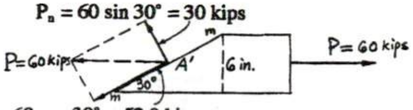
$$D^2 - 4.52 = 12.73$$

$$D = \sqrt{12.73 + 4.52} = 4.12\text{ in.}$$

$$\text{Use } D = 4\frac{1}{4}\text{ in.}\quad (4.25\text{ in.}) \quad ◄$$

9-31

$$P_n = 60\sin 30^\circ = 30\text{ kips}$$



$$P_s = 60\cos 30^\circ = 52.0\text{ kips}$$

$$\text{thickness } t = \frac{1}{2}\text{ in.}$$

$$b_{m\text{-}m} = \frac{6}{\cos 30^\circ} = 12\text{ in.}$$

$$A' = (12)\left(\frac{1}{2}\right) = 6\text{ in.}^2$$

$$\sigma_\theta = \frac{P_n}{A'} = \frac{30\text{ kips}}{6\text{ in.}^2} = 5\text{ ksi} \quad ◄$$

$$\tau_\theta = \frac{P_t}{A} = \frac{52\text{ kips}}{6\text{ in.}^2} = 8.67\text{ ksi} \quad ◄$$

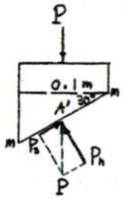
9-32

$$\tau_{max} = \frac{1}{2}\sigma = \frac{1}{2}\,\frac{P}{(2)(4\text{ in.}^2)} \geq 800\text{ psi}$$

$$P \geq (800)(2)(2)(4) = 12\ 800\text{ lb}$$

$$P_{min} = 12.8\text{ kips} \quad ◄$$

9-33



The square cross section is: 0.1 m x 0.1 m

$$b_{m\text{-}m} = \frac{0.1\text{ m}}{\cos 30^\circ} = 0.1155\text{ m}$$

$$A' = (0.1\text{ m})(0.1155\text{ m}) = 0.01155\text{ m}^2$$

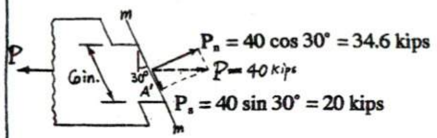
$$P_s = P\sin 30^\circ = 0.5\,P$$

$$\tau_\theta = \frac{P_s}{A'} = \frac{0.5\,P}{0.01155\text{ m}^2} = 2500\text{ kN/m}^2$$

From which we get

$$P = 57.8\text{ kN} \quad ◄$$

9-34



$$P_n = 40\cos 30^\circ = 34.6\text{ kips}$$

$$P_s = 40\sin 30^\circ = 20\text{ kips}$$

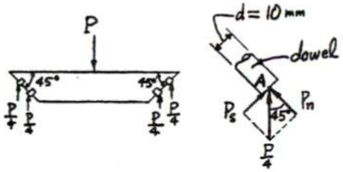
$$\text{thickness } t = \frac{1}{2}\text{ in.}$$

$$A' = (6\text{ in.})\left(\frac{1}{2}\text{ in.}\right) = 3\text{ in.}^2$$

$$\sigma_\theta = \frac{P_n}{A'} = \frac{34.6\text{ kips}}{3\text{ in.}^2} = 11.5\text{ ksi} \quad ◄$$

$$\tau_\theta = \frac{P_s}{A'} = \frac{20\text{ kips}}{3\text{ in.}^2} = 6.67\text{ ksi} \quad ◄$$

9-35



$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.01\text{ m})^2 = 7.85\times 10^{-5}\text{ m}^2$$

$$P_s = \frac{P}{4}\sin 45^\circ = 0.1768\,P$$

$$\tau = \frac{P_s}{A} = \frac{0.1768\,P}{7.85\times 10^{-5}\text{ m}^2} = \frac{P}{4.44\times 10^{-4}\text{ m}^2} \leq \tau_{allow} = 8000\text{ kN/m}^2$$

$$P \leq (8000\text{ kN/m}^2)(4.44\times 10^{-4}\text{ m}^2) = 3.55\text{ kN}$$

$$P_{max} = 3.55\text{ kN} \quad ◄$$

9-36

Solving p from Eq. 9-17 we find

$$p_{max} = \frac{t\,\sigma_{allow}}{r_i} = \frac{\left(\frac{1}{8}\text{ in.}\right)(8000\text{ lb / in.}^2)}{8\text{ in.}}$$

$$= 125\text{ psi}$$ ◀

9-37

From Eq. 9-19

$$t_{req} = \frac{p\,r_i}{\sigma_{allow}} = \frac{(3.5\text{ MPa})(0.300\text{ m})}{140\text{ MPa}}$$

$$= 0.0075\text{ m} = 7.5\text{ mm}$$

Use t = 8 mm ◀

9-38

Solving p from Eq. 9-20 we find

$$t_{req} = \frac{2t\sigma_{allow}}{r_i} = \frac{2\left(\frac{1}{4}\text{in.}\right)(6000\text{ psi})}{4\text{ in.}}$$

$$= 750\text{ psi}$$ ◀

9-39

$$p = \frac{P}{A} = \frac{P}{\frac{\pi}{4}d^2} = \frac{30\text{ kN}}{\frac{\pi}{4}(0.040\text{ m})^2}$$

$$= 23\,900\text{ kN/m}^2$$

$$= 23.9\text{ MPa}$$

From Eq. 9-19 we find

$$t_{req} = \frac{p\,r_i}{\sigma_{allow}} = \frac{(23.9\text{ MPa})(0.020\text{ m})}{140\text{ MPa}}$$

$$= 0.00341\text{ m} = 3.41\text{ mm}$$

Use t = 4 mm ◀

9-40

Solving p from Eq. 9-17 we find

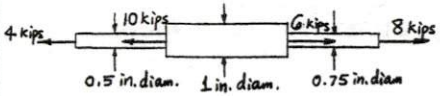
$$p_{max} = \frac{t\,\sigma_u}{r_i} = \frac{(0.5\text{ in.})(65\text{ ksi})}{7.5\text{ in.}}$$

$$= 4.33\text{ ksi}$$ ◀

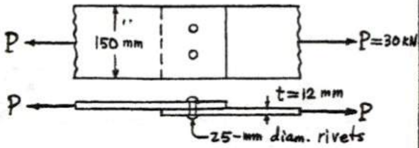
Test Problems for Chapter 9

The following problems may either be given to students for them to practice their problem solving skills or be used as test problems. The answers are provided on the next page.

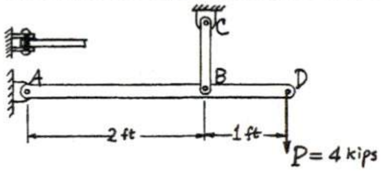
(1) For the stepped, circular bar subjected to the axial forces shown, find the normal stresses in the three segments.



(2) The lap joint shown is connected by two 25-mm diameter rivets. The load P is 30 kN. Determine (a) the average shear stress in the rivets, and (b) the bearing stress between the rivets and the plates.



(3) For the structure shown, the load P is 4 kips and the pin at A is in double shear. Determine the required diameters of the tie rod BC and the pin at A to the nearest sixteenth of an inch. The allowable tensile stress is 22 ksi and the allowable shear stress is 15 ksi.

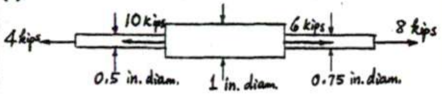


(4) A short concrete post have a rectangular section 50 mm x 100 mm. Determine the maximum compressive load that can be applied to the post without causing a maximum shear stress to exceed 6000 kPa.

(5) A cylindrical pressure vessel with inside diameter of 10 in. and a wall thickness of 1/2 in. is made of steel having an allowable tensile stress of 8000 psi. Determine the maximum internal pressure that the vessel can withstand.

Solutions to Test Problems for Chapter 9

(1)



$P_{AB} = +4$ kips
$P_{BC} = +14$ kips
$P_{AB} = +8$ kips

$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{+4 \text{ kips}}{\frac{\pi}{4}(0.5 \text{ in.})^2} = +15.92 \text{ ksi(T)}$$ ◄

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{+14 \text{ kips}}{\frac{\pi}{4}(1.0 \text{ in.})^2} = +17.8 \text{ ksi(T)}$$ ◄

$$\sigma_{CD} = \frac{P_{CD}}{A_{CD}} = \frac{+8 \text{ kips}}{\frac{\pi}{4}(0.75 \text{ in.})^2} = +18.1 \text{ ksi(T)}$$ ◄

(2)

(a)

$$\tau_{allow} = \frac{P}{2A_s} = \frac{30 \text{ kN}}{2\frac{\pi}{4}(0.025 \text{ m})^2}$$

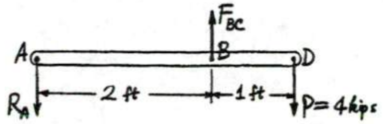
$$= 30\ 600 \text{ kN/m}^2 = 30.6 \text{ MPa}$$ ◄

(b)

$$\sigma_b = \frac{P}{A_b} = \frac{P}{td} = \frac{30 \text{ kN}}{(0.012 \text{ m})(0.025 \text{ m})}$$

$$= 100\ 000 \text{ kN/m}^2 = 100 \text{ MPa}$$ ◄

(3)



$$\sum M_C = F_{BC}(2 \text{ ft}) - (4 \text{ kips})(3 \text{ ft}) = 0$$
$F_{BC} = 6$ kips (T)

$$\sum F_y = -R_A + 6 \text{ kips} - 4 \text{ kips} = 0$$
$R_A = 2$ kips ↓

$$A_{BC} = \frac{F_{BC}}{\sigma_{allow}} = \frac{6 \text{ kips}}{22 \text{ kip/in.}^2} = 0.273 \text{ in.}^2$$

$$A_{BC} = \frac{\pi}{4}d_{BC}^2 = 0.7854\, d_{BC}^2 = 0.273 \text{ in.}^2$$

$$d_{BC} = \sqrt{\frac{0.273 \text{ in.}^2}{0.7854}} = 0.590 \text{ in.}$$

Use $d = \frac{5}{8}$ in. tie rod ◄

$$A_{pin} = \frac{\frac{1}{2}R_A}{\tau_{allow}} = \frac{\frac{1}{2}(2 \text{ kips})}{15 \text{ kip/in.}^2} = 0.0667 \text{ in.}^2$$

$$A_{pin} = \frac{\pi}{4}d_{pin}^2 = 0.7854\, d_{pin}^2 = 0.0667 \text{ in.}^2$$

$$d_{pin} = \sqrt{\frac{0.0667 \text{ in.}^2}{0.7854}} = 0.291 \text{ in.}$$

Use $d = \frac{5}{16}$ in. tie rod ◄

(4)

From Equation 9-16, the maximum shear stress at 45° inclined plane is one-half of the normal stress.

$$\tau_{max} = \frac{P}{2A} = \frac{P}{2(0.05 \text{ m})(0.10 \text{ m})} = 6000 \text{ ksi}$$

$$P = 2(6000 \text{ kN/m}^2)(0.005 \text{ m}^2) = 60 \text{ kN}$$ ◄

(5)

$$\sigma_c = \frac{pr_i}{t}$$

$$p_{max} = \frac{t\,\sigma_{allow}}{r_i} = \frac{(0.5 \text{in.})(8000 \text{ lb/in.}^2)}{10 \text{ in.}}$$

$$= 400 \text{ psi}$$ ◄

10-1

(a) $$\varepsilon = \frac{\sigma}{E} = \frac{+20 \text{ ksi}}{30 \times 10^3 \text{ ksi}} = +6.67 \times 10^{-4}$$ ◄

(b) $$\delta = \varepsilon L = (6.67 \times 10^{-4})(10 \text{ ft}) = 0.00667 \text{ ft}$$
$$= 0.08 \text{ in. (elongation)}$$ ◄

10-2

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.020 \text{ m})^2 = 3.14 \times 10^{-4} \text{ m}^2$$

Solving L from $\delta = \frac{PL}{AE}$ we get

$$L = \frac{\delta AE}{P} = \frac{(0.0035 \text{ m})(3.14 \times 10^{-4} \text{ m}^2)(70 \times 10^6 \text{ kN/m}^2)}{25 \text{ kN}}$$

$$= 3.08 \text{ m}$$ ◄

10-3

$$\sigma = \frac{P}{A} = \frac{3 \text{ kips}}{\frac{\pi}{4}\left(\frac{1}{2} \text{ in.}\right)^2} = 15.3 \text{ ksi (T)}$$ ◄

$$\varepsilon = \frac{\sigma}{E} = \frac{+15.3 \text{ ksi}}{29\ 000 \text{ ksi}} = +0.000\ 527$$ ◄

$$\delta = \varepsilon L = (0.000\ 527)(20 \text{ ft})$$
$$= 0.011 \text{ ft}$$

Elongated length = 20 + 0.011 = 20.011 ft ◄

10-4

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.002 \text{ m})^2 = 3.14 \times 10^{-6} \text{ m}^2$$

Solving E from $\delta = \frac{PL}{AE}$ we get

$$E = \frac{PL}{A\delta} = \frac{(400 \text{ N})(10 \text{ m})}{(3.14 \times 10^{-6} \text{ m}^2)(0.00606 \text{ m})}$$

$$E = 210 \times 10^9 \text{ N/m} = 210 \text{ GPa}$$ ◄

The wire is made of steel. ◄

10-5

Additional load $\Delta P = 20 \text{ lb} - 10 \text{ lb} = 10 \text{ lb}$

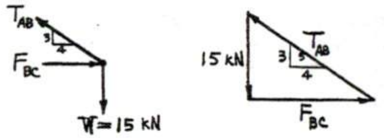
$$\Delta L = \frac{(\Delta P)L}{AE} = \frac{(10 \text{ lb})(100 \times 12 \text{ in.})}{\left(\frac{3}{8} \times \frac{1}{32} \text{ in.}^2\right)(30 \times 10^6 \text{ lb/in.}^2)}$$

$$= 0.034 \text{ in.} = +0.003 \text{ ft}$$

Stretched length = 100.003 ft ◄

10-6

Joint B



$$\frac{T_{AB}}{15 \text{ kN}} = \frac{5}{3}$$

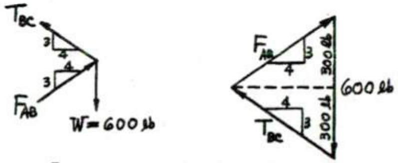
$$T_{AB} = \frac{5}{3}(15 \text{ kN}) = 25 \text{ kN (T)}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.010 \text{ m})^2 = 7.85 \times 10^{-5} \text{ m}^2$$

$$\delta = \frac{PL}{AE} = \frac{(25 \text{ kN})(0.5 \text{ m})}{(7.85 \times 10^{-5} \text{ m}^2)(210 \times 10^6 \text{ kN/m}^2)}$$

$$= 0.758 \times 10^{-3} \text{ m} = 0.758 \text{ mm}$$ ◄

10-7



$$\frac{T_{BC}}{300 \text{ lb}} = \frac{5}{3}$$

$$T_{BC} = \frac{5}{3}(300 \text{ lb}) = 500 \text{ lb}$$

$$\delta = \frac{PL}{AE} = \frac{(500 \text{ lb})(5 \times 12 \text{ in.})}{(0.025 \text{ in.}^2)(30 \times 10^6 \text{ lb/in.}^2)}$$

$$= 0.040 \text{ in. (elongation)}$$ ◄

10-8

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.030\ \text{m})^2 = 7.069 \times 10^{-4} \text{m}^2$

$E = 70\ \text{GPa} = 70 \times 10^9\ \text{N/m}^2$

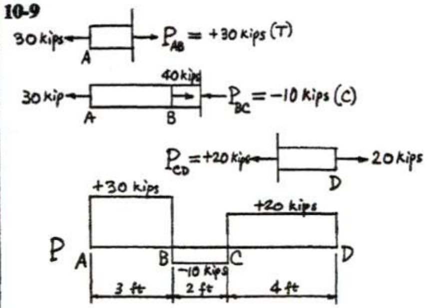
$AE = 4.95 \times 10^7\ \text{N}$

$\delta_C = \delta_{AC} = \delta_{AB} + \delta_{BC}$

$= \frac{(+30\,000\ \text{N})(2.0\ \text{m})}{4.95 \times 10^7\ \text{N}} + \frac{(+20\,000\ \text{N})(1.5\ \text{m})}{4.95 \times 10^7\ \text{N}}$

$= 0.00182\ \text{m} = 1.82\ \text{mm}$ ◄

10-9



$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD}$

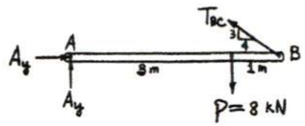
$= \frac{(+30\ \text{kips})(3 \times 12\ \text{in.})}{(2\ \text{in.}^2)(17\,000\ \text{kip/in.}^2)} + \frac{(-10\ \text{kips})(2 \times 12\ \text{in.})}{(2\ \text{in.}^2)(17\,000\ \text{kip/in.}^2)}$

$+ \frac{(+20\ \text{kips})(4 \times 12\ \text{in.})}{(2\ \text{in.}^2)(17\,000\ \text{kip/in.}^2)}$

$= +0.0318\ \text{in.} - 0.0071\ \text{in.} + 0.0282\ \text{in.}$

$= +0.0529\ \text{in.}$ (elongation) ◄

10-10



$\sum M_A = \left(\frac{3}{5} T_{BC}\right)(4\ \text{m}) - (8\ \text{kN})(3\ \text{m}) = 0$

$T = 10\ \text{kN (T)}$

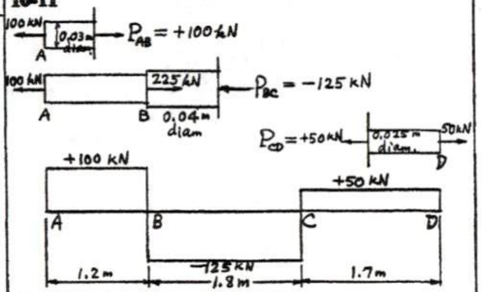
$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.010\ \text{m})^2 = 7.85 \times 10^{-5}\ \text{m}^2$

$\delta = \frac{PL}{AE} = \frac{(10\ \text{kN})(5\ \text{m})}{(7.85 \times 10^{-5}\ \text{m}^2)(210 \times 10^6\ \text{kN/m}^2)}$

$= 0.00303\ \text{m}$

$= 3.03\ \text{mm}$ (elongation) ◄

10-11



$E = 210\ \text{GPa} = 210 \times 10^6\ \text{kN/m}^2$

$A_{AB} = \frac{\pi}{4}(0.03\ \text{m})^2 = 7.07 \times 10^{-4}\ \text{m}^2$

$A_{BC} = \frac{\pi}{4}(0.04\ \text{m})^2 = 1.26 \times 10^{-3}\ \text{m}^2$

$A_{CD} = \frac{\pi}{4}(0.025\ \text{m})^2 = 4.91 \times 10^{-4}\ \text{m}^2$

$\delta_{AB} = \frac{(+100\ \text{kN})(1.2\ \text{m})}{(7.07 \times 10^{-4}\ \text{m}^2)(210 \times 10^6\ \text{kN/m}^2)}$

$= +8.08 \times 10^{-4}\ \text{m}$

$\delta_{BC} = \frac{(-125\ \text{kN})(1.8\ \text{m})}{(1.26 \times 10^{-3}\ \text{m}^2)(210 \times 10^6\ \text{kN/m}^2)}$

$= -8.50 \times 10^{-4}\ \text{m}$

$\delta_{CD} = \frac{(+50\ \text{kN})(1.7\ \text{m})}{(4.91 \times 10^{-4}\ \text{m}^2)(210 \times 10^6\ \text{kN/m}^2)}$

$= +8.24 \times 10^{-4}\ \text{m}$

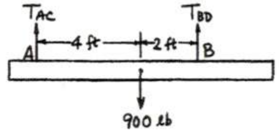
$\delta_{AD} = \delta_{AB} = \delta_{BC} + \delta_{CD}$

$= 8.08 \times 10^{-4}\ \text{m} - 8.50 \times 10^{-4}\ \text{m} + 8.24 \times 10^{-4}\ \text{m}$

$= +7.82 \times 10^{-4}\ \text{m} = 0.782 \times 10^{-3}\ \text{m}$

$= 0.782\ \text{mm}$ (elongation) ◄

10-12



$\sum M_B = -T_{AC}(6\ \text{ft}) + (900\ \text{lb})(2\ \text{ft}) = 0$

$T_{AC} = 300\ \text{lb (T)}$

$\sum M_A = T_{BD}(6\ \text{ft}) - (900\ \text{lb})(4\ \text{ft}) = 0$

$T_{BD} = 600\ \text{lb (T)}$

Check:

$\sum F_y = 300 + 600 - 900 = 0$ (Checks)

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{3}{8}\ \text{in.}\right)^2 = 0.1104\ \text{in.}^2$

$\delta_{AC} = \frac{PL}{AE} = \frac{(300\ \text{lb})(5 \times 12\ \text{in.})}{(0.1104\ \text{in.}^2)(30 \times 10^6\ \text{lb/in.}^2)}$

$= 0.00543\ \text{in.}$ (elongation) ◄

$\delta_{BD} = \frac{PL}{AE} = \frac{(600\ \text{lb})(5 \times 12\ \text{in.})}{(0.1104\ \text{in.}^2)(30 \times 10^6\ \text{lb/in.}^2)}$

$= 0.0109\ \text{in.}$ (elongation) ◄

10-13

From the solution to Prob. 10-12:

$P_{BD} = 600\ \text{lb}, \quad \delta_{AC} = 0.00543\ \text{in.}$

Solving A from $\delta = \frac{PL}{AE}$ we find

$A_{BD} = \frac{PL}{\delta E} = \frac{(600\ \text{lb})(60\ \text{in.})}{(0.00543\ \text{in.})(30 \times 10^6\ \text{lb/in.}^2)}$

$= 0.221\ \text{in.}^2$

$\frac{\pi}{4} d_{BD}^2 = 0.7854\ d_{BD}^2 = 0.221\ \text{in.}^2$

$d_{BD} = \sqrt{\frac{0.221\ \text{in.}^2}{0.7854}} = 0.530\ \text{in.}$ ◄

10-14

$A_1 = \frac{P}{\sigma_{allow}} = \frac{10\ \text{kN}}{150 \times 10^3\ \text{kN/m}^2} = 6.67 \times 10^{-5}\ \text{m}^2$

Solving A from $\delta = \frac{PL}{AE}$ we find

$A_2 = \frac{PL}{\delta E} = \frac{(10\ \text{kN})(1\ \text{m})}{(0.001\ \text{m})(210 \times 10^6\ \text{kN/m}^2)}$

$= 4.76 \times 10^{-5}\ \text{m}^2$

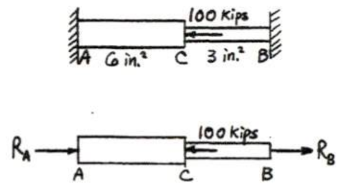
Since A_1 is greater, the stress controls.

$\frac{\pi}{4} d^2 = 0.7854\ d^2 = 6.67 \times 10^{-5}\ \text{m}^2$

$d = \sqrt{\frac{6.67 \times 10^{-5}\ \text{m}^2}{0.7854}} = 0.00922\ \text{m} = 9.22\ \text{m}$

Use d = 10 mm ◄

10-15



$\sum F_x = R_A + R_B - 100 = 0$ (a)

$\delta_{AB} = \delta_{AC} + \delta_{CB} = \frac{(-R_A)(10\ \text{in.})}{(6\ \text{in.}^2)(E)} + \frac{(+R_B)(8\ \text{in.})}{(3\ \text{in.}^2)(E)} = 0$

x 6E: $-10 R_A + 16 R_B = 0$ (b)

10(a) + (b): $26 R_B - 1000 = 0$

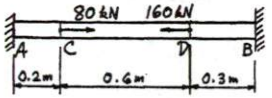
$R_B = 38.5\ \text{kips}$

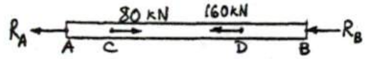
Substitute into (a): $R_A = 100 - 38.5 = 61.5\ \text{kips}$

$\sigma_{AC} = \frac{-R_A}{A_{AC}} = \frac{-61.5\ \text{kips}}{6\ \text{in.}^2} = -10.25\ \text{ksi (C)}$ ◄

$\sigma_{CB} = \frac{+R_B}{A_{CB}} = \frac{+38.5\ \text{kips}}{3\ \text{in.}^2} = +12.83\ \text{ksi(T)}$ ◄

10-16





$\sum F_x = -R_A - R_B + 80 - 160 = 0$

$R_A + R_B = -80$ (a)

$\delta_{AB} = \delta_{AC} + \delta_{CD} + \delta_{DB}$

$$= \frac{(+R_A)(0.2)}{AE} + \frac{(R_A - 80)(0.6)}{AE} + \frac{(-R_B)(0.3)}{AE} = 0$$

x 10AE: $2R_A + 6R_A - 480 - 3R_B = 0$

$8R_A - 3R_B = 480$ (b)

3(a) + (b): $11R_A = -240 + 480$

$R_A = 21.8 \text{ kN} \leftarrow$ ◀

Substitute into (a): $R_B = -80 - 21.8 = 101.8 \text{ kN}$

$= 101.8 \text{ kN} \rightarrow$ ◀

10-17

$P_{al} + P_{br} = 1000$

or $P_{br} = 1000 - P_{al}$ (a)

$\delta_{al} = \delta_{br} + 0.0003 \text{ m}$

$$\frac{P_{al}(0.1503 \text{ m})}{(0.0045 \text{ m}^2)(70 \times 10^6 \text{ kN/m}^2)} = \frac{P_{br}(0.1500 \text{ m})}{(0.0065 \text{ m}^2)(83 \times 10^6 \text{ kN/m}^2)} + 0.0003 \text{ m}$$

Multiplying each term by 10^6 we get

$0.477 P_{al} = 0.278 P_{br} + 300$ (b)

Substituting (a) into (b) gives

$0.477P_{al} = 0.278(1000 - P_{al}) + 300$

$(0.477 + 0.278)P_{al} = 578$

$P_{al} = 766 \text{ kN}$

$P_{br} = 1000 - 766 = 234 \text{ kN}$

$\sigma_{al} = \frac{766 \text{ kN}}{0.0045 \text{ m}^2} = 170 \times 10^3 \text{ kN/m}^2 = 170 \text{ MPa}$ ◀

$\sigma_{br} = \frac{234 \text{ kN}}{0.0065 \text{ m}^2} = 36.0 \times 10^3 \text{ kN/m}^2 = 36.0 \text{ MPa}$ ◀

10-18

Statics: $P_{st} + P_{wd} = P$ (a)

Deformation: $\delta_{st} = \delta_{wd}$

$$\frac{P_{st}(20 \text{ in.})}{(4 \times 0.715 \text{ in.}^2)(30 \times 10^3 \text{ kip/in.}^2)} = \frac{P_{wd}(20 \text{ in.})}{(100 \text{ in.}^2)(1.5 \times 10^3 \text{ kip/in.}^2)}$$

x 10^3: $0.233 P_{st} = 0.133 P_{wd}$

$P_{st} = 0.572 P_{wd}$ (b)

Substitute (b) into (a): $1.572 P_{wd} = P$

$P_{wd} = 0.636 P$

From (b): $P_{st} = 0.572(0.636 P) = 0.364 P$

$P_{st} = A_{st}(\sigma_{st})_{allow} = (4 \times 0.715 \text{ in.}^2)(23 \text{ kip/in.}^2)$

$= 0.364 P$

$P = 181 \text{ kips}$

$P_{wd} = A_{wd}(\sigma_{wd})_{allow} = (100 \text{ in.}^2)(1.7 \text{ kip/in.}^2) = 0.636 P$

$P = 267 \text{ kips}$

$P_{allow} = 181 \text{ kips}$ ◀

10-19

Let the forces in the two wires be P_1 and P_2, then

$P_1 + P_2 = 5 \text{ kips}$ (a)

$\delta_1 = \delta_2 + 0.002$

$$\frac{P_1(10.000 \text{ in.})}{(0.25 \text{ in.}^2)(30 \times 10^3 \text{ kip/in.}^2)} = \frac{P_2(10.002 \text{ in.})}{(0.25 \text{ in.}^2)(30 \times 10^3 \text{ kip/in.}^2)} + 0.002 \text{ in.}$$

Multiplying by 0.25(3000) we get

$P_1 - P_2 = 1.5$ (b)

(a) + (b): $2P_1 = 6.5$

$P_1 = 3.25 \text{ kips}$

From (b): $P_2 = 5 - 3.25 = 1.75 \text{ kips}$

$\sigma_1 = \frac{P_1}{A} = \frac{3.25 \text{ kips}}{0.25 \text{ in.}^2} = 13.0 \text{ ksi (T)}$ ◀

$\sigma_2 = \frac{P_2}{A} = \frac{1.75 \text{ kips}}{0.25 \text{ in.}^2} = 7.00 \text{ ksi (T)}$ ◀

10-20

$2P_{cu} + P_{st} = 50 \text{ kN}$ (a)

$\delta_{cu} = \delta_{st}$

$$\frac{P_{cu}(1.5 \text{ m})}{A(120 \text{ GPa})} = \frac{P_{st}(1 \text{ m})}{A(210 \text{ GPa})}$$

$P_{cu} = 0.381 P_{st}$ (b)

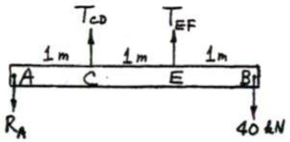
Substituting (b) in (a) gives

$(2 \times 0.381 + 1)P_{st} = 50 \text{ kN}$

$P_{st} = 28.4 \text{ kN}$ ◀

$P_{cu} = 0.381(28.4 \text{ kN}) = 10.8 \text{ kN}$ ◀

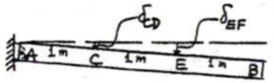
10-21



$\sum M_A = T_{CD}(1) + T_{EF}(2) - 40(3) = 0$

$T_{CD} + 2T_{EF} = 120$ (a)

Since the beam is rigid, the deflected position of the beam is



Thus

$\delta_{EF} = 2\delta_{CD}$, $\frac{T_{EF}L}{AE} = 2\frac{T_{CD}L}{AE}$

From which $T_{EF} = 2T_{CD}$ (b)

Substitute (b) into (a): $5T_{CD} = 120$

$T_{CD} = 24 \text{ kN}$ ◀

$T_{EF} = 48 \text{ kN}$ ◀

10-22

$\Delta T = 100°\text{F} - 60°\text{F} = 40°\text{F}$

$\sigma = E\varepsilon = E\alpha\Delta T$

$= (30 \times 10^6 \text{ lb/in.}^2)(6.5 \times 10^{-6} \text{ in./in./°F})(40 \text{ °F})$

$= 7800 \text{ psi (C)}$ ◀

10-23

$\Delta T = 40°\text{F}$

$L = 40 \text{ ft} = 480 \text{ in.}$

$\delta_T = \delta_P + \frac{1}{16} \text{ in.}$

$\alpha L\Delta T = \frac{PL}{AE} + \frac{1}{16} \text{ in.} = \frac{\sigma L}{E} + \frac{1}{16} \text{ in.}$

$(6.5 \times 10^{-6} \text{ in./in./°F})(480 \text{ in.})(40 \text{ °F})$

$$= \frac{\sigma(480 \text{ in.})}{30 \times 10^6 \text{ lb/in.}^2} + \frac{1}{16} \text{ in.}$$

From which we get

$\sigma = 3890 \text{ psi}$ ◀

10-24

The wire will become slack when the stretch due to temperature is just enough to offset the stretch due to the tensile force, i.e.,

$\delta_T = \delta_P$

$\alpha L\Delta T = \frac{PL}{AE} = \frac{\sigma L}{E}$

$\Delta T = \frac{\sigma}{\alpha E}$

$$= \frac{100 \times 10^6 \text{ N/m}^2}{(12 \times 10^{-6} \text{ m/m/°C})(210 \times 10^9 \text{ N/m}^2)}$$

$= 39.7 \text{ °C}$

$T = 10° + \Delta T = 49.7 \text{ °C}$ ◀

10-25

$\Delta T = -5 \text{ °C} - 25 \text{ °C} = -30 \text{ °C}$

When the temperature drops, the rod tends to shrink, since the support will not yield, it will pull the rod back to its original length with an axial force P. This requires

$|(\delta_{br})_T| + |(\delta_{st})_T| = (\delta_{br})_P + (\delta_{st})_P$

$$\alpha_{br}L_{br}|\Delta T| + \alpha_{st}L_{st}|\Delta T| = \frac{PL_{br}}{A_{br}E_{br}} + \frac{PL_{st}}{A_{st}E_{st}}$$

(Cont'd)

10-25 (Cont)

$(19 \times 10^{-6})(0.5)(30) + (12 \times 10^{-6})(0.75)(30)$

$$= \frac{P(0.5)}{\frac{\pi}{4}(0.10)^2(100 \times 10^6)} + \frac{P(0.75)}{\frac{\pi}{4}(0.05)^2(210 \times 10^6)}$$

Multiplying both sides by 10^6, we get

$$285 + 270 = 0.6366\,P + 1.819\,P$$

$$P = 226 \text{ kN (T)}$$

$$\sigma_{br} = \frac{P}{A_{br}} = \frac{226 \text{ kN}}{\frac{\pi}{4}(0.10 \text{ m})^2} = 28.8 \times 10^3 \text{ kN/m}^2$$

$$= 28.8 \text{ MPa (T)} \quad ◀$$

$$\sigma_{st} = \frac{P}{A_{st}} = \frac{226 \text{ kN}}{\frac{\pi}{4}(0.05 \text{ m})^2} = 115 \times 10^3 \text{ kN/m}^2$$

$$= 115 \text{ MPa (T)} \quad ◀$$

$$\sigma_{al} = \frac{25 \text{ kN}}{(0.0013 \text{ m}^2)} = 19.2 \times 10^3 \text{ kN/m}^2$$

$$= 19.2 \text{ MPa (C)} \quad ◀$$

$$\sigma_{st} = \frac{37.5 \text{ kN}}{(0.00065 \text{ m}^2)} = 57.7 \times 10^3 \text{ kN/m}^2$$

$$= 57.7 \text{ MPa (C)} \quad ◀$$

(b) Temperature drop for zero stress in alum. post

$P_{st} = 50$ kN, $P_{al} = 0$

$$|(\delta_{st})_P| + |(\delta_{st})_T| = |(\delta_{al})_T|$$

$$\frac{(50)L}{(0.00065)(210 \times 10^6)} + (12 \times 10^{-6})L\,|\Delta T|$$

$$= (23.4 \times 10^{-6})\,L\,|\Delta T|$$

Multiplying by $10^6/L$ we get

$$366.3 + 12\,|\Delta T| = 23.4\,|\Delta T|$$

$$\Delta T = 32.1\ ^\circ\text{C (decrease)} \quad ◀$$

10-26

(a) Stresses in posts

$$2P_{st} + P_{al} = 100 \text{ kN} \qquad \text{(a)}$$

$$\delta_{st} = \delta_{al}$$

$$\frac{P_{st}L}{(0.00065 \text{ m}^2)(210 \times 10^6 \text{ kN/m}^2)} = \frac{P_{al}L}{(0.0013 \text{ m}^2)(70 \times 10^6 \text{ kN/m}^2)}$$

Multiplying by $(0.00065)(210 \times 10^6)/L$, we get

$$P_{st} = 1.5\,P_{al} \qquad \text{(b)}$$

Substituting (b) in (a) gives

$$4P_{al} = 100 \text{ kN}$$

$$P_{al} = 25 \text{ kN}$$

$$P_{st} = 1.5\,(25 \text{ kN}) = 37.5 \text{ kN}$$

10-27

$\Delta T = 150°F - 60°F = 90°F$

$A_{st} = \frac{\pi}{4}(1 \text{ in.})^2 = 0.785 \text{ in.}^2$

$A_{al} = \frac{\pi}{4}[(1.5 \text{ in.})^2 - (1 \text{ in.})^2] = 0.982 \text{ in.}^2$

$(\delta_{st})_T + (\delta_{st})_P = (\delta_{al})_T - |(\delta_{al})_P|$

$$\alpha_{st} L\Delta T + \frac{PL}{A_{st}E_{st}} = \alpha_{al}L\Delta T - \frac{PL}{A_{al}E_{al}}$$

$$(6.5 \times 10^{-6})(90) + \frac{P}{(0.785)(30 \times 10^6)}$$

$$= (13 \times 10^{-6})(90) - \frac{P}{(0.982)(10 \times 10^6)}$$

Multiplying each term by 10^6 we get

$$585 + 0.0425P = 1170 - 0.1018P$$

$$P = 4054 \text{ lb}$$

$$\sigma_{st} = \frac{4054 \text{ lb}}{0.785 \text{ in.}^2} = 5160 \text{ psi (T)} \quad ◀$$

$$\sigma_{al} = \frac{4054 \text{ lb}}{0.982 \text{ in.}^2} = 4130 \text{ psi (C)} \quad ◀$$

10-28

$$\sigma = \frac{P}{A} = \frac{10 \text{ kN}}{\left(\frac{\pi}{4}\right)(0.01 \text{ m})^2} = 127\,300 \text{ kN/m}^2$$

$$\varepsilon_a = \frac{\delta}{L} = \frac{0.0544 \text{ mm}}{50 \text{ mm}} = 0.001\,088$$

$$E = \frac{\sigma}{\varepsilon_a} = \frac{127\,300 \text{ kN/m}^2}{0.001\,088}$$

$$= 117 \times 10^6 \text{ kN/m}^2$$

$$= 117 \text{ GPa} \quad ◀$$

$$\varepsilon_t = \frac{\delta_D}{D} = \frac{-0.0039 \text{ mm}}{10 \text{ mm}} = -0.00037$$

$$\mu = -\frac{\varepsilon_t}{\varepsilon_a} = -\frac{-0.00037}{0.001088} = 0.358 \quad ◀$$

$$G = \frac{E}{2(1 + \mu)}$$

$$= \frac{117 \text{ GPa}}{2(1 + 0.358)} = 43 \text{ GPa} \quad ◀$$

10-29

$$E = \frac{\sigma_p}{\varepsilon_p} = \frac{42.0 \text{ ksi}}{0.0014} = 30 \times 10^3 \text{ ksi}$$

$$\mu = \frac{E}{2G} - 1$$

$$= \frac{30 \times 10^3 \text{ ksi}}{2(11.6 \times 10^3 \text{ ksi})} - 1$$

$$= 0.293$$

$$\varepsilon_t = -\mu\varepsilon_a = -\mu\varepsilon_p = (-0.293)(+0.0014)$$

$$= -0.000410$$

$$\delta_D = \varepsilon_t D = (-0.000410)(0.505 \text{ in.}) = -2.07 \times 10^{-4} \text{ in.}$$

$$= -2.07 \times 10^{-4} \text{ in. (contraction)} \quad ◀$$

10-30

$$\mu = \frac{E}{2G} - 1$$

$$= \frac{70 \text{ GPa}}{(2)(26.3 \text{ GPa})} - 1 = 0.331$$

$$\sigma = \frac{P}{A} = \frac{P}{bt} = \frac{10 \text{ kN}}{(0.02 \text{ m})(0.005 \text{ m})}$$

$$= 100\,000 \text{ kN/m}^2 \text{ (T)}$$

$$\varepsilon_a = \frac{\sigma}{E} = \frac{+100\,000 \text{ kN/m}^2}{70 \times 10^6 \text{ kN/m}^2} = +0.001\,43$$

$$\delta_L = \varepsilon_a L = (+0.001\,43)(100 \text{ mm})$$

$$= +0.143 \text{ mm (elongation)} \quad ◀$$

$$\varepsilon_t = -\mu\varepsilon_a = (-0.331)(+0.00143) = -0.000\,473$$

$$\delta_b = \varepsilon_t b = (-0.000\,473)(20 \text{ mm})$$

$$= -0.00947 \text{ mm (contraction)} \quad ◀$$

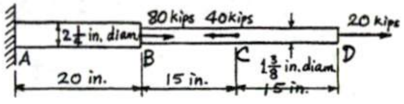
$$\delta_t = \varepsilon_t t = (-0.000\,473)(5 \text{ mm})$$

$$= -0.00237 \text{ mm (contraction)} \quad ◀$$

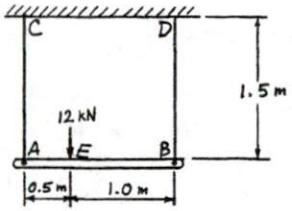
Test Problems for Chapter 10

The following problems may either be given to students for them to practice their problem solving skills or be used as test problems. The answers are provided on the next page.

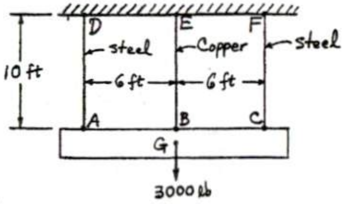
(1) For the stepped, steel bar of circular cross sections subjected to the axial forces shown, determine (*a*) the total axial deformation between *A* and *D*, and (*b*) the change of diameter in segment *AB*. For steel, E = 30 000 ksi, μ = 0.29.



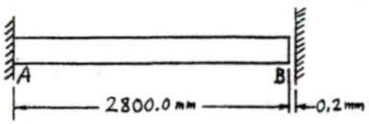
(2) The rigid beam *AB* is suspended by two steel wires *AC* and *BD* as shown. The wires are of 10 mm diameter and have a modulus of elasticity of E = 210 GPa. Determine the elongations of each wire due to the 12 kN load applied at point *E* as shown.



(3) The 3000-lb uniform concrete platform is suspended by three wires spaced as shown. The wires are 10 ft long and of 1/2 in. diameter. The two outer wires are steel and the middle one is copper. The modili of elasticity are: E_{st} = 30 000 ksi, E_{cu} = 17 000 ksi. Determine the stresses in the wires.



(4) A steel bar is placed between two rigid, unyielding supports as shown. At 20°C, the bar has a length of 2800.0 mm and a gap of 0.2 mm from the right support. Given that E_{st} = 210 GPa, α_{st} = 12 x 10^{-6}/°C. Determine the stress in the bar when the temperature is elevated to 60°C.



Solutions to Test Problems for Chapter 10

(1)

(a)

P_{AB} = +80 - 40 + 20 = +60 kips
P_{BC} = -40 + 20 = -20 kips
P_{CD} = +20 kips

$$A_{AB} = \frac{\pi}{4}(2.25\text{ in.})^2 = 3.98\text{ in.}^2$$

$$A_{BD} = \frac{\pi}{4}(1.375\text{ in.})^2 = 1.48\text{ in.}^2$$

$$\delta_{AB} = \frac{(+60\text{ kips})(20\text{in.})}{(3.98\text{in.}^2)(30\ 000\text{ kip/in.}^2)} = +0.01005\text{ in.}$$

$$\delta_{BC} = \frac{(-20\text{ kips})(15\text{ in.})}{(1.48\text{ in.}^2)(30\ 000\text{ kip/in.}^2)} = -0.00676\text{ in.}$$

$$\delta_{CD} = \frac{(+20\text{ kips})(15\text{ in.})}{(1.48\text{ in.}^2)(30\ 000\text{ kip/in.}^2)} = +0.00676\text{ in.}$$

δ_{AD} = +0.01005 in. - 0.00676 in. + 0.00676 in.
= +0.01005 in. (Elogation) ◀

(b)

$$\sigma_{AB} = \frac{+60\text{ kips}}{3.98\text{ in.}^2} = +15.1\text{ksi}$$

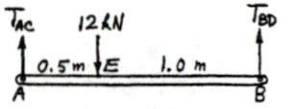
$$\varepsilon_{AB} = \frac{\sigma_{AB}}{E} = \frac{15.1\text{ ksi}}{30\ 000\text{ kip/in.}^2} = +0.000503$$

$\varepsilon_t = -\mu\varepsilon_{AB}$ = -0.29(+0.000503)

$$= -0.000146 = \frac{\delta_D}{D}$$

δ_D = -0.000146 (2.25 in.) = 0.000 328 in. ◀

(2)



$\Sigma M_A = T_{BD}(1.5\text{ m}) - (12\text{ kN})(0.5\text{ m}) = 0$
T_{BD} = 4 kN
$\Sigma F_y = T_{BC}$ - 12 kN + 4 kN = 0
T_{BC} = 8 kN

$$A = \frac{\pi}{4}(0.010\text{ m})^2 = 7.85 \times 10^{-5}\text{ m}^2$$

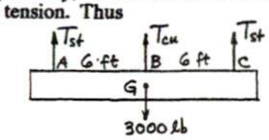
AE = (7.85 x 10^{-5} m²)(210 x 10^{5} kN/m²)
= 1.65 x 10^{4} kN

$$\delta_{AC} = \frac{(8\text{ kN})(1.5\text{ m})}{1.65\text{x}10^4\text{ kN}} = 0.000\ 727\text{ m} = 0.727\text{ mm}$$ ◀

$$\delta_{BD} = \frac{(4\text{ kN})(1.5\text{ m})}{1.65\text{x}10^4\text{ kN}} = 0.000\ 364\text{ m} = 0.364\text{ mm}$$ ◀

(3)

Due to symmetry, the two steel bars are subjected to the same tension. Thus



$\Sigma F_y = 2T_{st} + T_{cu} - 3000\text{ lb} = 0$ (a)

All three wires must have the same elongation. Thus

$$\frac{T_{st}L_{st}}{A_{st}E_{st}} = \frac{T_{cu}L_{cu}}{A_{cu}E_{cu}}$$

Since $A_{st} = A_{cu}$, $L_{st} = L_{cu}$, we have

$$T_{st} = \frac{E_{st}}{E_{cu}}T_{cu} = \frac{30\ 000}{17\ 000}T_{cu} = 1.76T_{cu}$$ (b)

Substituting into (a) gives
$2(1.76T_{cu}) + T_{cu} = 3000$
T_{cu} = 664 lb
T_{st} = 1.76(664 lb) = 1169 lb

$$A = \frac{\pi}{4}\left(\frac{1}{4}\text{in.}\right)^2 = 0.0491\text{ in.}^2$$

$$\sigma_{st} = \frac{1169\text{lb}}{0.0491\text{ in.}} = 23\ 800\text{ psi} = 23.8\text{ ksi}$$ ◀

$$\sigma_{cu} = \frac{664\text{ lb}}{0.0491\text{ in.}} = 13\ 500\text{ psi} = 13.5\text{ ksi}$$ ◀

(4)

$\delta_T = \alpha L\Delta T = (12 \times 10^{-6})(2800)(40)$ = 1.344 mm
δ_P = 1.344 mm- 0.2 mm = 1.144 mm = 0.001144 m

$$\delta_P = \frac{PL}{AE} = \frac{\sigma L}{E}$$

$$\sigma = \frac{\delta_P E}{L} = \frac{(0.001144\text{ m})(210\text{x}10^3\text{ MN/m}^2)}{2.8\text{ m}}$$

= 85.8 MPa (C) ◀

11-1

Elastic deformation is the deformation that can be recovered fully once the load is removed. Plastic deformation is permanent and cannot be recovered after the load is removed.

11-2

At the yield point, a mild steel specimen continues to elongate without any significant increase in load.

11-3

Necking is a drastic decrease in diameter at a localized area. Necking usually occurs a little beyond the ultimate strength of ductile materials.

11-4

Ultimate strength is the maximum stress that a meterial can resist before failure. After the ultimate strength is reached, the material will fail at any time, hence the ultimate strength is an important index of the strength of the material.

11-5

For bars of the same length and subjected to the same stress, the one with a lower value of modulus of elasticity stretches more. Hence, the aluminum bar, which has a lower value of modulus of elasticity than that of copper, stretches more.

11-6

The yield point at 0.2% offset is determined by drawing a line parallel to the straight-line portion of the stress-strain diagram, starting from the point on the abscissa with a strain value of 0.2% or 0.002. The intersection of the line and the stress-strain diagram is the yield point.

11-7

For ductile materials, the modulus of elasticity E and the yield strength σ_y obtained from the tension test and the compression test are the same. The tension test provides more information, such as ultimate strength, after the material has yielded. Tensile strength for ductile materials is utilized more than compressive strength. The tensile test is therefore more important than the compression test for ductile materials.

11-8

For brittle materials, the compressive strength is much greater than the tensile strength and hence is more important. Therefore the compression test is more important than the tension test for the brittle materials.

11-9

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.50\text{ in.})^2 = 0.1963\text{ in.}^2$$

$$A' = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.423\text{ in.})^2 = 0.1405\text{ in.}^2$$

(a) $\sigma_u = \dfrac{15\,200\text{ lb}}{0.1963\text{ in.}^2} = 77\,400\text{ psi}$ ◄

(b) $\%\text{ elongation} = \dfrac{2.59\text{ in.} - 2.00\text{ in.}}{2.00\text{ in.}} \times 100\%$

$= 29.5\%$ ◄

(c) % reduction in area

$= \dfrac{0.1963\text{ in.}^2 - 0.1405\text{ in.}^2}{0.1963\text{ in.}^2} \times 100\%$

$= 28.4\%$ ◄

11-10

$A = (0.050\text{ m})(0.015\text{ m}) = 7.50 \times 10^{-4}\text{ m}^2$

$\sigma = \dfrac{80\text{ kN}}{7.50 \times 10^{-4}\text{ m}^2} = 107 \times 10^3\text{ kPa} = 107\text{ MPa}$

$\varepsilon = \dfrac{0.305\text{ mm}}{200\text{ mm}} = 0.001\,53$

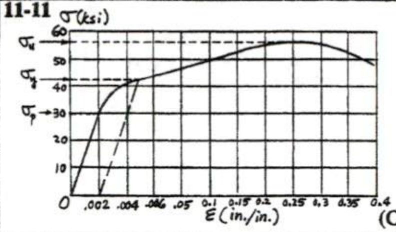
Since $\sigma = 107\text{ MPa} < \sigma_p = 190\text{ MPa}$, Hooke's Law applies.

$E = \dfrac{\sigma}{\varepsilon} = \dfrac{107\text{ MPa}}{0.001\,53}$

$= 70 \times 10^3\text{ MPa} = 70\text{ GPa}$ ◄

The bar is probably made of aluminum. ◄

11-11



(Cont'd)

11-11 (Cont)

(a) $\sigma_p = 30\text{ ksi}$ ◄

(b) $E = \dfrac{30\text{ ksi}}{0.002} = 15 \times 10^3\text{ ksi}$ ◄

(c) $(\sigma_y)_{0.2\%} = 42\text{ ksi}$ ◄

(d) $\sigma_u = 56\text{ ksi}$ ◄

(e) $\%\text{ elongation} = \dfrac{L' - L}{L} \times 100\%$

$= \dfrac{2.78 - 2.00}{2.00} \times 100\% = 39\%$ ◄

(f) $\%\text{ reduction in area} = \dfrac{A - A'}{A} \times 100\%$

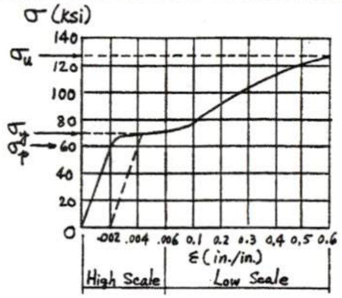
$= \dfrac{\frac{\pi}{4}(0.502)^2 - \frac{\pi}{4}(0.412)^2}{\frac{\pi}{4}(0.502)^2} \times 100\% = 32.6\%$ ◄

11-12

$A = \dfrac{\pi}{4}(0.505\text{ in.})^2 = 0.2003\text{ in.}^2$

Gage Length L= 2 in.

Load (lb)	Stress (ksi)	Elongation (in.)	Strain (in./in.)
200	1.0	0.0000	0.0000
1 000	5.0	0.0003	0.00015
2 000	10.0	0.0006	0.0003
4 000	20.0	0.0012	0.0006
6 000	30.0	0.0019	0.00095
8 000	39.9	0.0026	0.0013
10 000	49.9	0.0033	0.00165
12 000	59.9	0.0039	0.00195
13 400	66.9	0.045	0.00225
13 600	67.9	0.054	0.0027
13 800	68.9	0.063	0.00315
14 000	69.9	0.090	0.0045
14 400	71.9	0.0118	0.0059
15 200	75.9	0.0167	0.00835
16 000	79.9	0.0212	0.0106
16 800	83.9	0.0263	0.0132
17 600	87.9	0.0327	0.0164
18 400	91.9	0.0380	0.0190
19 200	95.9	0.0440	0.0220
20 000	99.9	0.0507	0.0254
20 800	104	0.0580	0.0290
21 600	108	0.0660	0.0330
22 400	112	0.0780	0.0390
25 400	127	specimen broke	



(a) $\sigma_p = 42\text{ ksi}$ ◄

(b) $E = \dfrac{\sigma}{\varepsilon} = \dfrac{60\text{ ksi}}{0.002\text{ in./in.}} = 30 \times 10^3\text{ ksi}$ ◄

(c) $(\sigma_y)_{0.2\%} = 70\text{ ksi}$ ◄

(d) $\sigma_u = 127\text{ ksi}$ ◄

(e) $\%\text{ elongation} = \dfrac{L' - L}{L} \times 100\%$

$= \dfrac{2.31 - 2.00}{2.00} \times 100\% = 15.5\%$ ◄

(f) $\%\text{ reduction in area} = \dfrac{A - A'}{A} \times 100\%$

$= \dfrac{\frac{\pi}{4}(0.505)^2 - \frac{\pi}{4}(0.450)^2}{\frac{\pi}{4}(0.505)^2} \times 100\%$

$= 20.6\%$ ◄

11-13

$\sigma_{allow} = \dfrac{\sigma_y}{F.S.} = \dfrac{36\text{ ksi}}{2} = 18\text{ ksi}$

$A = \dfrac{T}{\sigma_{allow}} = \dfrac{2.5\text{ kips}}{18\text{ kip / in.}^2} = 0.139\text{ in.}^2$

$\dfrac{\pi}{4}d^2 = 0.7854\, d^2 = 0.139\text{ in.}^2$

$d = \sqrt{\dfrac{0.139\text{ in.}^2}{0.7854}} = 0.421\text{ in.}$

Use $d = \dfrac{7}{16}$ in. (0.4375 in.) ◄

11-14

$P = \frac{1}{2}(800\ \text{kN}) = 400\ \text{kN}$

$\sigma_{allow} = \frac{\sigma_y}{F.S.} = \frac{700\ \text{MPa}}{2} = 350\ \text{MPa}$

$A = \frac{P}{\sigma_{allow}} = \frac{400\ \text{kN}}{350\ 000\ \text{kN/m}^2} = 0.001\ 143\ \text{m}^2$

$\frac{\pi}{4}d^2 = 0.7854\ d^2 = 0.001\ 143\ \text{m}^2$

$d = \sqrt{\frac{0.001\ 143\ \text{m}^2}{0.7854}} = 0.0381\ \text{m} = 38.1\ \text{mm}$

Use d = 39 mm ◀

11-15

$P = 40\ \text{kips}$

$\sigma_{allow} = \frac{\sigma_u}{F.S.} = \frac{90\ \text{ksi}}{4} = 22.5\ \text{ksi}$

$A = \frac{P}{\sigma_{allow}} = \frac{40\ \text{kips}}{22.5\ \text{kip/in.}^2} = 1.78\ \text{in}^2$

$A = a^2 = 1.78\ \text{in.}^2$

$a = \sqrt{1.78\ \text{in.}^2} = 1.33\ \text{in.}$

Use $d = 1\frac{3}{8}$ in. x $1\frac{3}{8}$ in. section ◀

11-16

For strength:

$A = \frac{P}{\sigma_{allow}} = \frac{75\ \text{kN}}{150\ 000\ \text{kN/m}^2} = 0.0005\ \text{m}^2$

For stiffness:

$A = \frac{PL}{\delta E} = \frac{(75\ \text{kN})(0.500\ \text{m})}{(0.00025\ \text{m})(210 \times 10^6\ \text{kN/m}^2)}$

$= 0.000\ 714\ \text{m}^2$

Stiffness controls.

$A = \frac{\pi}{4}d^2 = 0.7854\ d^2 = 0.000\ 714\ \text{m}^2$

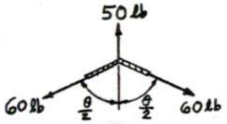
$d = \sqrt{\frac{0.000\ 714\ \text{m}^2}{0.7854}} = 0.0302\ \text{m} = 30.2\ \text{mm}$

Use d = 31 mm ◀

11-17

Tensile strength of the rope = 180 lb

$T_{allow} = \frac{\text{Tensile strength}}{F.S.} = \frac{180\ \text{lb}}{3} = 60\ \text{lb}$



$2(60\ \text{lb})\cos(\theta/2) = 50\ \text{lb}$

$\cos(\theta/2) = 0.417$

$\theta/2 = 65.4°$

$\theta = 130.8°$ ◀

11-18

Abrupt change in geometry of a member causes non-uniform distribution of stress. Stresses of high intensities (compared to the average stress) occur at some localized region. This phenomenon is called stress concentration. Stress concentration factor is the ratio of the maximum stress to the average stress over the net cross-sectional area.

11-19

Due to the lack of yield point in brittle materials, additional load causes continuous increase of the maximum stress at the point of stress concentration. The material will start to crack when σ_u is reached. Therefore, stress concentration is an important design factor for brittle materials even when the member is subjected to static load.

11-20

When a member is subjected to cyclic stress variations, progressive cracks are likely to start gradually from the point of maximum stress for both ductile and brittle materials. Therefore, in this case stress concentration becomes an important design factor for both type of materials.

11-21

(a) $F = \frac{1}{4}$ in., $b = 4\ \text{in.} - 2\left(\frac{1}{4}\right) = 3.5\ \text{in.}$

$\frac{r}{b} = \frac{0.25\ \text{in.}}{3.5\ \text{in.}} = 0.071$

From the chart, K = 2.6

$\sigma_{max} = K\,\sigma_{avg} = K\frac{P}{bt}$

$= (2.6)\frac{8000\ \text{lb}}{(3.5\ \text{in.})(0.5\ \text{in.})}$

$= 11\ 900\ \text{psi} = 11.9\ \text{ksi}$ ◀

(b) $r = \frac{1}{2}$ in., $b = 4\ \text{in.} - 2\left(\frac{1}{2}\right) = 3.0\ \text{in.}$

$\frac{r}{b} = \frac{0.5\ \text{in.}}{3.0\ \text{in.}} = 0.17,\quad k = 2.35$

$\sigma_{max} = K\,\sigma_{avg} = K\frac{P}{bt}$

$= (2.35)\frac{8000\ \text{lb}}{(3.0\ \text{in.})(0.5\ \text{in.})}$

$= 12\ 500\ \text{psi} = 12.5\ \text{ksi}$ ◀

(c) $r = 1$ in., $b = 4\ \text{in.} - 2(1\ \text{in.}) = 2\ \text{in.}$

$\frac{r}{b} = \frac{1\ \text{in.}}{2\ \text{in.}} = 0.5,\quad K = 2.12$

$\sigma_{max} = K\,\sigma_{avg} = K\frac{P}{bt}$

$= (2.12)\frac{8000\ \text{lb}}{(2.0\ \text{in.})(0.5\ \text{in.})}$

$= 17\ 000\ \text{psi} = 17.0\ \text{ksi}$ ◀

11-22

$\frac{r}{b} = \frac{10\ \text{mm}}{20\ \text{mm}} = 0.5,\quad K = 1.42$

$\sigma_{max} = K\frac{P}{A_{net}} = (1.42)\frac{5\ \text{kN}}{(0.02\ \text{m})(0.005\ \text{m})}$

$= 71 \times 10^3\ \text{kPa} = 71\ \text{MPa}$ ◀

11-23

$\frac{r}{b} = \frac{1}{4} = 0.25,\quad K = 1.9$

$\sigma_{max} = K\,\sigma_{avg} = K\frac{P}{bt}$

$= (1.9)\frac{P}{(4\ \text{in.})(0.5\ \text{in.})} = \frac{P}{1.05\ \text{in.}^2} \le 15\ \text{ksi}$

$P \le (15\ \text{kips/in.}^2)(1.05\ \text{in.}^2) = 15.8\ \text{kips}$

$P_{max} = 15.8\ \text{kips}$ ◀

11-24

Circular Hole:

$b = 100\ \text{mm} - 2(15\ \text{mm}) = 70\ \text{mm}$

$\frac{r}{b} = \frac{15\ \text{mm}}{70\ \text{mm}} = 0.214,\quad K = 2.3$

$\sigma_{max} = K\,\sigma_{avg} = K\frac{P}{bt}$

$= (2.3)\frac{P}{(0.070\ \text{m})(0.01\ \text{m})}$

$= \frac{P}{3.04 \times 10^{-4}\ \text{m}^2} \le 160\ \text{MPa} = 160\ 000\ \text{kN/m}^2$

$P \le (160\ 000\ \text{kN/m}^2)(3.04 \times 10^{-4}\ \text{m}^2) = 48.7\ \text{kN}$

(Cont'd)

11-24 (Cont)

Quarter-circular Fillet:

$$\frac{r}{b} = \frac{20\text{ mm}}{60\text{ mm}} = 0.33, \qquad K = 1.55$$

$$\sigma_{max} = K\,\sigma_{avg} = K\frac{P}{bt}$$

$$= (1.55)\,\frac{P}{(0.060\text{ m})(0.01\text{ m})}$$

$$= \frac{P}{3.87 \times 10^{-4}\text{ m}^2} \le 160\,000\text{ kN/m}^2$$

$$P \le (160\,000\text{ kN/m}^2)(3.87 \times 10^{-4}\text{ m}^2) = 61.9\text{ kN}$$

$$P_{max} = 48.7\text{ kN}$$ ◀

11-25

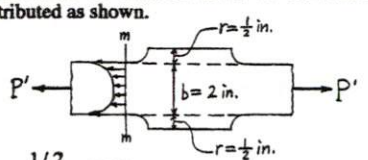
$$\sigma_{allow} = \frac{\sigma_y}{F.S.} = \frac{36\text{ ksi}}{3} = 12\text{ ksi}$$

For the straight link, there is no change in geometry, hence there is no stress concentration. We have

$$P = A\sigma_{allow} = bt\,\sigma_{allow}$$

$$= (2\text{ in.})\left(\frac{1}{4}\text{ in.}\right)(12\text{ ksi}) = 6\text{ ksi}$$

For the link with an enlarged section, stress distribution at section m-m is non-uniformly distributed as shown.



$$\frac{r}{b} = \frac{1/2}{2} = 0.25$$

From Fig. 11-11, $K = 1.6$

$$P' = \frac{A\,\sigma_{allow}}{K} = \frac{bt\,\sigma_{allow}}{K} = \frac{(2\text{ in.})(1/2\text{ in.})(12\text{ ksi})}{1.6}$$

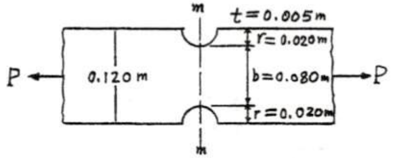
$$= 3.75\text{ ksi}$$

The ratio P/P' is

$$\frac{P}{P'} = \frac{6}{3.75} = 1.6$$

This means that the straight link is 60% stronger than the link with an enlarged section. ◀

11-26



At section m-m: $\frac{r}{b} = \frac{0.020}{0.080} = 0.25$

From Fig. 3-15, $K = 1.9$

(a) P = 40 kN

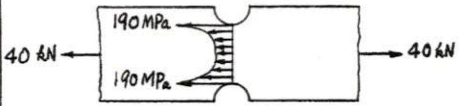
$$\sigma_{avg} = \frac{40\text{ kN}}{(0.080\text{ m})(0.005\text{ m})}$$

$$= 100 \times 10^3\text{ kPa} = 100\text{ MPa}$$

$$\sigma_{max} = K\sigma_{avg} = 1.9 \times 100\text{ MPa}$$

$$= 190\text{ MPa}$$ ◀

The stress distribution in section m-m is



(b) P = 70 kN

$$\sigma_{avg} = \frac{70\text{ kN}}{(0.080\text{ m})(0.005\text{ m})}$$

$$= 175 \times 10^3\text{ kPa} = 175\text{ MPa}$$

$$\sigma_{max} = k\sigma_{avg} = 1.9 \times 175\text{ MPa}$$

$$= 333\text{ MPa} > \sigma_y = 250\text{ MPa}$$

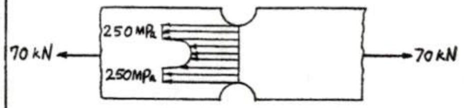
(Cont'd)

11-26 (Cont)

The mild steel yields and the maximum stress remains at σ_y. Thus,

$$\sigma_{max} = \sigma_y = 250\text{ MPa}$$ ◀

The stress distribution in section m-m is



11-27

(a) Allowable Load based on Elastic Design Approach

$$\frac{r}{b} = \frac{0.020}{0.080} = 0.25 \qquad K = 1.9$$

$$\sigma_{max} = \sigma_y = k\sigma_{avg} = k\frac{P_y}{bt}$$

From which

$$P_y = \frac{bt\,\sigma_y}{k}$$

$$= \frac{(0.080\text{ m})(0.005\text{ m})(250\,000\text{ kN/m}^2)}{1.9}$$

$$= 52.6\text{ kN}$$

$$(P_{allow})_{elastic} = \frac{P_y}{F.S.} = \frac{52.6\text{ kN}}{1.65} = 31.9\text{ kN}$$ ◀

(b) Allowable load based on Ultimate Strength Design Approach

$$P_p = A_{net}\sigma_y = (0.080\text{ m})(0.005\text{ m})(250\,000\text{ kN/m}^2)$$

$$= 100\text{ kN}$$

$$(P_{allow})_{plastic} = \frac{P_p}{\text{Load Factor}} = \frac{100\text{ kN}}{1.85}$$

$$= 54.1\text{ kN}$$ ◀

11-28

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\left(\frac{1}{2}\text{ in.}\right)^2 = 0.1963\text{ in.}^2$$

$$P = w(10\text{ ft}) = 10w\text{ (kips)}$$

(a) Allowable load based on Elastic Design Approach

Statics: $2P_{AB} + P_{CD} = P = 10\,w$ (a)

Deformation: $\delta_{AB} = \delta_{CD}$

$$\frac{P_{AB}(4\text{ ft})}{AE} = \frac{P_{CD}(5\text{ ft})}{AE}$$

$$P_{AB} = 1.25\,P_{CD}$$ (b)

Substitute into (a): $2(1.25\,P_{CD}) + P_{CD} = 10\,w$

$$P_{CD} = 2.857\,w$$

$$P_{AB} = 3.571\,w$$

$$P_y = A\sigma_y = (0.1963\text{ in.}^2)(36\text{ ksi}) = 7.067\text{ kips}$$

Equate P_{AB} to P_y

$$3.571\,w = 7.067\text{ kips}$$

$$w = 1.979\text{ kip/ft}$$

$$(w_{allow})_{elastic} = \frac{w}{F.S.} = \frac{1.979}{1.65} = 1.12\text{ kip/ft}$$ ◀

(b) Allowable load based on Ultimate Strength Design Approach

$$P_p = 3A\sigma_y = 3(0.1963\text{ in}^2)(36\text{ kip/in}^2)$$
$$= 21.2\text{ kips}$$

Equate $P = 10\,w$ to P_p

$$10\,w = 21.2\text{ kips}$$

$$w = 2.12\text{ kips}$$

$$(w_{allow})_{plastic} = \frac{w}{\text{Load Factor}} = \frac{2.12\text{ kips}}{1.85}$$

$$= 1.15\text{ kip/ft}$$ ◀

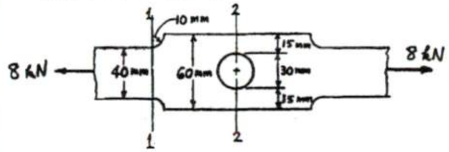
Test Problems for Chapter 11

The following problems may either be given to students for them to practice their problem solving skills or be used as test problems. The answers are provided on the next page.

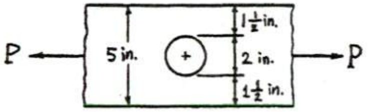
(1) A standard steel specimen having a diameter of 0.502 in. and a gage length of 2.00 in. is used in a tension test. At 6000 lb the extensometer read 0.00212 in. The load at the yield point is 7130 lb and the ultimate load is 11520 lb. The final elongated gage length is 2.53 in. and the final diameter at the necked down section is 0.441 in. Calculate (a) the yield strength, (b) the modulus of elasticity, (c) the ultimate strength, (d) the percent of elongation, and (e) the percent of reduction in area.

(2) A steel cable with yield strength of 350 MPa is used to support a load of 1000 kN. Select the diameter of the cable to the nearest mm using a factor of safety of 2.5 to guard against yielding.

(3) The bar shown is cut from a 6 mm thick steel plate. Determine the maximum normal stress in the bar due to an axial force of P = 8 kN.



(4) A steel machine member of 1/4 in thick has a circular hole in the middle as shown. The steel has a yield strength of 36 ksi. Determine the maximum allowable load that the member can carry based on (a) the elastic design approach using a factor of safety of 1.6, and (b) the ultimate strength design approach using a load factor of 1.85.



Solutions to Test Problems for Chapter 11

(1)

$A_{AB} = \frac{\pi}{4}(0.502 \text{ in.})^2 = 0.198 \text{ in.}^2$

(a) $\sigma_y = \frac{7130 \text{ lb}}{0.198 \text{ in.}^2} = 36\,000 \text{ psi} = 36 \text{ ksi}$ ◀

(b)

$\sigma = \frac{6000 \text{ lb}}{0.198 \text{ in.}^2} = 30\,300 \text{ psi} = 30.3 \text{ ksi}$

$\varepsilon = \frac{0.00202 \text{ in.}}{2.00 \text{ in.}} = 0.00101 \text{ in./in.}$

$E = \frac{30.3 \text{ ksi}}{0.00101 \text{ in./in.}} = 30\,000 \text{ ksi}$ ◀

(c) $\sigma_u = \frac{11\,520 \text{ lb}}{0.198 \text{ in.}^2} = 58\,200 \text{ psi} = 58.2 \text{ ksi}$ ◀

(d) $\% \text{ elongation} = \frac{2.5 \text{ in.} - 2.00 \text{ in.}}{2.00 \text{ in.}} \times 100\%$

$= 26.5\%$ ◀

(e) $A' = \frac{\pi}{4}(0.441 \text{ in.})^2 = 0.153 \text{ in.}^2$

% reduction in area

$= \frac{0.198 \text{ in.}^2 - 0.153 \text{ in.}^2}{0.198 \text{ in.}^2} \times 100\%$

$= 22.7\%$ ◀

(2)

$\sigma_{allow} = \frac{\sigma_y}{F.S.} = \frac{350 \text{ MPa}}{2.5} = 140 \text{ MPa}$

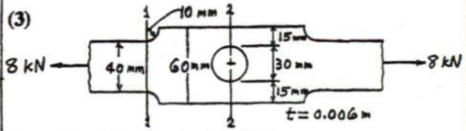
$A_{req} = \frac{P}{\sigma_{allow}} = \frac{1000 \text{ kN}}{140 \times 10^3 \text{ kN/m}^2}$

$= 0.00714 \text{ m}^2 = \frac{\pi}{4} d^2$

$d = \sqrt{\frac{4(0.00714)}{\pi}} = 0.0954 \text{ m} = 95.4 \text{ mm}$

Use d = 96 mm ◀

(3)



<u>At section 1-1, the circular fillet</u>

$\frac{r}{b} = \frac{10 \text{ mm}}{40 \text{ mm}} = 0.25, \quad K = 1.6$

$\sigma_{max} = K\sigma_{avg} = K\frac{P}{bt} = (1.6)\frac{8 \text{ kN}}{(0.04 \text{ m})(0.006 \text{ m})}$

$= 53 \times 10^3 \text{ kN/m}^2 = 53 \text{ MPa}$

<u>At section 2-2, the section with circular fillet</u>

$\frac{r}{b} = \frac{15 \text{ mm}}{30 \text{ mm}} = 0.50, \quad K = 2.12$

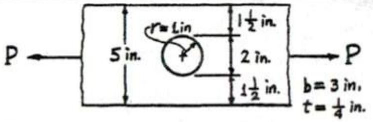
$\sigma_{max} = K\sigma_{avg} = K\frac{P}{bt} = (2.12)\frac{8 \text{ kN}}{(0.03 \text{ m})(0.006 \text{ m})}$

$= 94 \times 10^3 \text{ kN/m}^2 = 94 \text{ MPa}$

Thus the maximum stress is

$\sigma_{max} = 94 \text{ MPa}$ ◀

(4)



(a) <u>Allowable load based on elastic design approach</u>

$\frac{r}{b} = \frac{1 \text{ in.}}{3 \text{ in.}} = 0.33, \quad K = 2.2$

$\sigma_{max} = \sigma_y = K\sigma_{avg} = K\frac{P_y}{bt}$

$P_y = \frac{bt\sigma_y}{K} = \frac{(3 \text{ in.})(1/4 \text{ in.})(36 \text{ in.})}{2.2} = 12.3 \text{ ksi}$

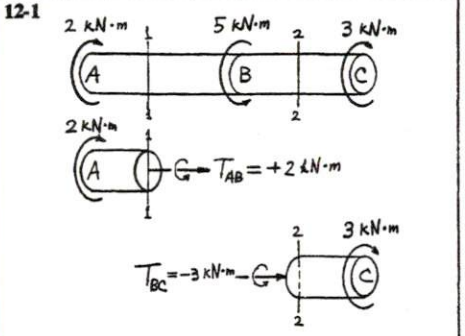
$(P_{allow})_{elastic} = \frac{P_y}{F.S.} = \frac{12.3 \text{ kips}}{1.6} = 7.7 \text{ ksi}$ ◀

(b) <u>Allowable load based on ultimate strength design approach</u>

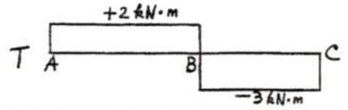
$P_p = A_{net}\sigma_y = (3 \text{ in.} \times 0.25 \text{ in.})(36 \text{ ksi}) = 27 \text{ kips}$

$(P_{allow})_{plastic} = \frac{P_p}{\text{Load Factor}} = \frac{27 \text{ kips}}{1.85} = 14.6 \text{ ksi}$ ◀

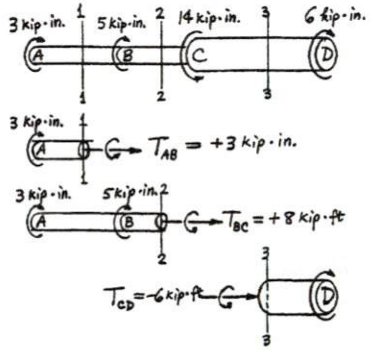
12-1



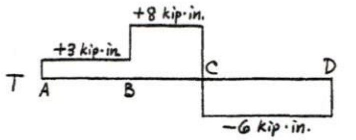
Internal Torque Diagram:



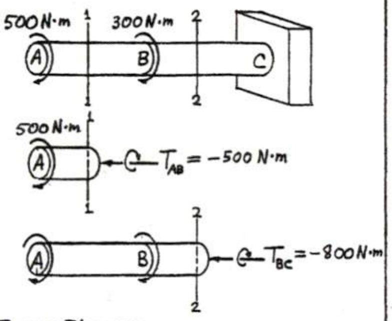
12-2



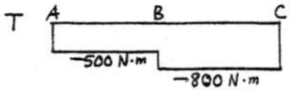
Internal Torque Diagram:



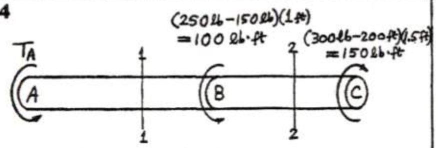
12-3



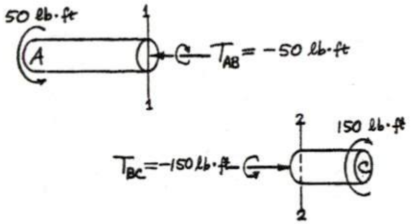
Internal Torque Diagram:



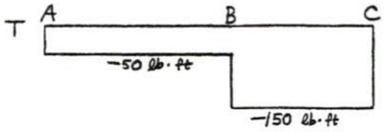
12-4



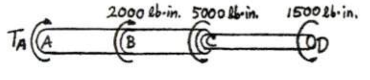
$$\sum M_x = +T_A + 100 - 150 = 0$$
$$T_A = +50 \text{ lb} \cdot \text{ft}$$



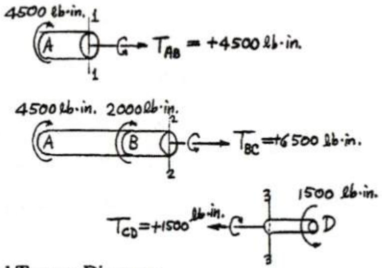
Internal Torque Diagram:



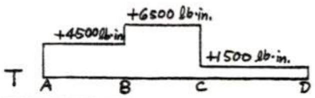
12-5



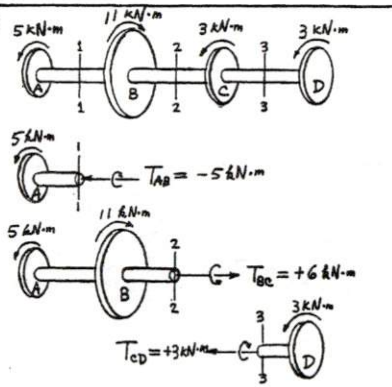
$$\sum M_x = +T_A + 2000 - 5000 - 1500 = 0$$
$$T_A = +4500 \text{ lb} \cdot \text{in.}$$



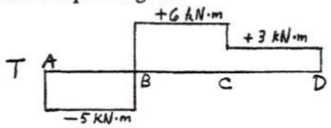
Internal Torque Diagram:



12-6



Internal Torque Diagram:



12-7

$T = 10 \text{ kip} \cdot \text{ft} = 120 \text{ kip} \cdot \text{in.}$

$$J = \frac{\pi}{32} d^4 = \frac{\pi}{32}(4.25 \text{ in.})^4 = 32.03 \text{ in.}^4$$

$$\tau_{max} = \frac{Tc}{J} = \frac{(120 \text{ kip} \cdot \text{in.})\left(\frac{1}{2} 4.25 \text{ in.}\right)}{32.03 \text{ in.}^4}$$
$$= 7.96 \text{ ksi} \blacktriangleleft$$

12-8

$$J = \frac{\pi}{32} d^4 = \frac{\pi}{32}(0.090 \text{ m})^4 = 6.441 \times 10^{-6} \text{ m}^4$$

$$\tau_{max} = \frac{Tc}{J} = \frac{(8.5 \text{ kN} \cdot \text{m})\left(\frac{1}{2} 0.090 \text{ m}\right)}{6.441 \times 10^{-6} \text{ m}^4}$$
$$= 822 \text{ kN/m}^2 = 8.22 \text{ MPa} \blacktriangleleft$$

12-9

$$J = \frac{\pi}{32}(d_o^4 - d_i^4) = \frac{\pi}{32}(4^4 - 3^4) = 7.18 \text{ in.}^4$$

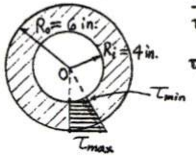
$$\tau_{max} = \frac{Tc}{J} = \frac{(80 \text{ kip} \cdot \text{in.})(2 \text{ in.})}{17.18 \text{ in.}^4}$$
$$= 9.31 \text{ ksi} \blacktriangleleft$$

12-10

$$J = \frac{\pi}{32}(d_o^4 - d_i^4) = \frac{\pi}{32}[(0.050 \text{ m})^4 - (0.040 \text{ m})^4]$$
$$= 3.62 \times 10^{-7} \text{ m}^4$$

$$\tau_{max} = \frac{Tc}{J} = \frac{(890 \text{ N} \cdot \text{m})(0.025 \text{ m})}{3.62 \times 10^{-7} \text{ m}^4}$$
$$= 61.4 \times 10^6 \text{ N/m}^2 = 61.4 \text{ MPa} \blacktriangleleft$$

12-11



$$\tau_{max} = 9000 \text{ psi}$$
$$\frac{\tau_{min}}{\tau_{max}} = \frac{R_i}{R_o} = \frac{4}{6}$$
$$\tau_{min} = \frac{R_i}{R_o}\tau_{max}$$
$$= \frac{4}{6}(9000 \text{ psi})$$
$$= 6000 \text{ psi} \blacktriangleleft$$

12-12

$T = (4000\ \text{lb})(10\ \text{in.}) = 40\ 000\ \text{lb} \cdot \text{in.}$

$$J = \frac{\pi}{32}(d_o^4 - d_i^4) = \frac{\pi}{32}(4^4 - 3^4) = 7.18\ \text{in.}^4$$

$$\tau_{max} = \frac{Tr_o}{J} = \frac{(40\ 000\ \text{lb}\cdot\text{in.})(2\ \text{in.})}{17.18\ \text{in.}^4}$$

$$= 4660\ \text{psi}$$ ◄

$$\tau_{min} = \frac{Tr_i}{J} = \frac{(40\ 000\ \text{lb}\cdot\text{in.})(1.5\ \text{in.})}{17.18\ \text{in.}^4}$$

$$= 3490\ \text{psi}$$ ◄

12-13

$T_{AB} = -80\ \text{N}\cdot\text{m}$

$$J_{AB} = \frac{\pi}{32}d^4 = \frac{\pi}{32}(0.050\ \text{m})^4 = 6.14 \times 10^{-7}\text{m}^4$$

$$(\tau_{max})_{AB} = \frac{Tc}{J} = \frac{(80\ \text{N}\cdot\text{m})(0.025\ \text{m})}{6.14 \times 10^{-7}\ \text{m}^4}$$

$$= 3.26 \times 10^6\ \text{N/m}^2 = 3.26\ \text{MPa}$$

$T_{BC} = -20\ \text{N}\cdot\text{m}$

$$J_{BC} = \frac{\pi}{32}d^4 = \frac{\pi}{32}(0.025\ \text{m})^4 = 3.83 \times 10^{-8}\ \text{m}^4$$

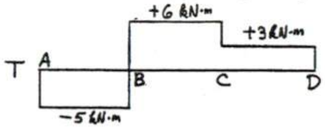
$$(\tau_{max})_{BC} = \frac{Tc}{J} = \frac{(20\ \text{N}\cdot\text{m})(0.0125\ \text{m})}{3.83 \times 10^{-8}\ \text{m}^4}$$

$$= 6.52 \times 10^6\ \text{N/m}^2 = 6.52\ \text{MPa}$$

$$\tau_{max} = (\tau_{max})_{BC} = 6.52\ \text{MPa}$$ ◄

12-14

From Prob. 12-6, the internal torque diagram is



$d_{AB} = 0.075\ \text{m}$ $d_{BC} = 0.085\ \text{m}$ $d_{CD} = 0.070\ \text{m}$

$$(\tau_{AB})_{max} = \frac{Tc}{J} = \frac{(5\ \text{kN}\cdot\text{m})(0.0375\ \text{m})}{\frac{\pi}{32}(0.075\ \text{m})^4}$$

$$= 60\ 400\ \text{kN/m}^2 = 60.4\ \text{MPa}$$

$$(\tau_{BC})_{max} = \frac{Tc}{J} = \frac{(6\ \text{kN}\cdot\text{m})(0.0425\ \text{m})}{\frac{\pi}{32}(0.085\ \text{m})^4}$$

$$= 49\ 800\ \text{kN/m}^2 = 49.8\ \text{MPa}$$

$$(\tau_{CD})_{max} = \frac{Tc}{J} = \frac{(3\ \text{kN}\cdot\text{m})(0.035\ \text{m})}{\frac{\pi}{32}(0.070\ \text{m})^4}$$

$$= 44\ 500\ \text{kN/m}^2 = 44.5\ \text{MPa}$$

$$\tau_{max} = (\tau_{AB})_{max} = 60\ 400\ \text{kN/m}^2 = 60.4\ \text{MPa}$$ ◄

12-15

$$J = \frac{\pi}{32}d^4 = \frac{\pi}{32}(2.5\ \text{in.})^4 = 3.835\ \text{in.}^4$$

$\tau_{allow} = 8000\ \text{psi}$

$$T_{allow} = \frac{\tau_{allow}J}{c} = \frac{(8000\ \text{lb/in.}^2)(3.835\ \text{in.}^4)}{1.25\ \text{in.}}$$

$$= 24\ 500\ \text{lb}\cdot\text{in.}$$

$$= 24.5\ \text{kip}\cdot\text{in.}$$ ◄

12-16

$$J = \frac{\pi}{32}d^4 = \frac{\pi}{32}(0.075\ \text{m})^4 = 3.106 \times 10^{-6}\ \text{m}^4$$

$\tau_{allow} = 60\ \text{MPa} = 60\ 000\ \text{kN/m}^2$

$$T_{allow} = \frac{\tau_{allow}J}{c}$$

$$= \frac{(60\ 000\ \text{kN/m}^2)(3.106 \times 10^{-6}\text{m}^4)}{0.0375\ \text{m}}$$

$$= 4.97\ \text{kN}\cdot\text{m}$$ ◄

12-17

For the solid shaft:

$$J_s = \frac{\pi}{32}d^4 = \frac{\pi}{32}(0.300\ \text{m})^4 = 7.95 \times 10^{-4}\ \text{m}^4$$

$$(T_{allow})_{solid} = \frac{\tau_{allow}J_s}{c} = \frac{\tau_{allow}(7.95 \times 10^{-4}\text{m}^4)}{0.150\ \text{m}}$$

$$= 5.30 \times 10^{-3}\ \tau_{allow}$$

For the hollow shaft:

$$J_h = \frac{\pi}{32}(d_o^4 - d_i^4) = \frac{\pi}{32}[(0.300\ \text{m})^4 - (0.200\ \text{m})^4]$$

$$= 6.38 \times 10^{-4}\ \text{m}^4$$

$$(T_{allow})_{hollow} = \frac{\tau_{allow}J_h}{c}$$

$$= \frac{\tau_{allow}(6.38 \times 10^{-4}\text{m}^4)}{0.150\ \text{m}}$$

$$= 4.25 \times 10^{-3}\ \tau_{allow}$$

% reduction of torsional strength

$$= \frac{(T_{allow})_{solid} - (T_{allow})_{hollow}}{(T_{allow})_{solid}}$$

$$= \frac{5.30 \times 10^{-3}\ \tau_{allow} - 4.25 \times 10^{-3}\ \tau_{allow}}{5.30 \times 10^{-3}\ \tau_{allow}}$$

$$= 0.198 = 19.8\ \%$$ ◄

$$A_s = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.300\ \text{m})^2 = 0.0707\ \text{m}^2$$

$$A_h = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}[(0.300\ \text{m})^2\ (0.200\ \text{m})^2]$$

$$= 0.0393\ \text{m}^2$$

% reduction of weight:

$$= \frac{W_{solid} - W_{hollow}}{W_{solid}} = \frac{A_s\gamma L - A_h\gamma L}{A_s\gamma L}$$

$$= \frac{A_s - A_h}{A_s} = \frac{0.0707 - 0.0393}{0.0707}$$

$$= 0.444 = 44.4\ \%$$ ◄

12-18

If one-half of the core material is removed, the inner circular area is equal to one-half of the cross-sectional area of the solid shaft. Thus

$$\frac{\pi}{4}d_i^2 = \frac{1}{2}\left(\frac{\pi}{4}d^2\right)$$

$$d_i = 0.7071\ d$$

For the solid shaft:

$$J_s = \frac{\pi}{32}d^4$$

$$(T_{allow})_{solid} = \frac{\tau_{allow}J_s}{c} = \frac{\tau_{allow}\frac{\pi}{32}d^4}{d/2}$$

$$= 0.1963\ \tau_{allow}\ d^3$$

(Cont'd)

12-18 (Cont)

For the hollow shaft:

$$J_h = \frac{\pi}{32}(d_o^4 - d_i^4) = \frac{\pi}{32}[d^4 - (0.7071\ d)^4] = 0.07363\ d^4$$

$$(T_{allow})_{hollow} = \frac{\tau_{allow} J_h}{c} = \frac{\tau_{allow}(0.07363\ d^4)}{d/2} = 0.1473\ \tau_{allow}\ d^3$$

% reduction of torsional strength

$$= \frac{(T_{allow})_{solid} - (T_{allow})_{hollow}}{(T_{allow})_{solid}} = \frac{(0.1963 - 0.1473)\tau_{allow}\ d^3}{0.1963\ \tau_{allow}\ d^3} = 0.25 = 25\ \% \quad ◀$$

12-19

$$d_{req} = \sqrt[3]{\frac{16\,T}{\pi\,\tau_{allow}}} = \sqrt[3]{\frac{16\ (1500\ N\cdot m)}{\pi\ (50 \times 10^6\ N/m^2)}} = 0.0535\ m = 53.5\ mm$$

Use d = 54 mm ◀

12-20

$T = 30\ kip\cdot ft = 360\ kip\cdot in.$

$$d_{req} = \sqrt[3]{\frac{16\,T}{\pi\,\tau_{allow}}} = \sqrt[3]{\frac{16\ (360\ kip\cdot in.)}{\pi\ (12\ kip/in.^2)}} = 5.35\ in.$$

Use $d = 5\frac{3}{8}$ in. (5.375 in.) ◀

12-21

From Eq. 12-7,

$\tau_y = 0.57\ \sigma_y = 0.57(36\ ksi) = 20.5\ ksi$

$$\tau_{allow} = \frac{\tau_y}{3} = \frac{20.5\ ksi}{3} = 6.84\ ksi$$

$$k = \frac{d_i}{d_o} = 0.75$$

$$(d_o)_{req} = \sqrt[3]{\frac{16\,T}{\pi\,\tau_{allow}(1-k^4)}} = \sqrt[3]{\frac{16\ (20\ kip\cdot in.)}{\pi\ (6.84\ kip/in.^2)(1-0.75^4)}} = 2.79\ in.$$

$(d_i)_{req} = 0.75(2.79\ in.) = 2.09\ in.$

Use $d_o = 2\frac{13}{16}$ in. (2.8125 in.) ◀

$d_i = 2\frac{1}{16}$ in. (2.0625 in.) ◀

12-22

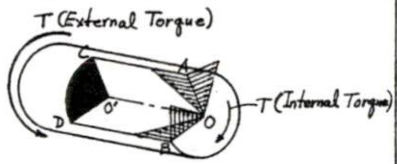
$$(d_o)_{req} = \sqrt[3]{\frac{16\,T}{\pi\,\tau_{allow}(1-k^4)}} = \sqrt[3]{\frac{16\ (12.5\ kN\cdot m)}{\pi\ (70 \times 10^3\ kN/m^2)(1-0.75^4)}} = 0.1100\ m = 110\ mm$$

$(d_i)_{req} = 0.75(110\ mm) = 82.5\ mm$

Use $d_o = 110$ mm ◀

$d_i = 82$ mm ◀

12-23



12-24

Since shear stresses in two perpendicular directions are equal, the allowable shear stress parallel to the grain, which is lower than the one perpendicular to the grain, governs the strength of the shaft.

$$T_{allow} = \frac{\tau_{allow} J}{c} = \frac{(140\ lb/in.^2)\frac{\pi}{32}(6\ in.)^4}{3\ in.} = 5940\ lb\cdot in. \quad ◀$$

12-25

$$T = \frac{63\ 000\ P}{n} = \frac{63\ 000\ (5)}{1500} = 210\ lb\cdot in. \quad ◀$$

12-26

$$T = \frac{9550\ P}{n} = \frac{9550\ (3.75)}{1200} = 29.8\ N\cdot m \quad ◀$$

12-27

$$T = \frac{63\ 000\ P}{n} = \frac{63\ 000\ (100)}{1000} = 6300\ lb\cdot in.$$

$$J = \frac{\pi}{32} d^4 = \frac{\pi}{32}(1.75\ in.)^4 = 0.921\ in.^4$$

$$\tau_{max} = \frac{Tc}{J} = \frac{(6300\ lb\cdot in.)\left(\frac{1}{2}1.75\ in.\right)}{0.921\ in.^4} = 5990\ psi \quad ◀$$

12-28

$$T = \frac{9550\ P}{n} = \frac{9550\ (75)}{1260} = 568\ N\cdot m$$

$$J = \frac{\pi}{32} d^4 = \frac{\pi}{32}(0.035\ m)^4 = 1.47 \times 10^{-7}\ m^4$$

$$\tau_{max} = \frac{Tc}{J} = \frac{(568\ N\cdot m)\left(\frac{1}{2}0.035\ m\right)}{1.47 \times 10^{-7}\ m^4} = 67.6 \times 10^6\ N/m^2 = 67.6\ MPa \quad ◀$$

12-29

$$J = \frac{\pi}{32} d^4 = \frac{\pi}{32}(1\ in.)^4 = 0.0982\ in.^4$$

$$T_{allow} = \frac{\tau_{allow} J}{c} = \frac{(8000\ lb/in.^2)(0.0982\ in.^4)}{0.5\ in.} = 1570\ lb\cdot in. \quad ◀$$

Solve P from: $T = \frac{63\ 000\ P}{n}$

$$P_{allow} = \frac{T_{allow}}{63\ 000} = \frac{(1570)(1000)}{63\ 000} = 25\ hp \quad ◀$$

12-30

$$J = \frac{\pi}{32} d^4 = \frac{\pi}{32}(4\ in.)^4 = 25.1\ in.^4$$

At 100 rpm

$$T = \frac{63\ 000\ P}{n} = \frac{63\ 000\ (200)}{100} = 126\ 000\ lb\cdot in. = 126\ kip\cdot in.$$

$$\tau = \frac{Tc}{J} = \frac{(126\ kip\cdot in.)(2\ in.)}{25.1\ in.^4} = 10.04\ ksi$$

At 300 rpm

$$T = \frac{63\ 000\ P}{n} = \frac{63\ 000\ (200)}{300} = 42\ 000\ lb\cdot in. = 42.0\ kip\cdot in.$$

$$\tau = \frac{Tc}{J} = \frac{(42.0\ kip\cdot in.)(2\ in.)}{25.1\ in.^4} = 3.35\ ksi$$

Thus when the angular velocity is increased by three times, the shear stress is only one-third of the original value, which represents a reduction of two-third in the shear stress.

12-31

$$J = \frac{\pi}{32}(d_o^4 - d_i^4) = \frac{\pi}{32}[(0.050\ m)^4 - (0.035\ m)^4] = 4.66 \times 10^{-7}\ m^4$$

$$\tau_{allow} = \frac{\tau_{allow} J}{c} = \frac{(50 \times 10^6\ N/m^2)(4.66 \times 10^{-7}\ m^4)}{0.025\ m} = 932\ N\cdot m$$

$$P_{allow} = \frac{T_{allow}\ n}{9550} = \frac{(932)(250)}{9550} = 24.4\ kW \quad ◀$$

12-32

$$J = \frac{\pi}{32}(d_o^4 - d_i^4) = \frac{\pi}{32}[(0.550\text{ m})^4 - (0.300\text{ m})^4]$$

$$= 8.19 \times 10^{-3}\text{ m}^4$$

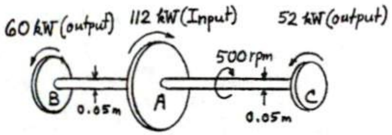
$$T = \frac{9550\,P}{n} = \frac{9550\,(30\,000\text{ kW})}{250\text{ rpm}}$$

$$= 1.146 \times 10^6\text{ N}\cdot\text{m}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{(1.146 \times 10^6\text{ N}\cdot\text{m})\left(\frac{1}{2}0.550\text{ m}\right)}{8.19 \times 10^{-3}\text{ m}^4}$$

$$= 38.5 \times 10^6\text{ N/m}^2 = 38.5\text{ MPa}$$ ◀

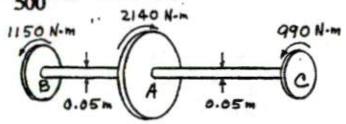
12-33

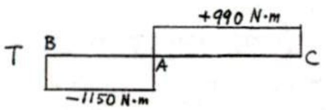


$$T_A = \frac{9550\,(112)}{500} = 2140\text{ N}\cdot\text{m}$$

$$T_B = \frac{9550\,(60)}{500} = 1150\text{ N}\cdot\text{m}$$

$$T_C = \frac{9550\,(52)}{500} = 990\text{ N}\cdot\text{m}$$



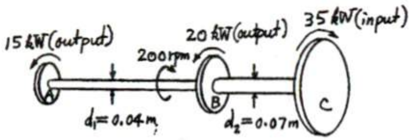


$$J = \frac{\pi}{32}d^4 = \frac{\pi}{32}(0.05\text{ m})^4 = 6.14 \times 10^{-7}\text{ m}^4$$

$$\tau_{max} = \frac{|T_{max}|\,c}{J} = \frac{(1150\text{ N}\cdot\text{m})(0.025\text{ m})}{6.14 \times 10^{-7}\text{ m}^4}$$

$$= 46.9 \times 10^6\text{ N/m} = 46.9\text{ MPa}$$ ◀

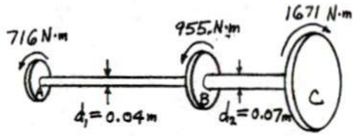
12-34

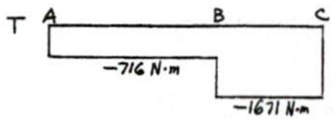


$$T_A = \frac{9550\,(15)}{200} = 716\text{ N}\cdot\text{m}$$

$$T_B = \frac{9550\,(20)}{200} = 955\text{ N}\cdot\text{m}$$

$$T_C = \frac{9550\,(35)}{200} = 1671\text{ N}\cdot\text{m}$$





$$(\tau_{max})_{AB} = \frac{Tc}{J} = \frac{(716\text{ N}\cdot\text{m})(0.020\text{ m})}{\frac{\pi}{32}(0.040\text{ m})^4}$$

$$= 57.0 \times 10^6\text{ N/m}^2 = 57.0\text{ MPa}$$

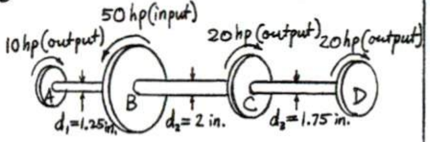
$$(\tau_{max})_{BC} = \frac{Tc}{J} = \frac{(1671\text{ N}\cdot\text{m})(0.035\text{ m})}{\frac{\pi}{32}(0.070\text{ m})^4}$$

$$= 24.8 \times 10^6\text{ N/m}^2 = 24.8\text{ MPa}$$

$$\tau_{max} = 57.0\text{ MPa}$$ ◀

which occurs on the surface of shaft AB.

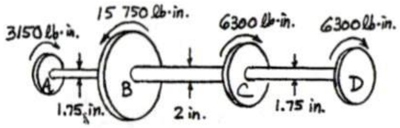
12-35

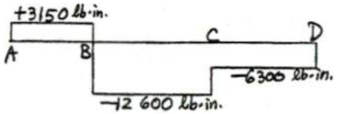


$$T_A = \frac{63\,000\,(10)}{200} = 3150\text{ lb}\cdot\text{in.}$$

$$T_B = \frac{63\,000\,(50)}{200} = 15\,750\text{ lb}\cdot\text{in.}$$

$$T_C = T_D = \frac{63\,000\,(20)}{200} = 6\,300\text{ lb}\cdot\text{in.}$$





$$(\tau_{max})_{AB} = \frac{Tc}{J} = \frac{(3150\text{ lb}\cdot\text{in.})(0.625\text{ in.})}{\frac{\pi}{32}(1.25\text{ in.})^4}$$

$$= 8210\text{ psi}$$

$$(\tau_{max})_{BC} = \frac{Tc}{J} = \frac{(12\,600\text{ lb}\cdot\text{in.})(1\text{ in.})}{\frac{\pi}{32}(2\text{ in.})^4}$$

$$= 8020\text{ psi}$$

$$(\tau_{max})_{CD} = \frac{Tc}{J} = \frac{(6300\text{ lb}\cdot\text{in.})(0.875\text{ in.})}{\frac{\pi}{32}(1.75\text{ in.})^4}$$

$$= 5990\text{ psi}$$

$$\tau_{max} = 8210\text{ psi}$$ ◀

which occurs on the surface of shaft AB.

12-36

$$\phi = \frac{TL}{JG} = \frac{(1200 \times 12\text{ lb}\cdot\text{in.})(12\text{ in.})}{\frac{\pi}{32}(2\text{ in.})^4(12 \times 10^6\text{ lb/in.}^2)}$$

$$= 0.009\,17\text{ rad} = 0.525^\circ$$ ◀

12-37

$$T = 40\text{ kip}\cdot\text{in.} = 40\,000\text{ lb}\cdot\text{in.}$$

$$J = \frac{\pi}{32}(d_o^4 - d_i^4) = \frac{\pi}{32}\left[(4\text{ in.})^4 - (3\text{ in.})^4\right]$$

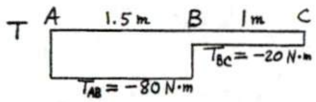
$$= 17.18\text{ in.}^4$$

The angle of twist over 2 ft length is

$$\phi = \frac{TL}{JG} = \frac{(40\,000\text{ lb}\cdot\text{in.})(24\text{ in.})}{(17.18\text{ in.}^4)(12 \times 10^6\text{ lb/in.}^2)}$$

$$= 0.004\,66\text{ rad} = 0.267^\circ$$ ◀

12-38



$$J_{AB} = \frac{\pi}{32}d^4 = \frac{\pi}{32}(0.050\text{ m})^4 = 6.14 \times 10^{-7}\text{ m}^4$$

$$J_{BC} = \frac{\pi}{32}d^4 = \frac{\pi}{32}(0.025\text{ m})^4 = 3.84 \times 10^{-8}\text{ m}^4$$

$$\phi_{AB} = \frac{TL}{JG} = \frac{(-80\text{ N}\cdot\text{m})(1.5\text{ m})}{(6.14 \times 10^{-7}\text{ m}^4)(84 \times 10^9\text{ N/m}^2)}$$

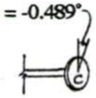
$$= -0.00233\text{ rad}$$

$$\phi_{BC} = \frac{TL}{JG} = \frac{(-20\text{ N}\cdot\text{m})(1\text{ m})}{(3.84 \times 10^{-8}\text{ m}^4)(84 \times 10^9\text{ N/m}^2)}$$

$$= -0.00620\text{ rad}$$

$$\phi_C = \phi_{C/A} = \phi_{AB} + \phi_{BC}$$

$$= -0.00853\text{ rad} = -0.489^\circ$$ ◀



12-39

From the solution to Prob 12-6,

$T_{AB} = -5 \text{ kN}\cdot\text{m}$
$T_{BC} = +6 \text{ kN}\cdot\text{m}$
$T_{CD} = +3 \text{ kN}\cdot\text{m}$

$$\phi_{AB} = \frac{TL}{JG} = \frac{(-5 \text{ kN}\cdot\text{m})(2 \text{ m})}{\frac{\pi}{32}(0.075 \text{ m})^4(84 \times 10^6 \text{ kN/m}^2)} = -0.0383 \text{ rad}$$

$$\phi_{BC} = \frac{TL}{JG} = \frac{(+6 \text{ kN}\cdot\text{m})(2 \text{ m})}{\frac{\pi}{32}(0.085 \text{ m})^4(84 \times 10^6 \text{ kN/m}^2)} = +0.0279 \text{ rad}$$

$$\phi_{CD} = \frac{TL}{JG} = \frac{(+3 \text{ kN}\cdot\text{m})(2 \text{ m})}{\frac{\pi}{32}(0.07 \text{ m})^4(84 \times 10^6 \text{ kN/m}^2)} = +0.0303 \text{ rad}$$

$$\phi_{D/A} = \phi_{AB} + \phi_{BC} + \phi_{CD}$$
$$= -0.0383 + 0.0279 + 0.0303$$
$$= +0.0199 \text{ rad} = +1.14° \quad ◄$$

12-40

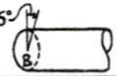
From the solution to Prob 12-33,

$T_{AB} = -1150 \text{ N}\cdot\text{m}$, $T_{AC} = +990 \text{ N}\cdot\text{m}$
$L_{AB} = 3 \text{ m}$, $L_{AC} = 4 \text{ m}$

$$J_{AB} = J_{BC} = \frac{\pi}{32} d^4 = \frac{\pi}{32}(0.05 \text{ m})^4 = 6.14 \times 10^{-7} \text{ m}^4$$
$$JG = (6.14 \times 10^{-7} \text{ m}^4)(84 \times 10^9 \text{ N/m}^2) = 5.16 \times 10^4 \text{ N}\cdot\text{m}^2$$

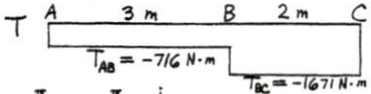
$$\phi_{AB} = \frac{TL}{JG} = \frac{(-1150 \text{ N}\cdot\text{m})(3 \text{ m})}{5.16 \times 10^4 \text{ N}\cdot\text{m}^2} = -0.0669 \text{ rad}$$
$$\phi_{AC} = \frac{TL}{JG} = \frac{(+990 \text{ N}\cdot\text{m})(4 \text{ m})}{5.16 \times 10^4 \text{ N}\cdot\text{m}^2} = +0.0767 \text{ rad}$$

$$\phi_{B/C} = \phi_{AB} + \phi_{AC} = -0.0669 + 0.0767$$
$$= +0.0098 \text{ rad} = +0.56° \quad ◄$$



12-41

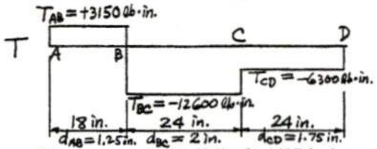
From the solution to Prob. 12-34,



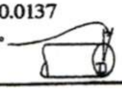
$$J_{AB} = \frac{\pi}{32} d^4 = \frac{\pi}{32}(0.040 \text{m})^4 = 2.51 \times 10^{-7} \text{ m}^4$$
$$J_{BC} = \frac{\pi}{32} d^4 = \frac{\pi}{32}(0.070 \text{ m})^4 = 2.36 \times 10^{-6} \text{ m}^4$$
$$\phi_{AB} = \frac{TL}{JG} = \frac{(-716 \text{ N}\cdot\text{m})(3 \text{ m})}{(2.51 \times 10^{-7} \text{ m}^4)(84 \times 10^9 \text{ N/m}^2)} = -0.0108 \text{ rad}$$
$$\phi_{BC} = \frac{TL}{JG} = \frac{(-1671 \text{ N}\cdot\text{m})(2 \text{ m})}{(2.36 \times 10^{-6} \text{ m}^4)(84 \times 10^9 \text{ N/m}^2)} = -0.0169 \text{ rad}$$
$$\phi_{C/A} = \phi_{AB} + \phi_{BC} = -0.0108 - 0.0169$$
$$= -0.0277 \text{ rad} = -1.59° \quad ◄$$

12-42

From the solution to Prob. 12-35, we have



$$\phi_{AB} = \frac{TL}{JG} = \frac{(+3150 \text{ lb}\cdot\text{in.})(18 \text{ in.})}{\frac{\pi}{32}(1.25 \text{ in.})^4(12 \times 10^6 \text{ lb/in.}^2)} = +0.0197 \text{ rad}$$
$$\phi_{BC} = \frac{TL}{JG} = \frac{(-12\,600 \text{ lb}\cdot\text{in.})(24 \text{ in.})}{\frac{\pi}{32}(2 \text{ in.})^4(12 \times 10^6 \text{ lb/in.}^2)} = -0.0160 \text{ rad}$$
$$\phi_{CD} = \frac{TL}{JG} = \frac{(-6300 \text{ lb}\cdot\text{in.})(24 \text{ in.})}{\frac{\pi}{32}(1.75 \text{ in.})^4(12 \times 10^6 \text{ lb/in.}^2)} = -0.0137 \text{ rad}$$
$$\phi_{D/A} = \phi_{AB} + \phi_{BC} + \phi_{CD}$$
$$= +0.0197 - 0.0160 - 0.0137$$
$$= -0.010 \text{ rad} = -0.573° \quad ◄$$



12-43

$$\phi_{allow} = 1° = 0.01745 \text{ rad}$$
$$J = \frac{TL}{\phi G} = \frac{(4 \text{ kN}\cdot\text{m})(1 \text{ m})}{(0.01745)(84 \times 10^6 \text{ kN/m}^2)} = 2.73 \times 10^{-6} \text{ m}^4$$
$$J = \frac{\pi}{32} d^4 = 2.73 \times 10^{-6} \text{ m}^4$$
$$d = \sqrt[4]{\frac{32(2.73 \times 10^{-6} \text{ m}^4)}{\pi}} = 0.0726 \text{ m} = 72.6 \text{ mm}$$

Use d = 73 mm ◄

12-44

$$T = \frac{63\,000\,P}{n} = \frac{63\,000\,(100)}{250} = 25\,200 \text{ lb}\cdot\text{in.}$$

For strength:

$$d_1 = \sqrt[3]{\frac{16\,T}{\pi \tau_{allow}}} = \sqrt[3]{\frac{16\,(25\,200 \text{ lb}\cdot\text{in.})}{\pi\,(8000 \text{ lb/in.}^2)}} = 2.52 \text{ in.}$$

For stiffness:

$$\phi_{allow} = 0.24° = 0.00419 \text{ rad}$$
$$J = \frac{TL}{\phi G} = \frac{(25\,200 \text{ lb}\cdot\text{in.})(12 \text{in.})}{(0.00419)(12 \times 10^6 \text{ lb/in.}^2)} = 6.01 \text{ in.}^4$$
$$J = \frac{\pi}{32} d^4 = 6.01 \text{ in.}^4$$
$$d_2 = \sqrt[4]{\frac{32(6.01 \text{ in.}^4)}{\pi}} = 2.80 \text{ in.}$$

Use $d = 2\frac{13}{16}$ in. (2.8125 in.) ◄

12-45

$$T = \frac{9550\,P}{n} = \frac{9550\,(186)}{450} = 3950 \text{ N}\cdot\text{m}$$

For strength:

$$d = \sqrt[3]{\frac{16\,T}{\pi \tau_{allow}}} = \sqrt[3]{\frac{16\,(3950 \text{ N}\cdot\text{m})}{\pi\,(70 \times 10^6 \text{ N/m}^2)}} = 0.0660 \text{ m} = 66 \text{ mm}$$

For stiffness:

$$\phi_{allow} = 1° = 0.01745 \text{ rad}$$
$$J = \frac{TL}{\phi G} = \frac{(3950 \text{ N}\cdot\text{m})(1 \text{ m})}{(0.01745)(84 \times 10^9 \text{ N/m}^2)} = 2.69 \times 10^{-6} \text{ m}^4$$
$$J = \frac{\pi}{32} d^4 = 2.69 \times 10^{-6} \text{ m}^4$$
$$d = \sqrt[4]{\frac{32(2.69 \times 10^{-6} \text{ m}^4)}{\pi}} = 0.0724 \text{ m} = 72.4 \text{ mm}$$

Use d = 73 mm ◄

12-46

$$T = \frac{63\,000\,P}{n} = \frac{63\,000\,(100)}{250} = 25\,200 \text{ lb}\cdot\text{in}$$
$$k = d_i/d_o = 0.8$$

For strength:

$$d = \sqrt[3]{\frac{16\,T}{\pi \tau_{allow}(1-k^4)}} = \sqrt[3]{\frac{16\,(25\,200 \text{ lb}\cdot\text{in.})}{\pi\,(8000 \text{ lb/in.}^2)(1-0.8^4)}} = 3.01 \text{ in.}$$

For stiffness:

$$\phi_{allow} = 0.24° = 0.00419 \text{ rad}$$
$$J = \frac{TL}{\phi G} = \frac{(25\,200 \text{ lb}\cdot\text{in.})(12 \text{ in.})}{(0.00419)(12 \times 10^6 \text{ lb/in.}^2)} = 6.01 \text{ in.}^4$$
$$J = \frac{\pi}{32}[d_o^4 - d_i^4] = \frac{\pi}{32}(1-k^4)\,d_o^4$$
$$= \frac{\pi}{32}(1-0.8^4)\,d_o^4 = 0.0580\,d_o^4 = 6.01 \text{ in.}^4$$
$$d_o = \sqrt[4]{\frac{6.01 \text{ in.}^4}{0.0580}} = 3.19 \text{ in.}$$
$$d_i = 0.8(3.19 \text{ in.}) = 2.55 \text{ in.}$$

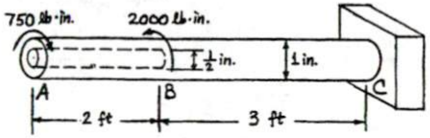
Use $d = 3\frac{1}{4}$ in. (3.25 in.) ◄

$d = 2\frac{1}{2}$ in. (2.5 in.) ◄

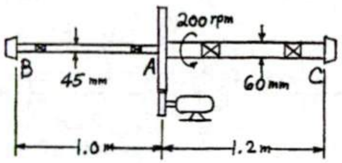
Test Problems for Chapter 12

The following problems may either be given to students for them to practice their problem solving skills or be used as test problems. The answers are provided on the next page.

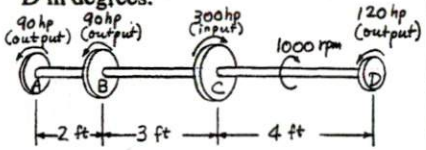
(1) The circular steel shaft is subjected to two torques shown. The segment *AB* is hollow, with an outside diameter of 1 in. and an inside diameter of 1/2. The segment *BC* is solid, with a diameter of 1 in. Determine (a) the maximum shear stress in the shaft, and (b) the angle of twist in degrees between *A* and *C*, using $G = 12 \times 10^6$ psi.



(2) An 80-kW motor drives gear A on a steel line shaft rotating at 200 rpm. A power of 20 kW is delivered to gear *B* and 60 kW is delivered to gear *C*. (a) Plot the internal torque diagram along the shaft. (b) Determine the angle of twist of *C* relative to *B* in degrees. The shear modulus of steel is $G = 83$ GPa.



(3) The input and output horsepowers of the steel shaft ($G = 12 \times 10^6$ psi) are indicated as shown. The shaft has a uniform diameter of 2 in. (a) Plot the internal torque diagram along the shaft. (b) Determine the maximum shear stress in the shaft. (c) Determine the relative angle of twist of the shaft between *A* and *D* in degrees.



(4) A solid circular shaft is used to transmit 40 kW at 1000 rpm. If the allowable shear stress is 41 MPa and the allowable angle of twist is 0.70°/m, determine the required diameter of the shaft to the nearest mm. The shear modulus of steel is $G = 83$ GPa.

Solutions to Test Problems for Chapter 12

(1)

$$J_{AB} = \frac{\pi}{32}\left[(1\text{ in.})^4 - (0.5\text{ in.})^4\right] = 0.0920\text{ in.}^4$$

$$J_{BC} = \frac{\pi}{32}(1\text{ in.})^4 = 0.0982\text{ in.}^4$$

$T_{AB} = -750$ lb·in.

$T_{BC} = -750 + 2000 = +1250$ lb·in.

(a)

$$\tau_{AB} = \frac{T_{AB}c}{J_{AB}} = \frac{(-750\text{ lb}\cdot\text{in.})\left(\frac{1}{2}\text{ in.}\right)}{0.0920\text{ in.}^4} = -4076\text{ psi}$$

$$\tau_{BC} = \frac{T_{BC}c}{J_{BC}} = \frac{(+1250\text{ lb}\cdot\text{in.})\left(\frac{1}{2}\text{ in.}\right)}{0.0920\text{ in.}^4} = +6365\text{ psi}$$

$\tau_{max} = \tau_{BC} = 6365$ psi ◄

(b)

$$\phi_{AB} = \frac{T_{AB}L_{AB}}{J_{AB}G} = \frac{(-750\text{ lb}\cdot\text{in.})(24\text{ in.})}{(0.0920\text{ in.}^4)(12\times10^6\text{ lb/in.}^2)}$$

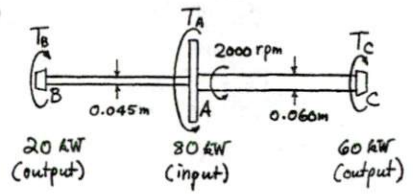
$= -0.0163$ rad

$$\phi_{BC} = \frac{T_{BC}L_{BC}}{J_{BC}G} = \frac{(+1250\text{ lb}\cdot\text{in.})(36\text{ in.})}{(0.0982\text{ in.}^4)(12\times10^6\text{ lb/in.}^2)}$$

$= +0.0382$ rad

$\phi_{BC} = -0.0163 + 0.0382$
$= +0.0219\text{ rad} = +1.25°$ ◄

(2)

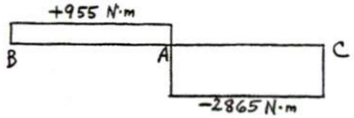


(a)

$$T_A = \frac{9550\ (20)}{200} = 955\text{ N}\cdot\text{m}$$

$$T_B = \frac{9550\ (80)}{200} = 3820\text{ N}\cdot\text{m}$$

$$T_C = \frac{9550\ (60)}{200} = 2865\text{ N}\cdot\text{m}$$



(b)

$$\phi_{BA} = \frac{(+955\text{ N}\cdot\text{m})(1.0\text{ m})}{\frac{\pi}{32}(0.045\text{ m})^4(83\times10^9\text{ N/m}^2)} = +0.0286\text{ rad}$$

$$\phi_{AC} = \frac{(-2865\text{ N}\cdot\text{m})(1.2\text{ m})}{\frac{\pi}{32}(0.060\text{ m})^4(83\times10^9\text{ N/m}^2)} = -0.0326\text{ rad}$$

$\phi_{C/B} = +0.0286 - 0.0326$
$= -0.0040\text{ rad} = -0.23°$ ◄

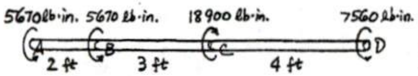
(3)

<u>External torques:</u>

$$T_A = T_B = \frac{63\ 000\ (90)}{1000} = 5670\text{ lb}\cdot\text{in.}$$

$$T_C = \frac{63\ 000\ (300)}{1000} = 18\ 900\text{ lb}\cdot\text{in.}$$

$$T_D = \frac{63\ 000\ (120)}{1000} = 7560\text{ lb}\cdot\text{in.}$$



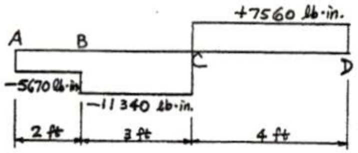
(a) <u>Internal torque diagram</u>

$T_{AB} = -5670$ lb·in.
$T_{BC} = -5670 - 5670 = -11340$ lb·in.
$T_{CD} = +7560$ lb·in.

(Cont'd)

Solutions to Test Problems for Chapter 12 (Cont'd)

(3) (Cont)



(b) Maximum shear stress:

$$J = \frac{\pi}{32}(2\ \text{in.})^4 = 1.571\ \text{in.}^4$$

$$\tau_{max} = \tau_{BC} = \frac{(11\,340\ \text{lb}\cdot\text{in.})(1\ \text{in.})}{1.571\ \text{in.}^4}$$

$$= 7220\ \text{psi} \blacktriangleleft$$

(c) Angle of twist:

$$JG = (1.571\ \text{in.}^4)(12\text{x}10^6\ \text{lb/in.}^2)$$

$$= 1.885\ \text{x}\ 10^7\ \text{lb}\cdot\text{in.}^2$$

$$\phi_{AB} = \frac{(-5670\ \text{lb}\cdot\text{in.})(24\ \text{in.})}{1.885\ \text{x}\ 10^7\ \text{lb}\cdot\text{in.}^2} = -0.00722\ \text{rad}$$

$$\phi_{BC} = \frac{(-11340\ \text{lb}\cdot\text{in.})(36\ \text{in.})}{1.885\ \text{x}\ 10^7\ \text{lb}\cdot\text{in.}^2} = -0.02166\ \text{rad}$$

$$\phi_{CD} = \frac{(+7560\ \text{lb}\cdot\text{in.})(48\ \text{in.})}{1.885\ \text{x}\ 10^7\ \text{lb}\cdot\text{in.}^2} = +0.01925\ \text{rad}$$

$$\phi_{AD} = -0.00722 - 0.02166 + 0.01925$$

$$= -0.00963\ \text{rad} = -0.552° \blacktriangleleft$$

(4)

$$T = \frac{9550\ (40)}{1000} = 382\ \text{N}\cdot\text{m}$$

For strength:

$$d_1 = \sqrt[3]{\frac{16\,T}{\pi\,\tau_{allow}}} = \sqrt[3]{\frac{16\ (382\ \text{N}\cdot\text{m})}{\pi\ (41\text{x}10^6\ \text{N/m}^2)}}$$

$$= 0.0362\ \text{m}$$

For stiffness:

$$\phi_{allow} = 0.70° = 0.0122\ \text{rad (per meter)}$$

$$J = \frac{TL}{\phi G} = \frac{(382\ \text{N}\cdot\text{m})(1\ \text{m})}{(0.0122\ \text{rad})(83\text{x}10^9\ \text{N/m}^2)}$$

$$= 3.77\ \text{x}\ 10^{-7}\ \text{m}^4$$

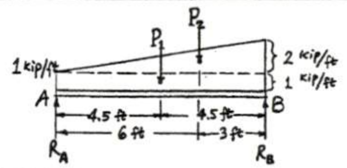
$$J = \frac{\pi}{32}d^4 = 3.77\ \text{x}\ 10^{-7}\ \text{m}^4$$

$$d_2 = \sqrt[4]{\frac{32(3.77\ \text{x}\ 10^{-7}\ \text{m}^4)}{\pi}} = 0.0443\ \text{m}$$

$$d_{req} = 0.0443\ \text{m} = 44.3\ \text{mm}$$

Use d = 45 mm ◀

13-1



$$P_1 = (1\ \text{kip/ft})(9\ \text{ft}) = 9\ \text{kips}$$

$$P_2 = \frac{1}{2}(2\ \text{kip/ft})(9\ \text{ft}) = 9\ \text{kips}$$

$$\sum M_B = -R_A\ (9\ \text{ft}) + (9\ \text{kips})(4.5\ \text{ft}) + (9\ \text{kips})(3\ \text{ft}) = 0$$

$$R_A = 7.5\ \text{kips} \uparrow \blacktriangleleft$$

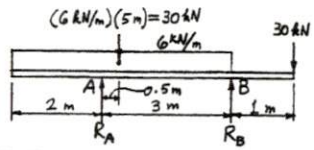
$$\sum M_A = R_B\ (9\ \text{ft}) - (9\ \text{kips})(4.5\ \text{ft}) - (9\ \text{kips})(6\ \text{ft}) = 0$$

$$R_B = 10.5\ \text{kips} \uparrow \blacktriangleleft$$

Check:

$$\sum F_y = 7.5 + 10.5 - 9 - 9 = 0 \quad \text{(Checks)}$$

13-2



$$\sum M_B = -R_A(3\ \text{m}) + (30\ \text{kN})(2.5\ \text{m}) + (30\ \text{kN})(1\ \text{m}) = 0$$

$$R_A = +15\ \text{kN} \uparrow \blacktriangleleft$$

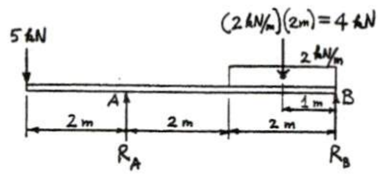
$$\sum M_A = R_B(3\ \text{m}) - (30\ \text{kN})(0.5\ \text{m}) - (30\ \text{kN})(4\ \text{m}) = 0$$

$$R_B = +45\ \text{kN} \uparrow \blacktriangleleft$$

Check:

$$\sum F_y = 45 + 15 - 30 - 30 = 0 \quad \text{(Checks)}$$

13-3



$$\sum M_B = -R_A\ (4\ \text{m}) + (5\ \text{kN})(6\ \text{m}) + (4\ \text{kN})(1\ \text{m}) = 0$$

$$R_A = +8.5\ \text{kN} \uparrow \blacktriangleleft$$

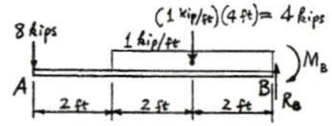
$$\sum M_A = R_B\ (4\ \text{m}) + (5\ \text{kN})(2\ \text{m}) - (4\ \text{kN})(3\ \text{m}) = 0$$

$$R_B = +0.5\ \text{kN} \uparrow \blacktriangleleft$$

Check:

$$\sum F_y = 8.5 + 0.5 - 5 - 4 = 0 \quad \text{(Checks)}$$

13-4



$$\sum F_y = R_B - 8\ \text{kips} - 4\ \text{kips} = 0$$

$$R_B = +12\ \text{kips} \uparrow \blacktriangleleft$$

$$\sum M_B = -M_B + (8\ \text{kips})(6\ \text{ft}) + (4\ \text{kips})(2\ \text{ft}) = 0$$

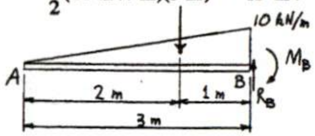
$$M_B = +56\ \text{kip}\cdot\text{ft} \curvearrowright \blacktriangleleft$$

Check:

$$\sum M_A = -4(4) + 12(6) - 56 = 0 \quad \text{(Checks)}$$

13-5

$$\frac{1}{2}(10\ \text{kN/m})(3\ \text{m}) = 15\ \text{kN}$$



$$\sum M_B = -M_B + (15\ \text{kN})(1\ \text{m}) = 0$$

$$M_B = +15\ \text{kN}\cdot\text{m} \curvearrowright \blacktriangleleft$$

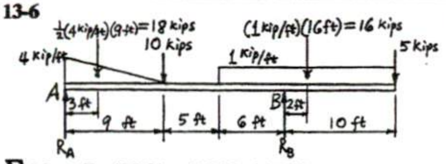
$$\sum F_y = R_B - 15\ \text{kN} = 0$$

$$R_B = +15\ \text{kN} \uparrow \blacktriangleleft$$

Check:

$$\sum M_A = -15(2) - 15 + 15(3) = 0 \quad \text{(Checks)}$$

13-6



$\sum M_B = -R_A\,(20\text{ ft}) + (18\text{ kips})(17\text{ ft})$
$+ (10\text{ kips})(11\text{ ft}) - (16\text{ kips})(2\text{ ft})$
$- (5\text{ kips})(10\text{ ft}) = 0$

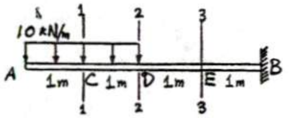
$R_A = +16.7\text{ kips}\uparrow$ ◀

$\sum M_A = R_B\,(20\text{ ft}) - (18\text{ kips})(3\text{ ft}) - (10\text{ kips})(9\text{ ft})$
$- (16\text{ kips})(22\text{ ft}) - (5\text{ kips})(30\text{ ft}) = 0$

$R_B = +32.3\text{ kips}\uparrow$ ◀

Check:
$\sum F_y = 16.7 + 32.3 - 18 - 10 - 16 - 5 = 0$ (Checks)

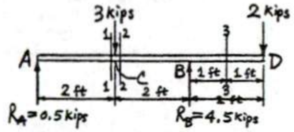
13-7



Using Eqs. 13-1 and 13-2 starting from the left-hand side of the beam we find

$V_{1\text{-}1} = -(10\text{ kN/m})(1\text{ m}) = -10\text{ kN}$ ◀
$V_{2\text{-}2} = -(10\text{ kN/m})(2\text{ m}) = -20\text{ kN}$ ◀
$V_{3\text{-}3} = -(10\text{ kN/m})(2\text{ m}) = -20\text{ kN}$ ◀
$M_{1\text{-}1} = -(10\text{ kN/m} \times 1\text{ m})(0.5\text{ m}) = -5\text{ kN}\cdot\text{m}$ ◀
$M_{2\text{-}2} = -(10\text{ kN/m} \times 2\text{ m})(1\text{ m}) = -20\text{ kN}\cdot\text{m}$ ◀
$M_{3\text{-}3} = -(10\text{ kN/m} \times 2\text{ m})(2\text{ m}) = -40\text{ kN}\cdot\text{m}$ ◀

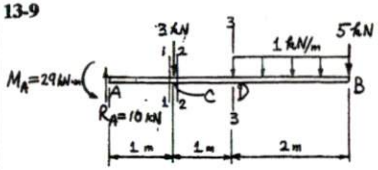
13-8



$V_{1\text{-}1} = +0.5\text{ kip}$ ◀
$V_{2\text{-}2} = +0.5\text{ kip} - 3\text{ kips} = -2.5\text{ kips}$ ◀
$V_{3\text{-}3} = +2\text{ kips (from right)}$ ◀

$M_{1\text{-}1} = +(0.5\text{ kip})(2\text{ ft}) = +1\text{ kip}\cdot\text{ft}$ ◀
$M_{2\text{-}2} = +(0.5\text{ kip})(2\text{ ft}) = +1\text{ kip}\cdot\text{ft}$ ◀
$M_{3\text{-}3} = -(2\text{ kips})(1\text{ ft}) = -2\text{ kip}\cdot\text{ft (from right)}$ ◀

13-9

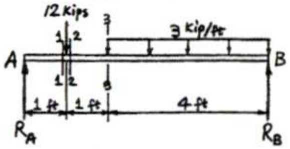


$V_{1\text{-}1} = +10\text{ kN}$ ◀
$V_{2\text{-}2} = +10\text{ kN} - 3\text{ kN} = +7\text{ kN}$ ◀
$V_{3\text{-}3} = +10\text{ kN} - 3\text{ kN} = +7\text{ kN}$ ◀

$M_{1\text{-}1} = M_{2\text{-}2} = -29\text{ kN/m} + (10\text{ kN})(1\text{ m}) = -19\text{ kN}\cdot\text{m}$ ◀

$M_{3\text{-}3} = -(5\text{ kN})(2\text{ m}) - (1\text{ kN/m} \times 2\text{ m})(1\text{ m})$
$= -12\text{ kN}\cdot\text{m (from right)}$ ◀

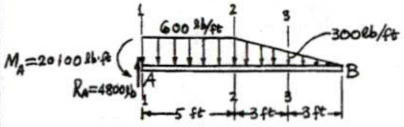
13-10



$V_{1\text{-}1} = +14\text{ kips}$ ◀
$V_{2\text{-}2} = +14 - 12 = +2\text{ kips}$ ◀
$V_{3\text{-}3} = +14 - 12 = +2\text{ kips}$ ◀

$M_{1\text{-}1} = M_{2\text{-}2} = +14 \times 1 = +14\text{ kip}\cdot\text{ft}$ ◀
$M_{3\text{-}3} = +14 \times 2 - 12 \times 1 = +16\text{ kip}\cdot\text{ft}$ ◀

13-11



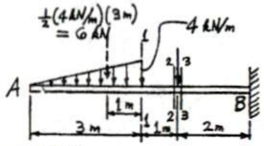
(Cont'd)

13-11 (Cont)

$V_{1\text{-}1} = +4800\text{ lb}$ ◀
$V_{2\text{-}2} = +4800 - 600 \times 5 = +1800\text{ lb}$ ◀
$V_{3\text{-}3} = +\frac{1}{2}(300 \times 3) = +450\text{ lb (from right)}$ ◀

$M_{1\text{-}1} = -20\,100\text{ lb}\cdot\text{ft}$ ◀
$M_{2\text{-}2} = -20\,100 + (4800 \times 5) - (600 \times 5)\left(\frac{5}{2}\right)$
$= -3600\text{ lb}\cdot\text{ft}$ ◀
$M_{3\text{-}3} = -\left(\frac{1}{2} \times 300 \times 3\right)(1)$
$= -450\text{ lb}\cdot\text{ft (from right)}$ ◀

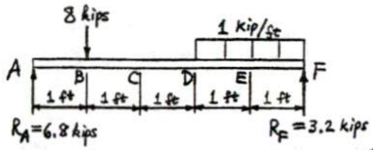
13-12



$V_{1\text{-}1} = V_{2\text{-}2} = -6\text{ kN}$ ◀
$V_{3\text{-}3} = -6\text{ kN} - 10\text{ kN} = -16\text{ kN}$ ◀

$M_{1\text{-}1} = -(6\text{ kN})(1\text{ m}) = -6\text{ kN}\cdot\text{m}$ ◀
$M_{2\text{-}2} = M_{3\text{-}3} = -(6\text{ kN})(2\text{ m}) = -12\text{ kN}\cdot\text{m}$ ◀

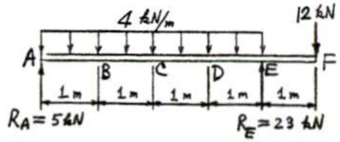
13-13



$V_{A^-} = 0$ ◀
$V_{A^+} = +6.8\text{ kips}$ ◀
$V_{B^-} = +6.8\text{ kips}$ ◀
$V_{B^+} = 6.8 - 8 = -1.2\text{ kips}$ ◀
$V_C = V_D = V_{B^+} = -1.2\text{ kips}$ ◀
$V_E = +6.8 - 8 - 1 \times 1 = 2.2\text{ kips}$ ◀
$V_{F^-} = -3.2\text{ kips (from right)}$ ◀
$V_{F^+} = 0$ ◀

$M_A = 0$ ◀
$M_B = +6.8 \times 1 = +6.8\text{ kip}\cdot\text{ft}$ ◀
$M_C = +6.8 \times 2 - 8 \times 1 = +5.6\text{ kip}\cdot\text{ft}$ ◀
$M_D = +6.8 \times 3 - 8 \times 2 = +4.4\text{ kip}\cdot\text{ft}$ ◀
$M_E = +3.2 \times 1 - (1 \times 1)(0.5)$
$= +2.7\text{ kip}\cdot\text{ft (from right)}$ ◀
$M_F = 0\text{ (from right)}$ ◀

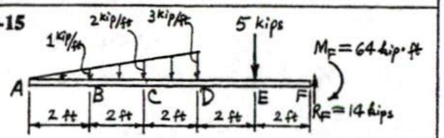
13-14



$V_{A^-} = 0$ ◀
$V_{A^+} = +5\text{ kN}$ ◀
$V_B = +5 - 4 \times 1 = +1\text{ kN}$ ◀
$V_C = +5 - 4 \times 2 = -3\text{ kN}$ ◀
$V_D = +5 - 4 \times 3 = -7\text{ kN}$ ◀
$V_{E^-} = +5 - 4 \times 4 = -11\text{ kN}$ ◀
$V_{E^+} = +12\text{ kN (from right)}$ ◀
$V_{F^-} = +12\text{ kN (from right)}$ ◀
$V_{F^+} = 0$ ◀

$M_A = 0$ ◀
$M_B = 5 \times 1 - (4 \times 1)(0.5) = +3\text{ kN}\cdot\text{m}$ ◀
$M_C = 5 \times 2 - (4 \times 2)(1) = +2\text{ kN}\cdot\text{m}$ ◀
$M_D = 5 \times 3 - (4 \times 3)(1.5) = -3\text{ kN}\cdot\text{m}$ ◀
$M_E = -(12)(1) = -12\text{ kN}\cdot\text{m (from right)}$ ◀
$M_F = 0\text{ (from right)}$ ◀

13-15



(Cont'd)

13-15 (Cont)

$V_A = 0$

$V_B = -\frac{1}{2}\,(1\text{ kip/ft})(2\text{ ft}) = -1\text{ kip}$

$V_C = -\frac{1}{2}\,(2\text{ kip/ft})(4\text{ ft}) = -4\text{ kips}$

$V_D = V_{E^-} = -\frac{1}{2}\,(3\text{ kip/ft})(6\text{ ft}) = -9\text{ kips}$

$V_{E^+} = V_{F^-} = -14$ kips (from right)

$V_{F^+} = 0$ (from right)

$M_A = 0$

$M_B = -\left(\frac{1}{2} \times 1\text{ kip/ft} \times 2\text{ft}\right)\left(\frac{2\text{ ft}}{3}\right)$

$= -0.667\text{ kip}\cdot\text{ft}$

$M_C = -\left(\frac{1}{2} \times 2\text{ kip/ft} \times 4\text{ft}\right)\left(\frac{4\text{ ft}}{3}\right)$

$= -5.33\text{ kip}\cdot\text{ft}$

$M_D = -\left(\frac{1}{2} \times 3\text{ kip/ft} \times 6\text{ ft}\right)\left(\frac{6\text{ ft}}{3}\right)$

$= -18\text{ kip}\cdot\text{ft}$

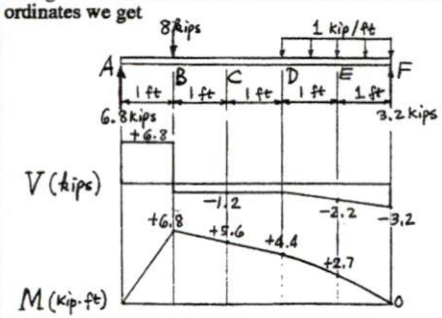
$M_E = -\left(\frac{1}{2} \times 3\text{ kip/ft} \times 6\text{ ft}\right)\left(\frac{6\text{ ft}}{3} + 2\text{ ft}\right)$

$= -36\text{ kip}\cdot\text{ft}$

$M_F = -64\text{ kip}\cdot\text{ft}$ (from right)

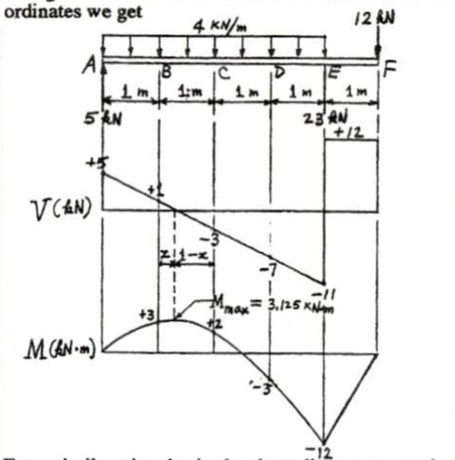
13-16

Using the results in the solution to Prob. 13-13 as ordinates we get



13-17

Using the results in the solution to Prob. 13-14 as ordinates we get



From similar triangles in the shear diagram, we write

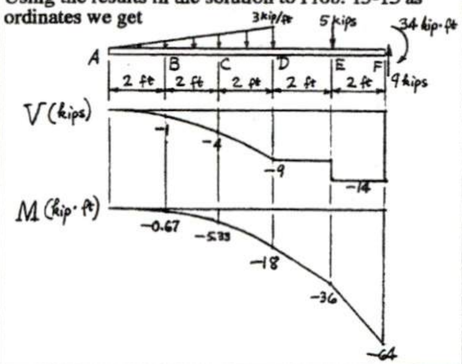
$\frac{x}{x-1} = \frac{1}{3}$ $\quad 3x = 1 - x$

$x = 0.25\text{ m}$

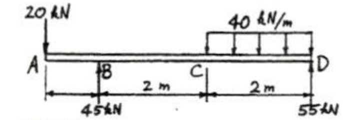
$M_{max} = 5(1.25) - (4 \times 1.25)\frac{1.25}{2} = 3.125\text{ kN}\cdot\text{m}$

13-18

Using the results in the solution to Prob. 13-15 as ordinates we get

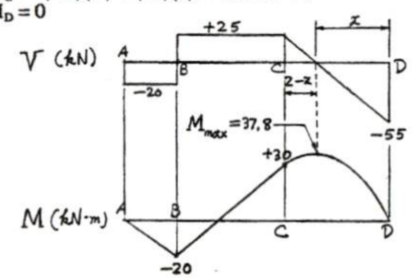


13-19



$V_{A^-} = 0$
$V_{A^+} = V_{B^-} = -20\text{ kN}$
$V_{B^+} = V_C = -20\text{ kN} + 45\text{ kN} = +25\text{ kN}$
$V_{D^-} = -55\text{ kN}$
$V_{D^+} = 0$

$M_A = 0$
$M_B = -(20)(1) = -20\text{ kN}\cdot\text{m}$
$M_C = -(20)(3) + (45)(2) = +30\text{ kN}\cdot\text{m}$
$M_D = 0$



From similar triangles in the shear diagram, we write

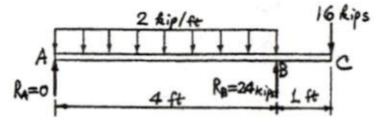
$\frac{x}{55} = \frac{2-x}{25}$ $\quad 25x = 55(2 - x)$

$x = 1.375\text{ m}$

$M_{max} = +(55)(1.375) - 40)(1.375)\frac{1.375}{2}$

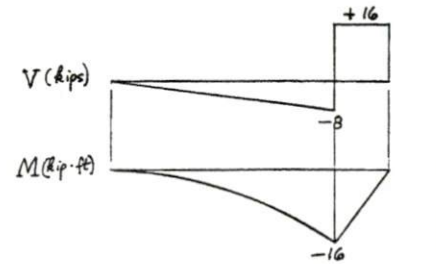
$= 37.8\text{ kN}\cdot\text{m}$ (from right)

13-20

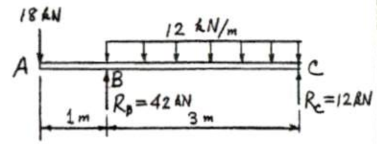


$V_{A^-} = V_{A^+} = 0$
$V_{B^-} = -2 \times 4 = -8\text{ kips}$
$V_{B^+} = V_{C^-} = +16$ kips (from right)
$V_{C^+} = 0$

$M_A = 0$
$M_B = -16 \times 1 = -16\text{ kip}\cdot\text{ft}$
$M_C = 0$ (from right)

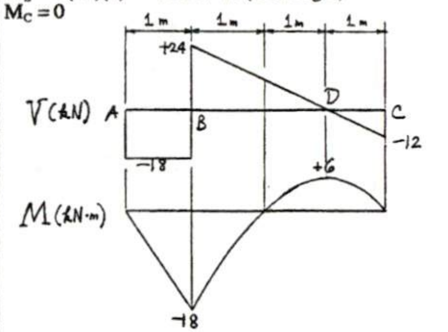


13-21



$V_{A^-} = 0$
$V_{A^+} = V_{B^-} = -18\text{ kN}$
$V_{B^+} = -18\text{kN} + 42\text{ kN} = +24\text{ kN}$
$V_{C^-} = -12\text{ kN}\cdot\text{ft}$ (from right)
$V_{C^+} = 0$

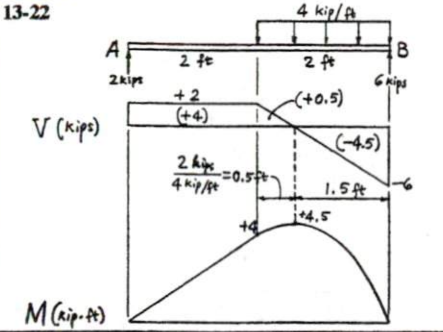
$M_A = 0$
$M_B = -(18)(1) = -18\text{ kN}\cdot\text{m}$ (from right)
$M_C = 0$



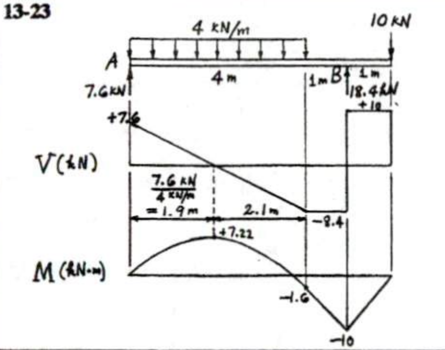
The shear force is zero at D:

$M_D = (12)(1) - (12 \times 1)(0.5)$
$= +6\text{ kip}\cdot\text{ft}$

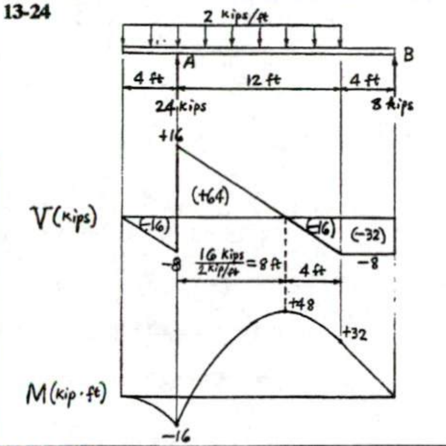
13-22



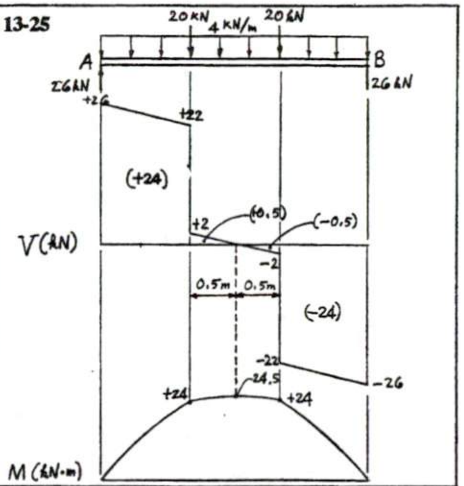
13-23



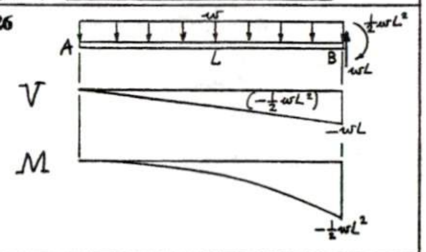
13-24



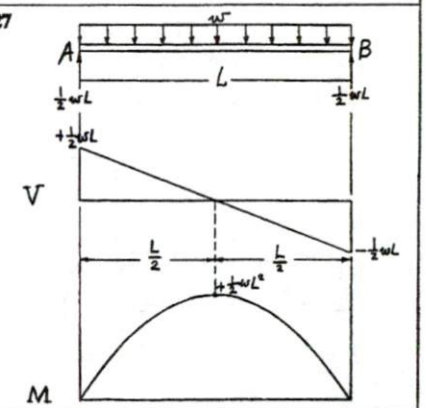
13-25



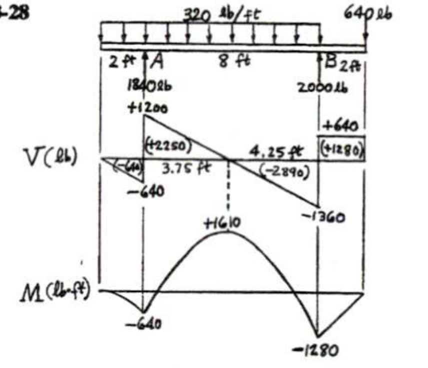
13-26



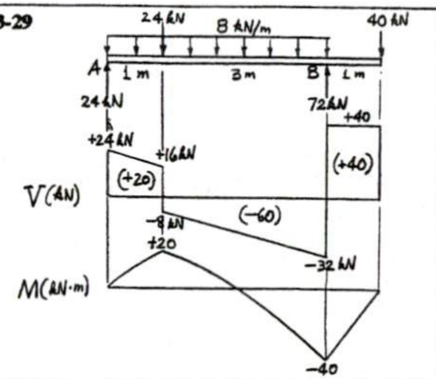
13-27



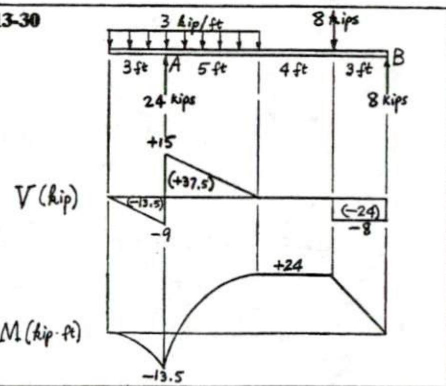
13-28



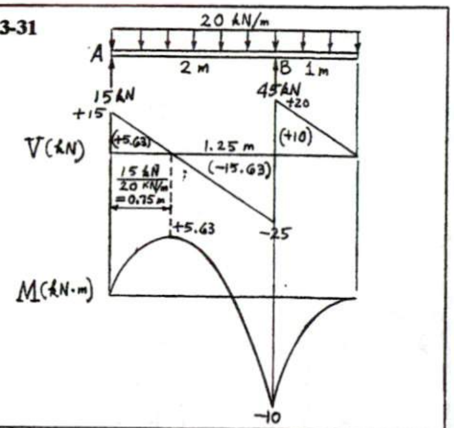
13-29



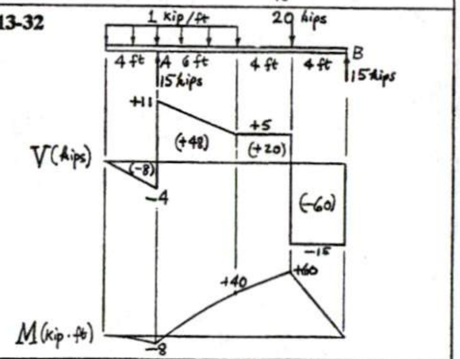
13-30



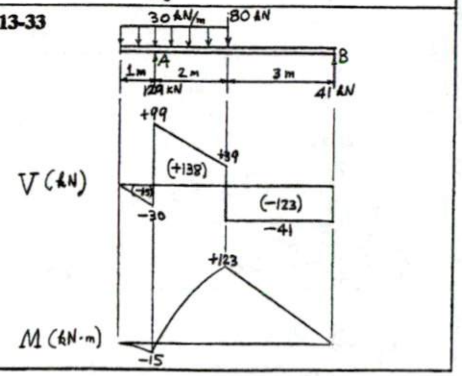
13-31



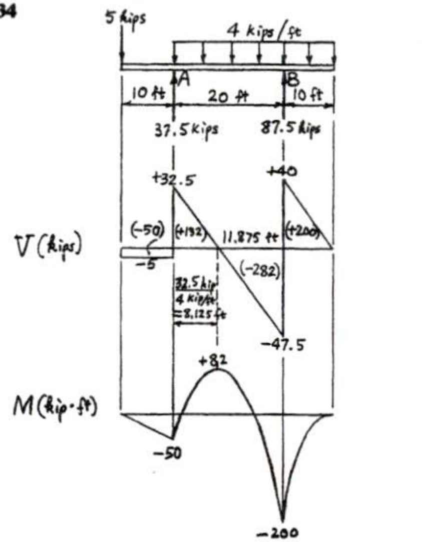
13-32

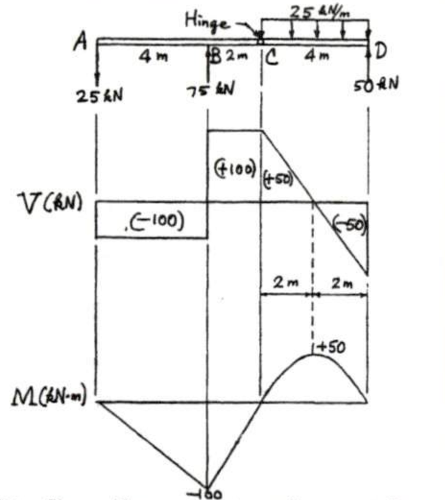


13-33



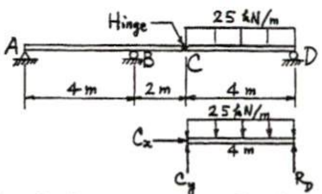
13-34





Note: Since a hinge can not transmit moment, the moment at the hinge must always be zero.

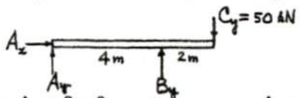
13-35



$\sum F_x = C_x + 0 = 0,\qquad C_x = 0$

$\sum M_D = C_y(4) - (25 \times 4)(2) = 0,\qquad C_y = 50 \text{ kN}$

$\sum F_y = R_D + 50 - 25 \times 4 = 0,\qquad R_D = 50 \text{ kN} \uparrow$



$\sum F_x = A_x + 0 = 0,\qquad A_x = 0$

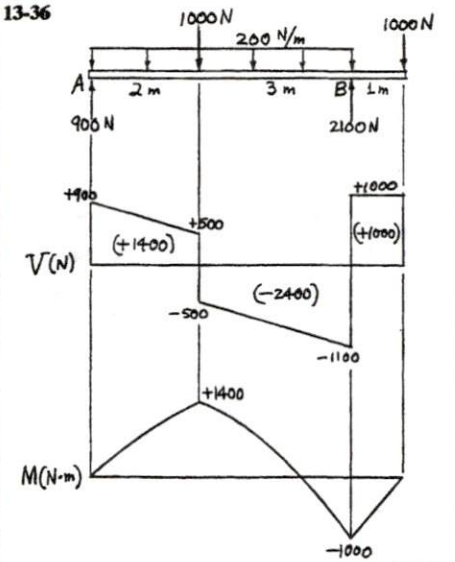
$\sum M_B = A_y(4) + (50)(2) = 0,\qquad A_y = -25 \text{ kN} \downarrow$

$\sum M_A = B_y(4) - (50)(6) = 0,\qquad B_y = +75 \text{ kN} \uparrow$

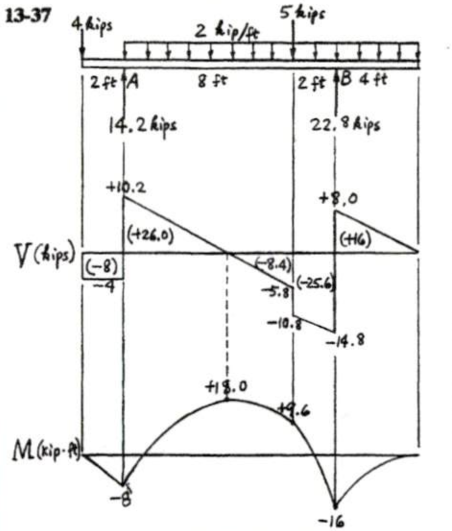
Check:

$\sum F_y = -25 + 75 - 50 = 0$ (Checks)

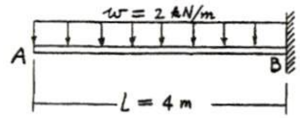
13-36



13-37



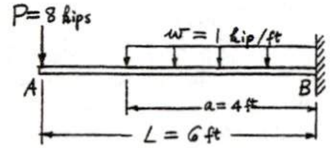
13-38



$V_{max}^{(-)} = -wL = -(2 \text{ kN/m})(4 \text{ m}) = -8 \text{ kN}$ ◀

$M_{max}^{(-)} = -\dfrac{wL^2}{2} = -\dfrac{(2 \text{ kN/m})(4 \text{ m})^2}{2} = -16 \text{ kN}\cdot\text{m}$ ◀

13-39



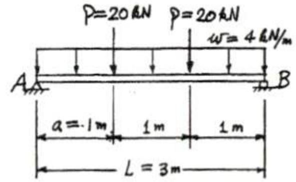
$V_{max}^{(-)} = -P - wa = -8 \text{ kips} - (1 \text{ kip/ft})(4 \text{ ft})$

$= -12 \text{ kips}$ ◀

$M_{max}^{(-)} = -PL - \dfrac{wa^2}{2} = -(8 \text{ kips})(6 \text{ ft}) - \dfrac{(1 \text{ kip/ft})(4 \text{ ft})^2}{2}$

$= -56 \text{ kip}\cdot\text{ft}$ ◀

13-40



$V_{max} = P + \dfrac{wL}{2}$

$= 20 \text{ kN} + \dfrac{(4 \text{ kN/m})(3 \text{ m})}{2}$

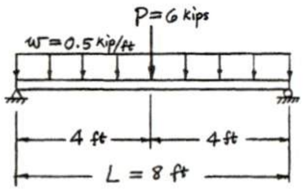
$= 26 \text{ kN}$ ◀

$M_{max} = Pa + \dfrac{wL^2}{8}$

$= (20 \text{ kN})(1 \text{ m}) + \dfrac{(4 \text{ kN/m})(3 \text{ m})^2}{8}$

$= 24.5 \text{ kN}\cdot\text{m}$ ◀

13-41



$V_{max} = \dfrac{P}{2} + \dfrac{wL}{2}$

$= \dfrac{6 \text{ kips}}{2} + \dfrac{(0.5 \text{kip/ft})(8 \text{ ft})}{2}$

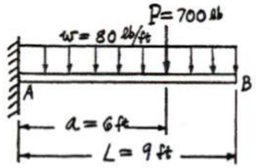
$= 5 \text{ kips}$ ◀

$M_{max} = \dfrac{PL}{4} + \dfrac{wL^2}{8}$

$= \dfrac{(6 \text{ kips})(8 \text{ ft})}{4} + \dfrac{(0.5 \text{ kip/ft})(8 \text{ ft})^2}{8}$

$= 19 \text{ kip}\cdot\text{ft}$ ◀

13-42



$V_{max}^{(-)} = -P - wL$

$= -700\text{ lb} - (80\text{ lb/ft})(9\text{ ft})$

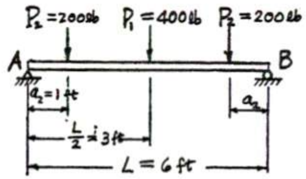
$= -1420\text{ lb}$ ◀

$M_{max}^{(-)} = -Pa - \dfrac{wL^2}{2}$

$= -(700\text{ lb})(6\text{ ft}) - \dfrac{(80\text{ lb/ft})(9\text{ ft})^2}{2}$

$= -7440\text{ lb}\cdot\text{ft}$ ◀

13-44



$V_{max} = \dfrac{P_1}{2} + P_2$

$= \dfrac{400\text{ lb}}{2} + 200\text{ lb}$

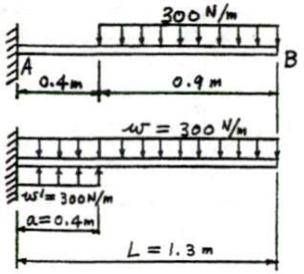
$= 400\text{ lb}$ ◀

$M_{max} = \dfrac{P_1 L}{4} + P_2 a_2$

$= \dfrac{(400\text{ lb})(6\text{ ft})}{4} + (200\text{ lb})(1\text{ ft})$

$= 800\text{ lb}\cdot\text{ft}$ ◀

13-43



$V_{max}^{(-)} = -wL + w'a$

$= -(300\text{ N/m})(1.3\text{ m}) + (300\text{ N/m})(0.4\text{ m})$

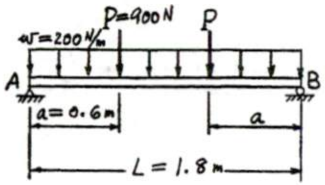
$= -270\text{ N}$ ◀

$M_{max}^{(-)} = -\dfrac{wL^2}{2} + \dfrac{w'a^2}{2}$

$= -\dfrac{(300\text{ N/m})(1.3\text{ m})^2}{2} + \dfrac{(300\text{ N/m})(0.4\text{ m})^2}{2}$

$= -230\text{ N}\cdot\text{m}$ ◀

13-45



$V_{max} = P + \dfrac{wL}{2}$

$= 900\text{ N} + \dfrac{(200\text{ N/m})(1.8\text{ m})}{2}$

$= 1080\text{ kN}$ ◀

$M_{max} = Pa + \dfrac{wL^2}{8}$

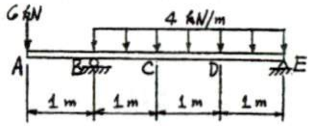
$= (900\text{ N})(0.6\text{ m}) + \dfrac{(200\text{ N/m})(1.8\text{ m})^2}{8}$

$= 621\text{ N}\cdot\text{m}$ ◀

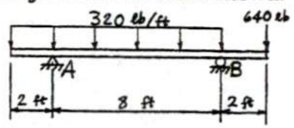
Test Problems for Chapter 13

The following problems may either be given to students for them to practice their problem solving skills or be used as test problems. The answers are provided on the next page.

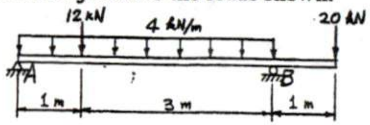
(1) For the overhanging beam subjected to the loads shown, determine the shear force and the bending moment at sections *A*, *B*, *C*, *D*, and *E*.



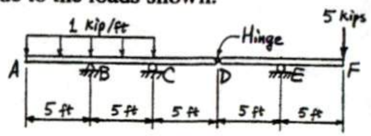
(2) Draw the shear force and the bending moment diagram for the overhanging beam subjected to the loads shown.



(3) Draw the shear force and the bending moment diagram for the overhanging beam subjected to the loads shown.

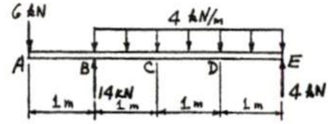


(4) The beam is on roller supports at *C* and *E* and on a hinge support at *B*. The two parts *AD* and *DF* are connected by a hinge at *D*. Draw the shear force and bending moment diagrams for the beam due to the loads shown.



Solutions to Test Problems for Chapter 13

(1)



$V_{A^-} = 0$ ◀

$V_{A^+} = -6$ kN ◀

$V_{B^-} = -6$ kN ◀

$V_{B^+} = -6 + 14 = +8$ kN ◀

$V_C = -6 + 14 - 4(1) = +4$ kN ◀

$V_D = -4 + 4(1) = 0$ (from right) ◀

$V_{E^-} = -4$ kN (from right) ◀

$V_{E^+} = 0$ ◀

$M_A = 0$ ◀

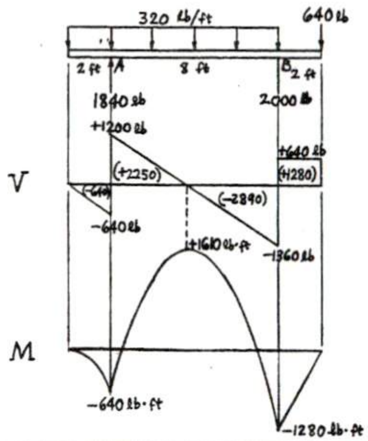
$M_B = +6.8(1) = +6$ kN • m ◀

$M_C = -6(2) + (14)(1) - (4 \times 1)(0.5) = 0$ ◀

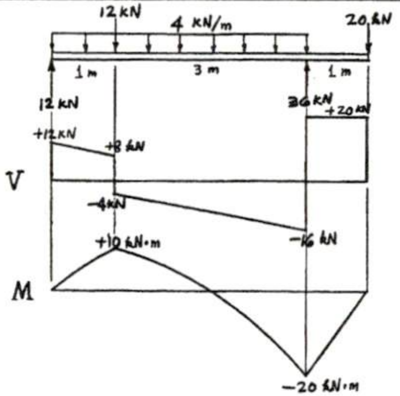
$M_D = 4(1) - (4 \times 1)(0.5) = 2$ kN • m ◀

$M_E = +0$ (from right) ◀

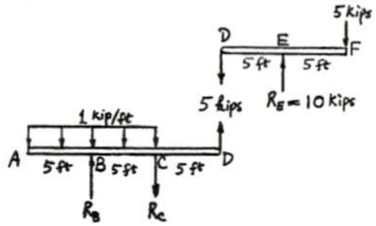
(2)



(3)



(4)

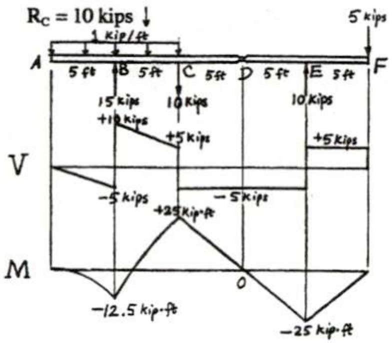


$$\sum M_C = R_B(5) + (1 \times 10)(5) - 5(5) = 0$$

$R_B = 15$ kips ↑ ◀

$$\sum M_B = R_C(5) + (1 \times 10)(0) - 5(10) = 0$$

$R_C = 10$ kips ↓



14-1

From Table 13-1 Case 1:

$$M_{max} = \frac{PL}{4} = \frac{(800 \text{ lb})(10 \text{ ft})}{4}$$

$= 2000$ lb • ft $= 24\,000$ lb • in.

$$S = \frac{bh^2}{6} = \frac{(4 \text{ in.})(6 \text{ in.})^2}{6} = 24 \text{ in.}^3$$

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{24\,000 \text{ lb} \cdot \text{in.}}{24 \text{ in.}^3} = 1000 \text{ psi}$$ ◀

14-2

From Appendix Table A-6(a), a 4 in. x 6 in. has a section modulus of

$S = 17.6$ in.^{3}

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{24\,000 \text{ lb} \cdot \text{in.}}{17.6 \text{ in.}^3} = 1360 \text{ psi}$$ ◀

14-3

From Table 13-1 Case 5:

$M_{max} = PL = (5 \text{ kN})(3 \text{ m}) = 15$ kN • m

$$I = \frac{\pi}{32} d^3 = \frac{\pi}{32}(0.100 \text{ m})^3 = 9.82 \times 10^{-5} \text{m}^3$$

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{15 \text{ kN} \cdot \text{m}}{9.82 \times 10^{-5} \text{ m}^3}$$

$= 153 \times 10^3$ kN/m² $= 153$ MPa ◀

14-4

(a) From Appendix Table A-1(a), for W 18 x 35:

$d = 17.70$ in.
$I_x = 510$ in.^{4}
$S_x = 57.6$ in.^{3}

$$S_x = \frac{I_x}{c} = \frac{I_x}{d/2} = \frac{510 \text{ in.}^4}{(17.70 \text{ in.})/2} = 57.6 \text{ in.}^3$$ ◀

(b) From Appendix Table A-1(b), for W 250 x 0.71:

$b_f = 0.254$ m
$I_y = 38.9 \times 10^{-6}$ m^4,
$S_y = 0.306 \times 10^{-3}$ m^3

$$S_y = \frac{I_y}{c} = \frac{I_y}{b_f/2} = \frac{38.9 \times 10^{-6} \text{ m}^4}{(0.254 \text{ m})/2}$$

$= 0.306 \times 10^{-3}$ m^3 ◀

(c) From Appendix Table A-2(a), for S 12 x 31.8:

$d = 12.00$ in.
$b_f = 5.00$ in.
$I_x = 218$ in.^{4}
$S_x = 36.4$ in.^{3}
$I_y = 9.36$ in.^{4}
$S_y = 3.74$ in.^{3}

$$S_x = \frac{I_x}{c} = \frac{I_x}{d/2} = \frac{218 \text{ in.}^4}{(12 \text{ in.})/2} = 36.3 \text{ in.}^3$$ ◀

$$S_y = \frac{I_y}{c} = \frac{I_y}{b_f/2} = \frac{9.36 \text{ in}^4}{(5 \text{ in.})/2} = 3.74 \text{ in.}^3$$ ◀

14-5

From Table 13-1 Case 3,

$M_{max} = Pa = (20 \text{ kips})(8 \text{ ft})$
$= 160$ kip • ft $= 1920$ kip • in.

From Appendix Table A-1(a), for W 16 x 50:
$S_x = 81.0$ in.^{3}

$$\sigma_{max} = \frac{M_{max}}{S_x} = \frac{1920 \text{ kip} \cdot \text{in.}}{81.0 \text{ in.}^3} = 23.7 \text{ ksi}$$ ◀

14-6

From Table 13-1 Case 3,

$$M_{max} = Pa = P\left(\frac{L}{3}\right) = (90 \text{ kN})\left(\frac{8 \text{ m}}{3}\right) = 240 \text{ kN} \cdot \text{m}$$

From Appendix Table A-1(b), for W 410 x 0.73:

$S_x = 1.33 \times 10^{-3}$ m^3

(Cont'd)

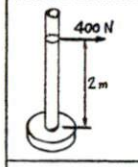
14-6 (Cont)

$$\sigma_{max} = \frac{M_{max}}{S_x} = \frac{240\ \text{kN}\cdot\text{m}}{1.33 \times 10^{-3}\ \text{m}^3}$$

$= 180 \times 10^3\ \text{kN/m}^2 = 180\ \text{MPa}$ ◀

14-7

For 50-mm steel pipe, $S = 9.19 \times 10^{-6}\ \text{m}^3$



$M_{max} = (400\ \text{N})(2\ \text{m}) = 800\ \text{N}\cdot\text{m}$

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{800\ \text{N}\cdot\text{m}}{9.19 \times 10^{-6}\ \text{m}^3}$$

$= 87 \times 10^6\ \text{N/m}^2 = 87\ \text{MPa}$ ◀

14-8

From Table 13-1 Case 4,

$$M_{max} = \frac{wL^2}{8} = \frac{(200\ \text{lb/ft})(15\ \text{ft})^2}{8}$$

$= 5625\ \text{lb}\cdot\text{ft} = 67\,500\ \text{lb}\cdot\text{in.}$

$$S = \frac{\pi}{32}d^3 = \frac{\pi}{32}(10\ \text{in.})^3 = 98.2\ \text{in.}^3$$

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{67\,500\ \text{lb}\cdot\text{in.}}{98.2\ \text{in.}^3} = 688\ \text{psi}$$ ◀

14-9

From Table 13-1, Cases 3 and 4,

$$M_{max} = Pa + \frac{wL^2}{8}$$

$$= (10\ \text{kips})(6\ \text{ft}) + \frac{(3\ \text{kip/ft})(18\ \text{ft})^2}{8}$$

$= 181.5\ \text{kip}\cdot\text{ft} = 2178\ \text{kip}\cdot\text{in.}$

From Appendix Table A-1(a), for W 18 x 50:

$S_x = 88.9\ \text{in.}^3$

$$\sigma_{max} = \frac{M_{max}}{S_x} = \frac{2178\ \text{kip}\cdot\text{in.}}{88.9\ \text{in.}^3} = 24.5\ \text{ksi}$$ ◀

14-10

From Table 13-1, Cases 1 and 4,

$$M_{max} = \frac{PL}{4} + \frac{wL^2}{8}$$

$$= \frac{(16\ \text{kN})(5\ \text{m})}{4} + \frac{(4.5\ \text{kN/m})(5\ \text{m})^2}{8}$$

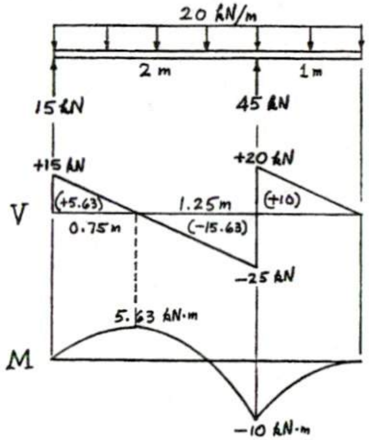
$= 34.1\ \text{kN}\cdot\text{m}$

From Appendix Table A-6(b), for a 150 mm x 410 mm rectangular section: $S_x = 3.61 \times 10^{-3}\ \text{m}^3$

$$\sigma_{max} = \frac{M_{max}}{S_x} = \frac{34.1\ \text{kN}\cdot\text{m}}{3.61 \times 10^{-3}\ \text{m}^3}$$

$= 9450\ \text{kPa} = 9.45\ \text{MPa}$ ◀

14-11



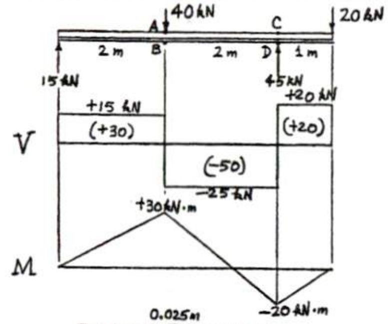
$|M_{max}| = 10\ \text{kN}\cdot\text{m}$

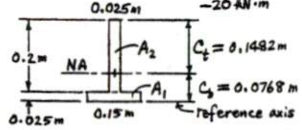
From Appendix Table A-6(b), for 100 mm x 300 mm nominal section: $S_x = 1.21 \times 10^{-3}\ \text{m}^3$

$$\sigma_{max} = \frac{M_{max}}{S_x} = \frac{10\ \text{kN}\cdot\text{m}}{1.21 \times 10^{-3}\ \text{m}^3}$$

$= 8260\ \text{kN/m}^2 = 8.26\ \text{MPa}$ ◀

14-12





$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2}$$

$$= \frac{(0.15 \times 0.025)(0.0125) + (0.025 \times 0.2)(0.125)}{(0.15 \times 0.025) + (0.025 \times 0.2)}$$

$= 0.0768\ \text{m}$

$$\bar{I}_x = \sum\left[\bar{I} + A(\bar{y} - y)^2\right]$$

$$= \left[\frac{(0.15)(0.025)^3}{12} + (0.15 \times 0.025)(0.0768 - 0.0125)^2\right]$$

$$+ \left[\frac{(0.025)(0.2)^3}{12} + (0.025 \times 0.2)(0.0768 - 0.125)^2\right]$$

$= 4.40 \times 10^{-5}\ \text{m}^4$

$$\sigma_A = \frac{M^{(+)}_{max}c_t}{I} = \frac{(30\ \text{kN}\cdot\text{m})(0.1482\ \text{m})}{4.40 \times 10^{-5}\ \text{m}^4}$$

$= 101 \times 10^3\ \text{kN/m}^2 = 101\ \text{MPa (C)}$

$$\sigma_B = \frac{M^{(+)}_{max}c_b}{I} = \frac{(30\ \text{kN}\cdot\text{m})(0.0768\ \text{m})}{4.40 \times 10^{-5}\ \text{m}^4}$$

$= 52.4 \times 10^3\ \text{kN/m}^2 = 52.4\ \text{MPa (T)}$

$$\sigma_C = \frac{M^{(-)}_{max}c_t}{I} = \frac{(20\ \text{kN}\cdot\text{m})(0.1482\ \text{m})}{4.40 \times 10^{-5}\ \text{m}^4}$$

$= 67.4 \times 10^3\ \text{kN/m}^2 = 67.4\ \text{MPa (T)}$

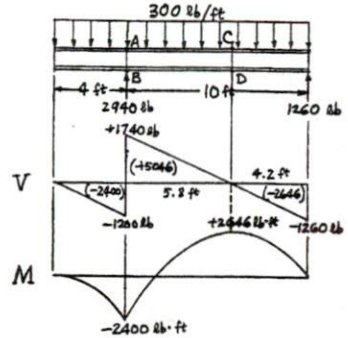
$$\sigma_D = \frac{M^{(-)}_{max}c_b}{I} = \frac{(20\ \text{kN}\cdot\text{m})(0.0768\ \text{m})}{4.40 \times 10^{-5}\ \text{m}^4}$$

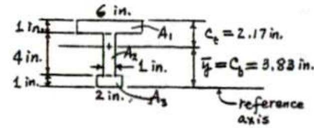
$= 34.9 \times 10^3\ \text{kN/m}^2 = 34.9\ \text{MPa (C)}$

$\sigma^{(T)}_{max} = \sigma_C = 67.4\ \text{MPa}$ ◀

$\sigma^{(C)}_{max} = \sigma_A = 101\ \text{MPa}$ ◀

14-13





$\Sigma A = 6(1) + 1(4) + 2(1) = 12\ \text{in.}^2$

$\Sigma Ay = 6(5.5) + 4(3) + 2(0.5) = 46\ \text{in.}^3$

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{46}{12} = 3.83\ \text{in.}$$

$$\bar{I}_x = \sum\left[\bar{I} + A(\bar{y} - y)^2\right]$$

$$= \left[\frac{(6)(1)^3}{12} + 6(3.83 - 5.5)^2\right] + \left[\frac{1(4)^3}{12} + 4(3.83 - 3)^2\right]$$

$$+ \left[\frac{2(1)^3}{12} + 2(3.83 - 0.5)^2\right] = 47.7\ \text{in.}^4$$

$$\sigma_A = \frac{M^{(-)}_{max}c_t}{I} = \frac{(2400 \times 12\ \text{lb}\cdot\text{in.})(2.17\ \text{in.})}{47.7\ \text{in.}^4}$$

$= 1310\ \text{psi (T)}$

$$\sigma_B = \frac{M^{(-)}_{max}c_b}{I} = \frac{(2400 \times 12\ \text{lb}\cdot\text{in.})(3.83\ \text{in.})}{47.7\ \text{in.}^4}$$

$= 2310\ \text{psi (C)}$

$$\sigma_C = \frac{M^{(+)}_{max}c_t}{I} = \frac{(2646 \times 12\ \text{lb}\cdot\text{in.})(2.17\ \text{in.})}{47.7\ \text{in.}^4}$$

$= 1440\ \text{psi (C)}$

$$\sigma_D = \frac{M^{(+)}_{max}c_b}{I} = \frac{(2646 \times 12\ \text{lb}\cdot\text{in.})(3.83\ \text{in.})}{47.7\ \text{in.}^4}$$

$= 2550\ \text{psi (T)}$

$\sigma^{(T)}_{max} = \sigma_D = 2550\ \text{psi}$ ◀

$\sigma^{(C)}_{max} = \sigma_B = 2310\ \text{psi}$ ◀

14-14

From Appendix Table A-6(b), for a 50 x 100 section,

$S_x = 0.0502 \times 10^{-3}\ m^3$

$$M_{allow} = \sigma_{allow} S$$
$$= (10 \times 10^6\ N/m^2)(0.0502 \times 10^{-3}\ m^3)$$
$$= 502\ N \cdot m$$ ◀

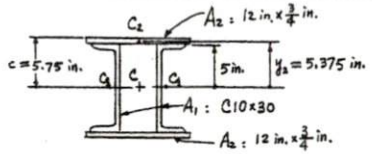
14-15

For W 16 x 50,

$S_x = 81.0\ in.^3$

$$M_{allow} = S\sigma_{allow}$$
$$= (81.0\ in.^3)(24\ kip\text{-}in.^2)$$
$$= 1944\ kip \cdot in. = 162\ kip \cdot ft$$ ◀

14-16



From Appendix Table A-3(a), for C 10 x 30:

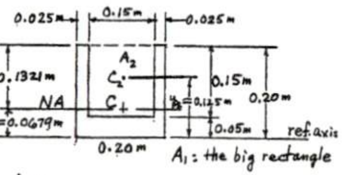
d = 10 in.
$(I_x)_1 = 103\ in.^4$

$$I_x = 2(I_x)_1 + 2[(I_x)_2 + A_2 y_2^2]$$
$$= 2(103) + 2\left[\frac{12(0.75)^3}{12} + (12 \times 0.75)(5.375)^2\right]$$
$$= 727\ in.^4$$

$$S = \frac{I_x}{c} = \frac{727\ in.^4}{5.75\ in.} = 126.4\ in.^3$$

$$M_{allow} = S\sigma_{allow}$$
$$= (126.4\ in.^3)(24\ kip\text{-}in.^2)$$
$$= 3034\ kip \cdot in. = 253\ kip \cdot ft$$ ◀

14-17



$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$$
$$= \frac{(0.20 \times 0.20)(0.10) - (0.15 \times 0.15)(0.125)}{(0.20 \times 0.20) - (0.15 \times 0.15)}$$
$$= 0.0679\ m$$

$$I_x = \left[I_1 + A_1(\bar{y} - y_1)^2\right] - \left[I_2 + A_2(\bar{y} - y_2)^2\right]$$
$$= \left[\frac{(0.20)(0.20)^3}{12} + (0.20 \times 0.20)(0.0679 - 0.10)^2\right]$$
$$- \left[\frac{(0.15)(0.15)^3}{12} + (0.15 \times 0.15)(0.0679 - 0.125)^2\right]$$
$$= 5.90 \times 10^{-5} m^4$$

Due to positive moment, the maximum tensile stress occurs at the bottom of the section. Thus

$$M_{allow}^{(+)} = \frac{I_x \sigma_{allow}^{(T)}}{c_b}$$
$$= \frac{(5.90 \times 10^{-5}\ m^4)(21 \times 10^6\ N/m^2)}{0.0679\ m}$$
$$= 18.2 \times 10^3\ N \cdot m = 18.2\ kN \cdot m$$

Due to positive moment, the maximum compressive stress occurs at the top of the section. Thus

$$M_{allow}^{(+)} = \frac{I_x \sigma_{allow}^{(C)}}{c_t}$$
$$= \frac{(5.90 \times 10^{-5}\ m^4)(84 \times 10^6\ N/m^2)}{0.1321\ m}$$
$$= 37.5 \times 10^3\ N \cdot m = 37.5\ kN \cdot m$$

$$M_{allow}^{(+)} = 18.2\ kN \cdot m$$ ◀

14-18

From the solution to Prob. 14-17, we have
$I_x = 5.90 \times 10^{-5}\ m^4$
$c_b = 0.0679\ m, \quad c_t = 0.1321\ m$

Due to negative moment, the maximum tensile stress occurs at the top of the section. Thus

$$M_{allow}^{(-)} = \frac{I_x \sigma_{allow}^{(T)}}{c_t}$$
$$= \frac{(5.90 \times 10^{-5}\ m^4)(21 \times 10^6\ N/m^2)}{0.1321\ m}$$
$$= 9.38 \times 10^3\ N \cdot m = 9.38\ kN \cdot m$$

Due to negative moment, the maximum compressive stress occurs at the bottom of the section. Thus

$$M_{allow}^{(-)} = \frac{I_x \sigma_{allow}^{(C)}}{c_b}$$
$$= \frac{(5.90 \times 10^{-5}\ m^4)(84 \times 10^6\ N/m^2)}{0.0679\ m}$$
$$= 73.0 \times 10^3\ N \cdot m = 73.0\ kN \cdot m$$
$$M_{allow}^{(-)} = 9.38\ kN \cdot m$$ ◀

14-19

$$M_{max} = \frac{PL}{4} = \frac{P(10 \times 12\ in.)}{4} = 30P$$

For W14 x 32, $S_x = 123\ in.^3$

$$M_{allow} = S_x\sigma_{allow} = (123\ in.^3)(33\ kip \cdot in.^2)$$
$$= 4060\ kip \cdot in. = 30\ P$$
$$P_{allow} = 135.3\ kips$$ ◀

14-20

From Appendix Table A-2(a), for S 15 x 50:
$S_x = 64.8\ in.^3$

$$M_{allow} = S_x\sigma_{allow} = (64.8\ in.^3)(24\,000\ lb/in.^2)$$
$$= 1.56 \times 10^6\ lb \cdot in.$$

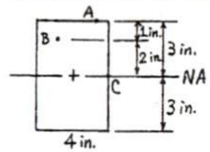
From Table 13-1 Case 5, $M_{max} = \dfrac{wL^2}{2}$
$M_{max} = 1.56 \times 10^6\ lb \cdot in. = 1.30 \times 10^5\ lb \cdot ft$

$$w = \frac{2M_{max}}{L^2} = \frac{2(1.30 \times 10^{-5}\ lb \cdot ft)}{(8\ ft)^2} = 4060\ lb/ft$$

Subtracting the beam weight, we get

$$w_{allow} = 4060\ lb/ft - 50\ lb/ft = 4010\ lb/ft$$ ◀

14-21



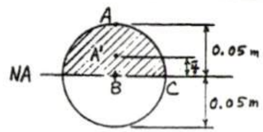
V = 1900 lb

$$I = \frac{4 \times 6^3}{12} = 72\ in.^4$$
$$\tau_A = 0$$ ◀
$$\tau_B = \frac{VQ}{It} = \frac{(1900\ lb)[(4 \times 1)(2.5)\ in.^3]}{(72\ in.^4)(4\ in.)}$$
$$= 66.0\ psi$$ ◀
$$\tau_C = \frac{VQ}{It} = \frac{(1900\ lb)[(4 \times 3)(1.5)\ in.^3]}{(72\ in.^4)(4\ in.)}$$
$$= 118.8\ psi$$ ◀

or

$$\tau_C = 1.5\frac{V}{A} = 1.5\frac{1900\ lb}{4 \times 6\ in.^2} = 118.8\ psi$$

14-22



V = 15 kN
$$\tau_A = 0$$ ◀
$$I = \frac{\pi}{4}R^4 = \frac{\pi}{4}(0.05\ m)^4 = 4.91 \times 10^{-6}\ m^4$$

(Cont'd)

14-22 (Cont)

$$A' = \frac{\pi}{2}R^2 = \frac{\pi}{2}(0.05\text{ m})^2 = 3.93 \times 10^{-3}\text{ m}^2$$

$$\bar{y} = \frac{4R}{3\pi} = \frac{4\ (0.05\text{ m})}{3\pi} = 2.12 \times 10^{-2}\text{ m}$$

$$\tau_B = \tau_C = \frac{VQ}{It}$$

$$= \frac{(15\text{ kN})[(3.93 \times 10^{-3}\text{ m}^2)(2.12 \times 10^{-2}\text{ m})]}{(4.91 \times 10^{-6}\text{ m}^4)(0.10\text{ m})}$$

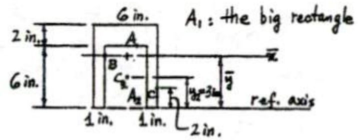
$$= 2.55 \times 10^3\text{ kN/m}^2 = 2.55\text{ MPa}$$ ◀

or from Eq. 14-12, we have

$$\tau_B = \tau_C = \tau_{max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{15\text{ kN}}{\pi(0.05\text{ m})^2}$$

$$= 2550\text{ kN/m}^2 = 2.55\text{ MPa}$$

14-23



$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{(6 \times 8)(4) - (4 \times 6)(3)}{6 \times 8 - 4 \times 6} = 5\text{ in.}$$

$$I_x = [I_1 + A_1(\bar{y} - y_1)^2] - [I_2 + A_2(\bar{y} - y_2)^2]$$

$$= \left[\frac{6(8)^3}{12} + (6 \times 8)(5-4)^2\right] - \left[\frac{4(6)^3}{12} + (4 \times 6)(5-3)^2\right]$$

$$= 136\text{ in.}^4$$

V = 10 kips

For point A, taking the area above the level through point A as A', we have

$$Q = A'\bar{y}' = (6 \times 2)(2) = 24\text{ in.}^3$$

$$\tau_A = \frac{VQ}{It} = \frac{(10\text{ kips})(24\text{ in.}^3)}{(136\text{ in.}^4)(6\text{ in.})}$$

$$= 0.294\text{ ksi} = 294\text{ psi}$$ ◀

For point B, taking the area below the N.A. as A', we have

$$Q = A'\bar{y}' = 2(1 \times 5)(2.5) = 25\text{ in.}^3$$

$$\tau_B = \frac{VQ}{It} = \frac{(10\text{ kips})(25\text{ in.}^3)}{(136\text{ in.}^4)(2\text{ in.})}$$

$$= 0.919\text{ ksi} = 919\text{ psi}$$ ◀

For point C, taking the area below the level through point C as A', we have

$$Q = A'\bar{y}' = 2(1 \times 2)(4) = 16\text{ in.}^3$$

$$\tau_C = \frac{VQ}{It} = \frac{(10\text{ kips})(16\text{ in.}^3)}{(136\text{ in.}^4)(2\text{ in.})}$$

$$= 0.588\text{ ksi} = 588\text{ psi}$$ ◀

14-24

$$M_{max} = PL$$

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{PL}{S} = \sigma_{allow}$$

$$P = \frac{S\sigma_{allow}}{L} = \frac{\left[\frac{1}{6}(0.1\text{ m})(0.15\text{ m})^2\right](10 \times 10^3\text{ kN/m}^2)}{1\text{ m}}$$

$$= 3.75\text{ kN}$$

$$V_{max} = P$$

For the rectangular section,

$$\tau_{max} = 1.5\frac{V_{max}}{A} = 1.5\frac{P}{A} = \sigma_{allow}$$

$$P = \frac{A\,\sigma_{allow}}{1.5} = \frac{(0.10\text{ m} \times 0.15\text{ m})(800\text{ kN/m}^2)}{1.5}$$

$$= 8\text{ kN}$$

$$P_{allow} = 3.75\text{ kN}$$ ◀

14-25

For the rectangular section,

$$S = \frac{bh^2}{6} = \frac{(3\text{ in.})(12\text{ in.})^2}{6} = 72\text{ in.}^3$$

For the uniformly loaded simple beam,

$$V_{max} = \frac{wL}{2}$$

$$\tau_{allow} = \tau_{max} = 1.5\frac{V_{max}}{A} = 1.5\frac{wL}{2A}$$

$$w = \frac{2A\tau_{allow}}{1.5\ L} = \frac{2(3\text{ in.} \times 12\text{ in.})(145\text{ lb/in.}^2)}{1.5(120\text{ in.})}$$

$$= 58\text{ lb/in.} = 696\text{ lb/ft}$$

$$M_{max} = \frac{wL^2}{8}$$

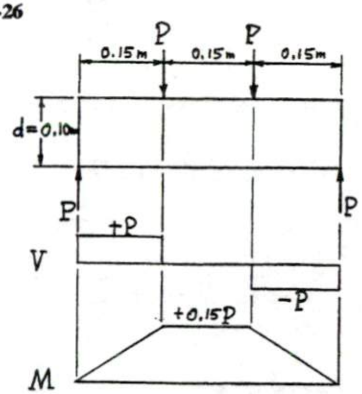
$$\sigma_{allow} = \sigma_{max} = \frac{M_{max}}{S} = \frac{wL^2}{8S}$$

$$w = \frac{8S\sigma_{allow}}{L^2} = \frac{8(72\text{ in.}^3)(1900\text{ lb/in.}^2)}{(120\text{ in.})^2}$$

$$= 76\text{ lb/in.} = 912\text{ lb/ft}$$

$$w_{allow} = 696\text{ lb/ft}$$ ◀

14-26



$$S = \frac{\pi}{32}(0.1\text{ m})^3 = 9.82 \times 10^{-5}\text{ m}^3$$

$$\sigma_{allow} = \sigma_{max} = \frac{M_{max}}{S} = \frac{0.15\,P}{S}$$

$$P = \frac{S\,\sigma_{allow}}{0.15}$$

$$= \frac{(9.82 \times 10^{-5}\text{ m}^3)(9 \times 10^3\text{ kN/m}^2)}{0.15\text{ m}}$$

$$= 5.89\text{ kN}$$

$$A = \frac{\pi}{4}(0.1\text{ m})^2 = 7.85 \times 10^{-3}\text{ m}^2$$

$$\tau_{allow} = \tau_{max} = \frac{4V_{max}}{3A} = \frac{4P}{3A}$$

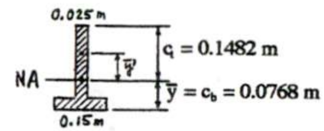
$$P = \frac{3A\,\tau_{allow}}{4}$$

$$= \frac{3(7.85 \times 10^{-3}\text{ m}^2)(850\text{ kN/m}^2)}{4}$$

$$= 5.00\text{ kN}$$

$$P_{allow} = 5.00\text{ kN}$$ ◀

14-27



$$A' = (0.025\text{ m})(0.1482\text{ m}) = 3.705 \times 10^{-3}\text{ m}^2$$

$$\bar{y}' = \frac{0.1482\text{ m}}{2} = 0.0741\text{ m}$$

$$Q = A'\bar{y}' = (3.705 \times 10^{-3}\text{ m}^2)(0.0741\text{ m})$$
$$= 2.745 \times 10^{-4}\text{ m}^2$$

From the solution to Problem 14-12,

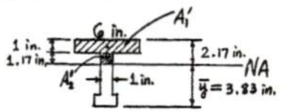
$$I = 4.40 \times 10^{-5}\text{ m}^4$$
$$|V_{max}| = 25\text{ kN}$$

$$\tau_{max} = \tau_{NA} = \frac{VQ}{It}$$

$$= \frac{(25\text{ kN})(2.745 \times 10^{-4}\text{ m}^3)}{(4.40 \times 10^{-5}\text{ m}^4)(0.025\text{ m})}$$

$$= 6240\text{ kN/m}^2 = 6.24\text{ MPa}$$ ◀

14-28



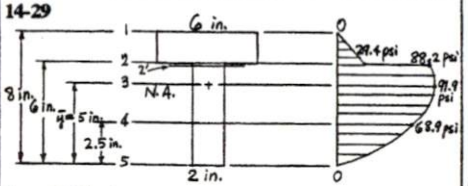
$Q = A_1\bar{y}_1' + A_2\bar{y}_2' = (6)(1)(1.17+0.5) + (1)(1.17)\left(\frac{1.17}{2}\right) = 10.70 \text{ in.}^3$

From the solution to Problem 14-13,

$I = 47.7 \text{ in.}^4$
$|V_{max}| = 1740 \text{ lb}$

$$\tau_{max} = \tau_{NA} = \frac{VQ}{It} = \frac{(1740 \text{ lb})(10.70 \text{ in.}^2)}{(47.7 \text{ in.}^4)(1 \text{ in.})} = 390 \text{ psi}$$

14-29



$I_{NA} = 136 \text{ in.}^4$

At Section A-A, $V = -1000$ lb

$\tau_1 = 0$

$$\tau_2 = \frac{VQ}{It} = \frac{(1000)[(6 \times 2)(2)]}{(136)(6)} = 29.4 \text{ psi}$$

$$\tau_{2'} = \frac{VQ}{It} = \frac{(1000)[(6 \times 2)(2)]}{(136)(2)} = 88.2 \text{ psi}$$

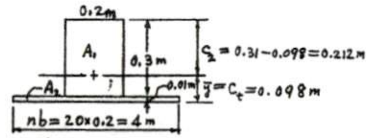
$$\tau_3 = \frac{VQ}{It} = \frac{(1000)[(2 \times 5)(2.5)]}{(136)(2)} = 91.9 \text{ psi}$$

$$\tau_4 = \frac{VQ}{It} = \frac{(1000)[(2 \times 2.5)(3.75)]}{(136)(2)} = 68.9 \text{ psi}$$

$\tau_5 = 0$

14-30

Transformed section in wood:



$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(0.2 \times 0.3)(0.01+0.15) + (4 \times 0.01)(0.005)}{0.2 \times 0.3 + 4 \times 0.01} = 0.098 \text{ m}$$

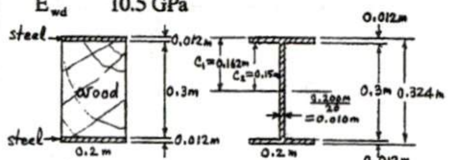
$$I_x = \left[I_1 + A_1(\bar{y} - y_1)^2\right] + \left[I_2 + A_2(\bar{y} - y_2)^2\right]$$
$$= \left[\frac{(0.2)(0.3)^3}{12} + (0.2 \times 0.3)(0.098 - 0.16)^2\right] + \left[\frac{(4)(0.01)^3}{12} + (4 \times 0.01)(0.098 - 0.005)^2\right]$$
$$= 0.001027 \text{ m}^4$$

$$(\sigma_{wd})_{max} = \frac{Mc_2}{I_{wd}} = \frac{(50 \text{ kN}\cdot\text{m})(0.212 \text{ m})}{0.001027 \text{ m}^4} = 10.3 \times 10^3 \text{ kN/m}^2 = 10.3 \text{ MPa (C)}$$

$$(\sigma_{st})_{max} = n\frac{Mc_1}{I_{wd}} = 20\frac{(50 \text{ kN}\cdot\text{m})(0.098 \text{ m})}{0.001027 \text{ m}^4} = 95.4 \times 10^3 \text{ kN/m}^2 = 95.4 \text{ MPa (T)}$$

14-31

$$n = \frac{E_{st}}{E_{wd}} = \frac{210 \text{ GPa}}{10.5 \text{ GPa}} = 20$$



Section — Transformed section (in steel)

(Cont'd)

14-31 (Cont)

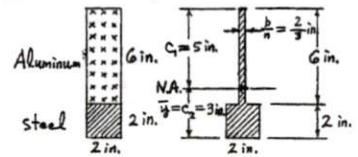
$$I_{st} = \frac{0.2(0.324)^3}{12} - \frac{0.19(0.3)^3}{12} = 1.394 \times 10^{-4} \text{ m}^4$$

$$(\sigma_{st})_{max} = \frac{Mc_2}{I_{st}} = \frac{(100 \text{ kN}\cdot\text{m})(0.162 \text{ m})}{1.394 \times 10^{-4} \text{ m}^4} = 116 \times 10^3 \text{ kN/m}^2 = 116 \text{ MPa}$$

$$(\sigma_{wd})_{max} = \frac{1}{n}\frac{Mc_2}{I_{st}} = \frac{1}{20}\frac{(100 \text{ kN}\cdot\text{m})(0.15 \text{ m})}{1.394 \times 10^{-4} \text{ m}^4} = 5.38 \times 10^3 \text{ kN/m}^2 = 5.38 \text{ MPa}$$

14-32

$$n = \frac{E_{st}}{E_{al}} = \frac{30\,000 \text{ ksi}}{10\,000 \text{ ksi}} = 3$$



Section — Transformed section (in steel)

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(2 \times 2)(1) + \left(\frac{2}{3}\right)(6)(5)}{2 \times 2 + \left(\frac{2}{3}\right)(6)} = 3 \text{ in.}$$

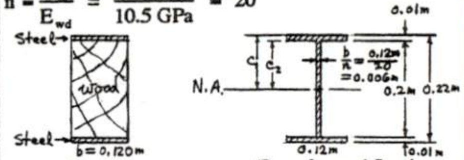
$$I_{st} = \left[I_1 + A_1(\bar{y} - y_1)^2\right] + \left[I_2 + A_2(\bar{y} - y_2)^2\right]$$
$$= \left[\frac{1}{2}(2)(2)^3 + (2 \times 2)(3-1)^2\right] + \left[\frac{1}{12}\left(\frac{2}{3}\right)(6)^3 + \left(\frac{2}{3} \times 6\right)(3-5)^2\right]$$
$$= 45.33 \text{ in.}^4$$

$$(\sigma_{st})_{max} = \frac{Mc_2}{I_{st}} = \frac{(300 \text{ kip}\cdot\text{in.})(3 \text{ in.})}{45.33 \text{ in.}^4} = 19.9 \text{ ksi (T)}$$

$$(\sigma_{al})_{max} = \frac{1}{n}\frac{Mc_1}{I_{st}} = \frac{1}{3}\frac{(300 \text{ kip}\cdot\text{in.})(5 \text{ in.})}{45.33 \text{ in.}^4} = 11.03 \text{ ksi (C)}$$

14-33

$$n = \frac{E_{st}}{E_{wd}} = \frac{210 \text{ GPa}}{10.5 \text{ GPa}} = 20$$



Section — Transformed Section (in steel)

$$I_{st} = \frac{0.12(0.22)^3}{12} - \frac{0.114(0.20)^3}{12} = 3.05 \times 10^{-5} \text{ m}^4$$

$$(\sigma_{st})_{max} = \frac{Mc_1}{I_{st}} = \frac{M(0.11 \text{ m})}{3.05 \times 10^{-5} \text{ m}^4} = \frac{M}{2.77 \times 10^{-4} \text{ m}^3} \le (\sigma_{st})_{allow} = 160 \times 10^3 \text{ kN/m}^2$$

$M \le (160 \times 10^3 \text{ kN/m}^2)(2.77 \times 10^{-4} \text{ m}^3) = 44.3 \text{ kN}\cdot\text{m}$

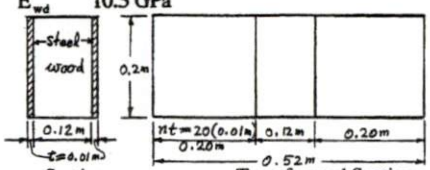
$$(\sigma_{wd})_{max} = \frac{1}{n}\frac{Mc_2}{I_{st}} = \frac{1}{20}\left[\frac{M(0.10 \text{ m})}{3.05 \times 10^{-5} \text{ m}^4}\right] = \frac{M}{6.10 \times 10^{-3} \text{ m}^3} \le (\sigma_{wd})_{allow} = 8.5 \times 10^3 \text{ kN/m}^2$$

$M \le (8.5 \times 10^3 \text{ kN/m}^2)(6.10 \times 10^{-3} \text{ m}^3) = 51.9 \text{ kN}\cdot\text{m}$

$M_{allow} = 44.3 \text{ kN}\cdot\text{m}$

14-34

$$n = \frac{E_{st}}{E_{wd}} = \frac{210 \text{ GPa}}{10.5 \text{ GPa}} = 20$$



Section — Transformed Section (in wood)

(Cont'd)

14-34 (Cont)

$$S_{wd} = \frac{bh^2}{6} = \frac{0.52(0.2)^2}{6} = 3.47 \times 10^{-3}\ m^3$$

$$(\sigma_{wd})_{max} = \frac{M}{S} = \frac{M}{3.47 \times 10^{-3}\ m^3}$$

$$\leq (\sigma_{wd})_{allow} = 8.5 \times 10^3\ kN/m^2$$

$$M \leq (8.5 \times 10^3\ kN/m^2)(3.47 \times 10^{-3}\ m^3) = 29.5\ kN \cdot m$$

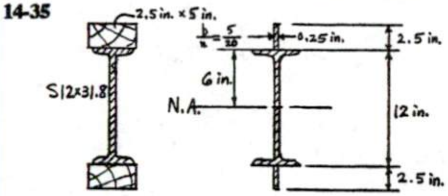
$$(\sigma_{st})_{max} = n\frac{M}{S} = 20\left[\frac{M}{3.47 \times 10^{-3}\ m^3}\right] = \frac{M}{1.735 \times 10^{-4}\ m^3}$$

$$\leq (\sigma_{st})_{allow} = 160 \times 10^3\ kN/m^2$$

$$M \leq (160 \times 10^3\ kN/m^2)(1.735 \times 10^{-4}\ m^3) = 27.8\ kN \cdot m$$

$$M_{allow} = 27.8\ kN \cdot m$$ ◀

14-35



Section

Transformed Section (in steel)

$$n = \frac{E_{st}}{E_{wd}} = \frac{30 \times 10^6\ psi}{1.5 \times 10^6\ psi} = 20$$

$$I_{st} = 218 + 2\left[\frac{0.25(2.5)^3}{12} + (0.25 \times 2.5)(7.25)^2\right]$$

$$= 284\ in.^4$$

$$(\sigma_{st})_{max} = \frac{My}{I_{st}} = \frac{M(6\ in.)}{284\ in.^4} = \frac{M}{47.3\ in.^3}$$

$$\leq (\sigma_{st})_{allow} = 24\ ksi$$

$$M_{max} = (24\ kip/in.^2)(47.3\ in.^3)$$

$$= 1136\ kip \cdot in. = 94.7\ kip \cdot ft$$

$$(\sigma_{wd})_{max} = \frac{1}{n}\frac{Mc}{I_{st}} = \frac{1}{20}\left[\frac{M(8.5 in.)}{284 in.^4}\right] = \frac{M}{668\ in.^3}$$

$$\leq (\sigma_{wd})_{allow} = 1.2\ ksi$$

$$M_{max} = (1.2\ kip/in.^2)(668\ in.^3)$$

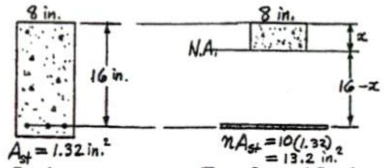
$$= 802\ kip \cdot in. = 66.8\ kip \cdot ft$$

Equating the smaller one of M_{max} to $wL^2/8$, we get

$$M_{max} = \frac{WL^2}{8} = \frac{w(20)^2}{8} = 66.8$$

$$w_{allow} = 1.34\ kip/ft$$ ◀

14-36



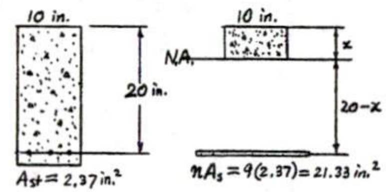
Section Transformed Section

$$8x\left(\frac{x}{2}\right) = 13.2\,(16 - x)$$

$$4x^2 + 13.2x - 211.2 = 0$$

$$x = \frac{-13.2 + \sqrt{(13.2)^2 + 4(4)(211.2)}}{2(4)} = 5.80\ in.$$ ◀

14-37



Section Transformed Section

$$10x\left(\frac{x}{2}\right) = 21.33\,(20 - x)$$

$$5x^2 + 21.33x - 427 = 0$$

$$x = \frac{-21.33 + \sqrt{(21.33)^2 + 4(5)(427)}}{2(5)} = 7.35\ in.$$

$$I = \frac{10(7.35)^3}{3} + 21.33\,(20 - 7.35)^2$$

$$= 4740\ in.^4$$

$$(\sigma_{cn})_{max} = \frac{Mx}{I} = \frac{(70 \times 12\ kip \cdot in.)(7.35\ in.)}{4740\ in.^4}$$

$$= 1.30\ ksi\ (C)$$ ◀

(Cont'd)

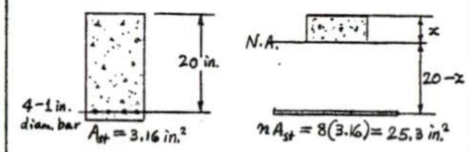
14-37 (Cont)

$$(\sigma_{st})_{max} = n\frac{M(20 - x)}{I}$$

$$= 9\ \frac{(70 \times 12\ kip \cdot in.)(20\ in. - 7.35\ in.)}{4740\ in.^4}$$

$$= 20.2\ ksi\ (T)$$ ◀

14-38



Section Transformed Section

$$12x\left(\frac{x}{2}\right) = 25.3\,(20 - x)$$

$$6x^2 + 25.3x - 506 = 0$$

$$x = \frac{-25.3 + \sqrt{(25.3)^2 + 4(6)(506)}}{2(6)} = 7.31\ in.$$

$$I = \frac{12(7.31)^3}{3} + 25.3\,(20 - 7.31)^2 = 5640\ in.^4$$

$$(\sigma_{cn})_{max} = \frac{Mx}{I} = \frac{M(7.31\ in.)}{5640\ in.^4} = \frac{M}{772\ in.^3}$$

$$\leq (\sigma_{cn})_{allow} = 1800\ psi$$

$$M \leq (1800\ lb/in.^2)(772\ in.^3) = 1.39 \times 10^6\ lb \cdot in.$$

$$(\sigma_{st})_{max} = n\frac{M(20 - x)}{I} = 8\left[\frac{M(20\ in. - 7.31\ in.)}{5640\ in.^4}\right]$$

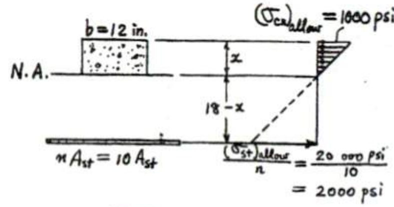
$$= \frac{M}{55.6\ in.^3} \leq (\sigma_{st})_{allow} = 20\,000\ psi$$

$$M \leq (20\,000\ lb/in.^2)(55.6\ in.^3) = 1.11 \times 10^6\ lb \cdot in.$$

$$M_{allow} = 1.11 \times 10^6\ lb \cdot in.$$

$$= 92.5\ kip \cdot ft$$ ◀

14-39



(a)

$$\frac{x}{18 - x} = \frac{1000}{2000} = \frac{1}{2}$$

$$2x = 18 - x$$

$$x = 6\ in.$$ ◀

(b)

$$12(6)\left(\frac{6}{2}\right) = (10A_{st})(18 - 6)$$

$$A_{st} = 1.80\ in.^2$$ ◀

(c)

$$I = \frac{12(6)^3}{3} + (10 \times 1.8)(18 - 6)^2 = 3460\ in.^4$$

$$(\sigma_{cn})_{max} = \frac{Mx}{I} = \frac{M(6\ in.)}{3460\ in.^4} = \frac{M}{577\ in.^3}$$

$$\leq (\sigma_{cn})_{allow} = 1000\ psi$$

$$M \leq (1000\ lb/in.^2)(577\ in.^3) = 5.77 \times 10^5\ lb \cdot in.$$

$$(\sigma_{st})_{max} = n\frac{M(d - x)}{I} = 10\left[\frac{M(18\ in. - 6\ in.)}{3460\ in.^4}\right]$$

$$= \frac{M}{28.8\ in.^3} \leq (\sigma_{st})_{allow} = 20\,000 psi$$

$$M \leq (20\,000\ lb/in.^2)(28.8\ in.^3) = 5.77 \times 10^5\ lb \cdot in.$$

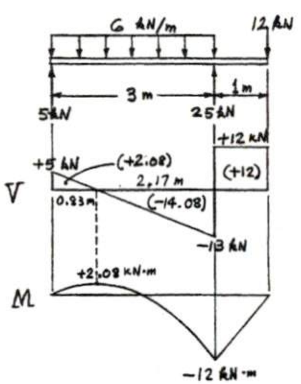
$$M_{allow} = 5.77 \times 10^5\ lb \cdot in.$$

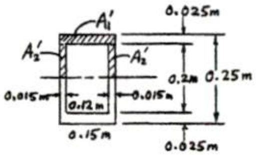
$$= 4.81 \times 10^4\ lb \cdot ft$$

$$w_{allow} = \frac{8M_{allow}}{L^2} = \frac{8(4.81 \times 10^4\ lb \cdot ft)}{(16\ ft)^2}$$

$$= 1503\ lb/ft$$ ◀

14-40





$$I = \frac{(0.15)(0.25)^3}{12} - \frac{(0.12)(0.2)^3}{12} = 1.153 \times 10^{-4}\ m^4$$

$$Q = A_1'\bar{y}_1' + A_2'\bar{y}_2' = (0.15 \times 0.025)(0.1125)$$
$$+ 2(0.015 \times 0.10)(0.05) = 5.72 \times 10^{-4}\ m^3$$

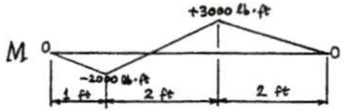
$$\tau_{max} = \frac{VQ}{It} = \frac{(13\ kN)(5.72 \times 10^{-4}\ m^3)}{(1.153 \times 10^{-4}\ m^4)(2 \times 0.015\ m)}$$

$= 2150\ kN/m^2 = 2.15\ MPa$ ◄

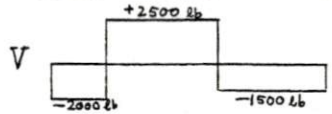
$$\sigma_{max} = \frac{Mc}{I} = \frac{(12\ kN \cdot m)(0.125\ m)}{1.153 \times 10^{-4}\ m^4}$$

$= 13\ 000\ kN/m^2 = 13.0\ MPa$ ◄

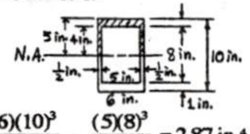
14-41



$$V = \frac{\Delta M}{\Delta x} \quad \frac{-2000-0}{1} = -2000\ lb \quad \frac{3000-(-2000)}{2} = +2500\ lb \quad \frac{0-3000}{2} = -1500\ lb$$



$V_{max} = 2500$ lb
$M_{max} = 3000$ lb-ft



$$I = I_1 - I_2 = \frac{(6)(10)^3}{12} - \frac{(5)(8)^3}{12} = 2.87\ in.^4$$

$$Q = A_1'\bar{y}_1' - A_2'\bar{y}_2' = (6 \times 5)(2.5) - (5 \times 4)(2) = 35\ in.^3$$

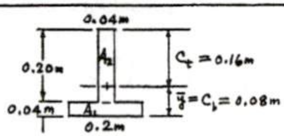
$$\sigma_{max} = \frac{Mc}{I} = \frac{(3000 \times 12\ lb \cdot in.)(5\ in.)}{287\ in.^4}$$

$= 627$ psi ◄

$$\tau_{max} = \frac{VQ}{It} = \frac{(2500\ lb)(35\ in.^3)}{(287\ in.^4)(2 \times \frac{1}{2}\ in.)}$$

$= 305$ psi ◄

14-42



$\Sigma A = A_1 + A_2 = 0.2 \times 0.04 + 0.04 \times 0.20 = 0.016\ m^2$
$\Sigma Ay = A_1 y_1 + A_2 y_2$
$= (0.2 \times 0.04)(0.02) + (0.04 \times 0.2)(0.14)$
$= 0.00128\ m^3$

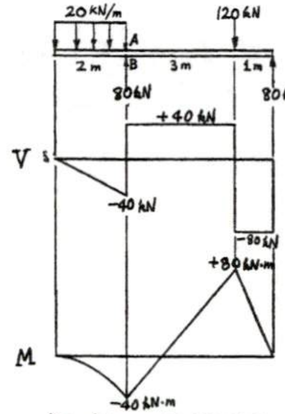
(Cont'd)

14-42 (Cont)

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{0.00128\ m^3}{0.016\ m^2} = 0.08\ m$$

$$I_x = \sum[I + A(\bar{y} - y)^2]$$
$$= \left[\frac{(0.2)(0.04)^3}{12} + (0.2 \times 0.04)(0.08-0.02)^2\right]$$
$$+ \left[\frac{(0.04)(0.2)^3}{12} + (0.04 \times 0.2)(0.08-0.14)^2\right]$$
$$= 8.53 \times 10^{-5}\ m^4$$

$Q_{NA} = (0.04 \times 0.16)(0.08) = 5.12 \times 10^{-4}\ m^3$



$$\tau_{max} = \frac{|V_{max}|Q}{It} = \frac{(80\ kN)(5.12 \times 10^{-4}\ m^3)}{(8.53 \times 10^{-5}\ m^4)(0.04\ m)}$$

$= 12\ 000\ kN/m^2 = 12.0\ MPa$ ◄

$$\sigma_A = \frac{M^{(-)}_{max} c_t}{I} = \frac{(40\ kN \cdot m)(0.16\ m)}{8.53 \times 10^{-5}\ m^4}$$

$= 75\ 000\ kN/m^2 = 75.0\ MPa$ (T)

$$\sigma_B = \frac{M^{(-)}_{max} c_b}{I} = \frac{(40\ kN \cdot m)(0.08\ m)}{8.53 \times 10^{-5}\ m^4}$$

$= 37\ 500\ kN/m^2 = 37.5\ MPa$ (C)

$$\sigma_C = \frac{M^{(+)}_{max} c_t}{I} = \frac{(80\ kN \cdot m)(0.16\ m)}{8.53 \times 10^{-5}\ m^4}$$

$= 150\ 000\ kN/m^2 = 150\ MPa$ (C)

$$\sigma_D = \frac{M^{(+)}_{max} c_b}{I} = \frac{(80\ kN \cdot m)(0.08\ m)}{8.53 \times 10^{-5}\ m^4}$$

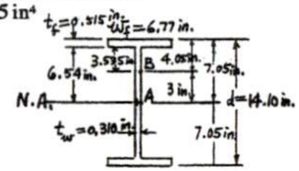
$= 75\ 000\ kN/m^2 = 75.0\ MPa$ (T)

$\sigma^{(T)}_{max} = \sigma_A = \sigma_D = 75.0\ MPa$ ◄

$\sigma^{(C)}_{max} = \sigma_C = 150\ MPa$ ◄

14-43

For W 14 x 35 section:
$I = 385\ in^4$



The reaction at A is:

$$R_A = \frac{wL}{2} = \frac{(3\ kips/ft)(16\ ft)}{2} = 24\ kips$$

At section 1-1:
$V = +24 - 3 \times 6 = +6$ kips
$M = 24(6) - (3 \times 6)(3) = +90$ kip·ft

For the element at A:

$\sigma_A = 0$ (at the N.A.) ◄

$$Q_{NA} = (6.77 \times 0.515)\left(7.05 - \frac{0.515}{2}\right)$$
$$+ (0.310 \times 6.54)\left(\frac{6.54}{2}\right) = 30.3\ in.^3$$

$$\tau_A = \frac{VQ}{It} = \frac{(6\ kips)(30.3\ in.^3)}{(385\ in.^4)(0.310\ in.)}$$

$= 1.52$ ksi ◄

$\sigma_A = 0$
$\tau_A = 1.52$ ksi

(Cont'd)

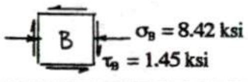
14-43 (Cont)

For the element at B:

$$\sigma_B = \frac{My}{I} = \frac{(90 \times 12 \text{ kip}\cdot\text{in.})(3 \text{ in.})}{385 \text{ in.}^4}$$
$$= 8.42 \text{ ksi (C)} \blacktriangleleft$$

$$Q = (6.77 \times 0.515)\left(7.05 - \frac{0.515}{2}\right) + (0.310 \times 3.535)\left(3 + \frac{3.535}{2}\right)$$
$$= 28.9 \text{ in.}^3$$

$$\tau_B = \frac{VQ}{It} = \frac{(6 \text{ kips})(28.9 \text{ in.}^3)}{(385 \text{ in.}^4)(0.310 \text{ in.})}$$
$$= 1.45 \text{ ksi} \blacktriangleleft$$



14-44

From Table 13-1 Case 4,

$$V_{max} = \frac{wL}{2}$$
$$M_{max} = \frac{wL^2}{8}$$

From Appendix Table A-1(a), for W 18 x 50:
$d = 17.99$ in., $t_w = 0.355$ in., $S_x = 88.9 \text{ in.}^3$

$$\sigma_{max} = \frac{M_{max}}{S_x} = \frac{wL^2}{8S_x} = \sigma_{allow}$$

$$w = \frac{8S_x\sigma_{allow}}{L^2} = \frac{8(88.9 \text{ in.}^3)(24\,000 \text{ lb/in.}^2)}{(8 \times 12 \text{ in.})^2}$$
$$= 1850 \text{ lb/in.} = 22\,200 \text{ lb/ft}$$

$$\tau_{avg} = \frac{V_{max}}{dt_w} = \frac{wL}{2dt_w} = \frac{(22\,000 \text{ lb/ft})(8 \text{ ft})}{2(17.94 \text{ in.})(0.355 \text{ in.})}$$
$$= 13\,900 \text{ lb/in.}^2 = 13.9 \text{ ksi}$$
$$< \tau_{allow} = 14.5 \text{ ksi} \quad \text{(O.K.)}$$

$$w_{allow} = 22\,200 - 50 = 22\,150 \text{ lb/ft} \blacktriangleleft$$

14-45

From Table 13-1 Case 4,

$$V_{max} = \frac{wL}{2}$$
$$M_{max} = \frac{wL^2}{8}$$

From Appendix Table A-1(b), for W 410 x 0.83:
$d = 0.417$ m, $t_w = 0.0109$ m, $S_x = 1.51 \times 10^{-3} \text{ m}^3$

$$\sigma_{max} = \frac{M_{max}}{S_x} = \frac{wL^2}{8S_x} = \sigma_{allow}$$

$$w = \frac{8S_x\sigma_{allow}}{L^2} = \frac{8(1.51 \times 10^{-3} \text{ m}^3)(165 \times 10^3 \text{ kN/m}^2)}{(4 \text{ m})^2}$$
$$= 124.6 \text{ kN/m}$$

$$\tau_{avg} = \frac{V_{max}}{dt_w} = \frac{wL}{2dt_w} = \frac{(124.6 \text{ kN/m})(4 \text{ m})}{2(0.417 \text{ m})(0.0109 \text{ m})}$$
$$= 54.8 \times 10^3 \text{ kN/m}^2 = 54.8 \text{ MPa}$$
$$< \tau_{allow} = 100 \text{ MPa} \quad \text{(O.K.)}$$

$$w_{allow} = 124.6 - 0.83 = 124 \text{ kN/m} \blacktriangleleft$$

14-46

From Appendix Table A-6(b), for a 100 mm x 250 mm section,
$A = 20.9 \times 10^3 \text{ m}^2$
$S_x = 0.818 \times 10^{-3} \text{ m}^3$
$w = 0.130$ kN/m

$$M_{allow} = S_x\,\sigma_{allow} = (0.818 \times 10^{-3} \text{ m}^3)(10\,000 \text{ kN/m}^2)$$
$$= 8.18 \text{ kN}\cdot\text{m}$$

$$V_{allow} = \frac{A\,\tau_{allow}}{1.5} = \frac{(20.9 \times 10^{-3} \text{ m}^2)(660 \text{ kN/m}^2)}{1.5}$$
$$= 9.20 \text{ kN}$$

From Table 13-1, Cases 3 and 4,

$$M_{max} = Pa + \frac{wL^2}{8}$$

$$P = \frac{M_{allow} - \frac{wL^2}{8}}{a}$$

$$= \frac{8.18 \text{ kN}\cdot\text{m}^2 - \frac{(0.130 \text{ kN/m})(4\text{m})^2}{8}}{1 \text{ m}} = 7.92 \text{ kN}$$

(Cont'd)

14-46 (Cont)

$$V_{max} = P + \frac{wL}{2}$$
$$P = V_{allow} - \frac{wL}{2}$$
$$= 9.20 \text{ kN} - \frac{(0.130 \text{ kN/m})(4 \text{ m})}{2}$$
$$= 8.94 \text{ kN}$$

$$P_{allow} = 7.92 \text{ kN} \blacktriangleleft$$

14-47

From Appendix Table A-6(a), for a 3 x 10 joist,
$A = 23.1 \text{ in.}^2$
$S = 35.7 \text{ in.}^3$
wt = 6.42 lb/ft

(a)

$$M_{allow} = S\,\sigma_{allow} = (35.7 \text{ in.}^3)(1450 \text{ lb/in.}^2)$$
$$= 51\,770 \text{ lb}\cdot\text{in.} = 4314 \text{ lb}\cdot\text{ft}$$
$$= \frac{wL^2}{8}$$
$$w = \frac{8(4314 \text{ lb}\cdot\text{ft})}{(15 \text{ ft})^2} = 153 \text{ lb/ft}$$

$$V_{allow} = \frac{A\,\tau_{allow}}{1.5} = \frac{(23.1 \text{ in.}^2)(95 \text{ lb/in.}^2)}{1.5}$$
$$= 1463 \text{ lb} = \frac{wL}{2}$$
$$w = \frac{2(1463 \text{ lb})}{15 \text{ ft}} = 195 \text{ lb/ft}$$

$$w_{allow} = 153 - 6.42 = 147 \text{ lb/ft} \blacktriangleleft$$

(b)

$$\text{Allowable load per unit area} = \frac{w_{allow}}{S} = \frac{147 \text{ lb/ft}}{2 \text{ ft}}$$
$$= 73.5 \text{ lb/ft}^2$$

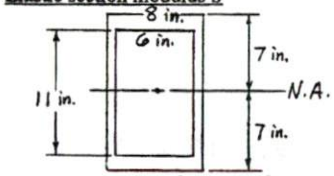
$$\text{Dead wt.} = (45 \text{ lb/ft}^3)\left(\frac{2}{12}\text{ft}\right) = 7.5 \text{ lb/ft}^2$$

Allowable Live Load

$$= 73.5 - 7.5 = 66 \text{ lb/ft}^2 \blacktriangleleft$$

14-48

(a) Elastic section modulus S



$$I = S_1 - S_2 = \frac{8(14)^3}{12} - \frac{6(11)^3}{12} = 1664 \text{ in.}^4$$

$$S = \frac{I}{c} = \frac{1664 \text{ in.}^4}{7 \text{ in.}} = 166 \text{ in}^3 \blacktriangleleft$$

(b) Plastic section modulus Z

$$Z = Z_1 - Z_2 = \frac{8(14)^2}{4} - \frac{6(11)^2}{4}$$
$$= 211 \text{ in.}^3 \blacktriangleleft$$

$$k = \frac{Z}{S} = \frac{211 \text{ in.}^3}{166 \text{ in.}^3} = 1.27 \blacktriangleleft$$

14-49

(a)

$$M_y = S\sigma_y = (166 \text{ in.}^3)(36 \text{ kip/in.}^2)$$
$$= 5980 \text{ kip}\cdot\text{in.} = 498 \text{ kip}\cdot\text{ft} \blacktriangleleft$$

(b)

$$M_p = Z\sigma_y = (211 \text{ in.}^3)(36 \text{ kip/in.}^2)$$
$$= 7600 \text{ kip}\cdot\text{in.} = 633 \text{ kip}\cdot\text{ft} \blacktriangleleft$$

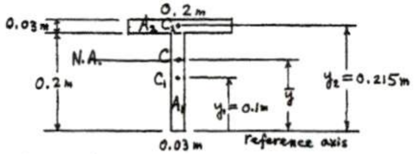
(c)

$$M_y = \frac{w_yL^2}{8}$$
$$w_y = \frac{8M_y}{L^2} = \frac{8(498 \text{ kip}\cdot\text{ft})}{(30 \text{ ft})^2}$$
$$= 4.43 \text{ kip/ft} \blacktriangleleft$$

$$M_p = \frac{w_pL^2}{8}$$
$$w_p = \frac{8M_p}{L^2} = \frac{8(633 \text{ kip}\cdot\text{ft})}{(30 \text{ ft})^2}$$
$$= 5.63 \text{ kip/ft} \blacktriangleleft$$

14-50

Elastic section modulus S:



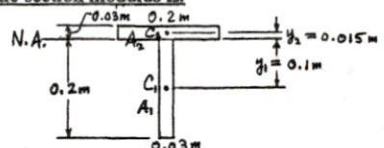
$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2}$$
$$= \frac{(0.3 \times 0.2)(0.1) + (0.2 \times 0.03)(0.215)}{(0.03)(0.2) + (0.2)(0.03)}$$
$$= 0.1575 \text{ m}$$

$$I_x = \sum\left[I + A(\bar{y} - y)^2\right]$$
$$= \left[\frac{0.03(0.2)^3}{12} + (0.03 \times 0.2)(0.1575 - 0.1)^2\right]$$
$$+ \left[\frac{0.2(0.03)^3}{12} + (0.2 \times 0.03)(0.1575 - 0.215)^2\right]$$
$$= 6.013 \times 10^{-5} \text{ m}^4$$

$c = \bar{y} = 0.1575$ m

$$S = \frac{I_x}{c} = \frac{6.013 \times 10^{-5} \text{ m}^4}{0.1575 \text{ m}}$$
$$= 3.82 \times 10^{-4} \text{ m}^3 \quad ◀$$

Plastic section modulus Z:



Since $A_1 = A_2$, the N.A. is located at the junction of A_1 and A_2, as shown.

$$Z = A_1y_1 + A_2y_2$$
$$= (0.03 \times 0.2)(0.1) + (0.2 \times 0.03)(0.015)$$
$$= 6.9 \times 10^{-4} \text{ m}^3 \quad ◀$$

Shape factor k:

$$k = \frac{Z}{S} = \frac{6.9 \times 10^{-4} \text{ m}^3}{3.82 \times 10^{-4} \text{ m}^3} = 1.81 \quad ◀$$

14-51

(a)

$$M_y = S\sigma_y$$
$$= (3.82 \times 10^{-4} \text{ m}^3)(250 \times 10^3 \text{ kN/m}^2)$$
$$= 95.5 \text{ kN} \cdot \text{m} \quad ◀$$

(b)

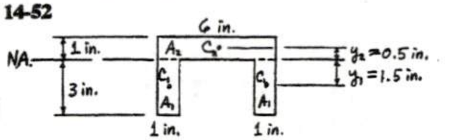
$$M_p = Z\sigma_y$$
$$= (6.9 \times 10^{-4} \text{ m}^3)(250 \times 10^3 \text{ kN/m}^2)$$
$$= 172.5 \text{ kN} \cdot \text{m} \quad ◀$$

(c)

$$M_y = \frac{w_y L^2}{8}$$
$$w_y = \frac{8 M_y}{L^2} = \frac{8(95.5 \text{ kN} \cdot \text{m})}{(12 \text{ m})^2}$$
$$= 5.31 \text{ kN/m} \quad ◀$$

$$M_p = \frac{w_p L^2}{8}$$
$$w_p = \frac{8 M_p}{L^2} = \frac{8(172.5 \text{ kN} \cdot \text{m})}{(12 \text{ m})^2}$$
$$= 9.58 \text{ kN/m} \quad ◀$$

14-52



Since $2A_1 = A_2$, the N.A. is located is located at the junction of A_1 and A_2, as shown.

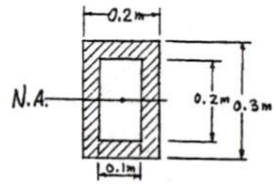
$$Z = 2A_1y_1 + A_2y_2$$
$$= 2(1 \times 3)(1.5) + (6 \times 1)(0.5)$$
$$= 12 \text{ in.}^3$$

(Cont'd)

14-52 (Cont)

$$M_p = Z\sigma_y$$
$$= (12 \text{ in.}^3)(36 \text{ kip/in.}^2)$$
$$= 432 \text{ kip} \cdot \text{in.} = 36 \text{ kip} \cdot \text{ft} = \frac{w L^2}{8}$$
$$w = \frac{8 M_p}{L^2} = \frac{8(36 \text{ kip} \cdot \text{ft})}{(12 \text{ ft})^2}$$
$$= 2 \text{ kip/ft} \quad ◀$$

14-53



$$Z = Z_1 - Z_2 = \frac{b_1h_1^2}{4} - \frac{b_2h_2^2}{4}$$
$$= \frac{(0.2 \text{ m})(0.3 \text{ m})^2}{4} - \frac{(0.1 \text{ m})(0.2 \text{ m})^2}{4}$$
$$= 0.0035 \text{ m}^3 \quad ◀$$

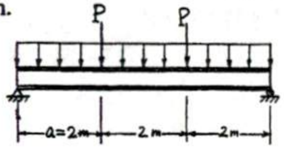
$$M_p = Z\sigma_y$$
$$= (0.0035 \text{ m}^3)(250 \times 10^3 \text{ kN/m}^2)$$
$$= 875 \text{ kN} \cdot \text{m} = Pa$$

$$P = \frac{875 \text{ kN} \cdot \text{m}}{6 \text{ m}} = 149 \text{ kN} \quad ◀$$

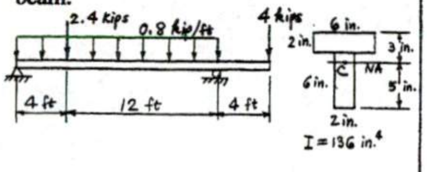
Test Problems for Chapter 14

The following problems may either be given to students for them to practice their problem solving skills or be used as test problems. The answers are provided on the next page.

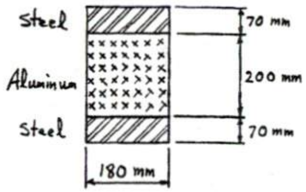
(1) The structural steel beam of W360 x 1.08 section is subjected to the loads shown. The weight of the beam is included in the uniform load. If the allowable flexural stress is 165 MPa, determine the allowable cencentrated loads P in kN that can be applied to the beam.



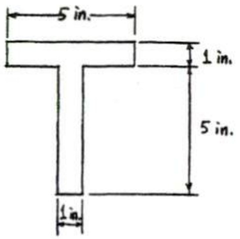
(2) The overhanging beam having a T-section is subjected to the loads shown. Determine (a) the maximum tensile and compressive stresses in the beam, and (b) the maximum shear stress in the beam.



(3) A composite beam consists of steel and aluminum has the cross section shown. The beam has a simple span of 8 m and is subjected to a uniform load of 60 kN/m. Determine the maximum normal stress in each material. The moduli of elasticity are E_{st} = 210 GPa, E_{al} = 70 GPa.



(4) The simple beam of 20-ft. span has a cross-section shown. Determine the maximum uniform load that the beam can carry (a) at yield, and (b) at full plastification. Assume that the beam is made of elastoplastic material for which σ_y = 36 ksi.



Solutions to Test Problems for Chapter 14

(1)

From Appendix Table A-1(b), for W360 x 1.08:

$S_x = 1.84 \times 10^{-3}\ m^3$

$M_{allow} = S\sigma_{allow} = (1.84 \times 10^{-3}\ m^3)(165 \times 10^3\ kN/m^3)$
$= 303.6\ kN \cdot m$

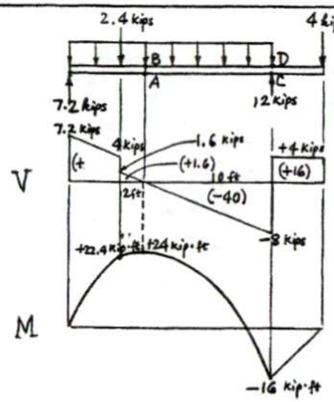
$$M_w = \frac{wL^2}{8} = \frac{(3\ kN/m)(6\ m)^2}{8} = 13.5\ kN \cdot m$$

$M_P = 303.6 - 13.5 = 290.1\ kN \cdot m$

Equating M_P to Pa, we get

$$P = \frac{M_P}{a} = \frac{290.1\ kN \cdot m}{2\ m} = 145\ kN$$ ◀

(2)



$$\sigma_A = \frac{M^{(+)}_{max} c_b}{I} = \frac{(24 \times 12\ lb \cdot in.)(5\ in.)}{136\ in.^4}$$
$= 10.6$ ksi (T)

$$\sigma_B = \frac{M^{(+)}_{max} c_t}{I} = \frac{(24 \times 12\ lb \cdot in.)(3\ in.)}{136\ in.^4}$$
$= 6.35$ ksi(C)

$$\sigma_C = \frac{M^{(-)}_{max} c_b}{I} = \frac{(16 \times 12\ lb \cdot in.)(5\ in.)}{136\ in.^4}$$
$= 7.06$ ksi (C)

$$\sigma_D = \frac{M^{(-)}_{max} c_t}{I} = \frac{(16 \times 12\ lb \cdot in.)(3\ in.)}{136\ in.^4}$$
$= 4.23$ ksi(T)

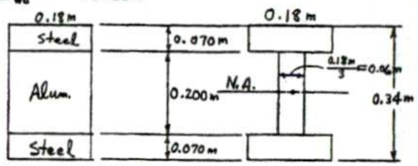
$\sigma^{(T)}_{max} = \sigma_A = 10.6$ ksi ◀

$\sigma^{(C)}_{max} = \sigma_C = 7.06$ ksi ◀

$$\tau_{max} = \tau_{NA} = \frac{VQ}{It} = \frac{(8\ kip)(2 \times 5 \times 2.5\ in.^3)}{(136\ in.^4)(2\ in.)}$$
$= 0.735$ ksi $= 735$ psi ◀

(3)

$$n = \frac{E_{st}}{E_{wd}} = \frac{210\ GPa}{70\ GPa} = 3$$



Section — Transformed section (in steel)

$$I_{st} = \frac{(0.180\ m)(0.34\ m)^3}{12} - \frac{(0.120\ m)(0.200\ m)^3}{12}$$
$= 5.10 \times 10^{-4}\ m^4$

$$M_{max} = \frac{wL^2}{8} = \frac{(60\ kN/m)(8\ m)^2}{8} = 480\ kN \cdot m$$

$$(\sigma_{st})_{max} = \frac{M_{max} c}{I_{st}} = \frac{(480\ kN \cdot m)(0.170\ m)}{5.10 \times 10^{-4}\ m^4}$$

$= 160 \times 10^3\ kN/m^2 = 160$ MPa ◀

(Cont'd)

Solutions to Test Problems for Chapter 14 (Cont'd)

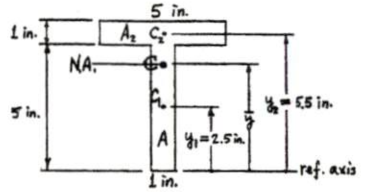
(3) (Cont)

$$(\sigma_{al})_{max} = \frac{1}{n}\frac{M_{max}y}{I_{st}} = \frac{1}{3}\left[\frac{(480\ \text{kN}\cdot\text{m})(0.100\ \text{m})}{5.10 \times 10^{-4}\ \text{m}^4}\right]$$

$$= 31.4 \times 10^3\ \text{kN/m}^2 = 31.4\ \text{MPa}$$ ◄

(4)

(a) Elastic analysis:



$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{5(2.5) + 5(5.5)}{5 + 5} = 4\ \text{in.}$$

$$I_x = \sum\left[I + A(\bar{y} - y)^2\right]$$

$$= \left[\frac{1(5)^3}{12} + 5(4-2.5)^2\right] + \left[\frac{5(1)^3}{12} + 5(4-5.5)^2\right]$$

$$= 33.3\ \text{in.}^4$$

$$S = \frac{I_x}{c} = \frac{33.3\ \text{in.}^4}{4\ \text{in.}} = 8.33\ \text{in.}^3$$

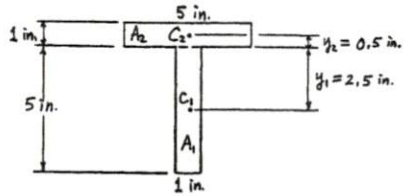
$$M_y = S\sigma_y = (8.33\ \text{in.}^3)(136\ \text{kip/in.}^2)$$

$$= 300\ \text{kip}\cdot\text{in.} = 25\ \text{kip}\cdot\text{ft}$$

$$w_y = \frac{8M_y}{L^2} = \frac{8(25\ \text{kip}\cdot\text{ft})}{(20\ \text{ft})^2}$$

$$= 0.5\ \text{kip/ft} = 500\ \text{lb/ft}$$ ◄

(b) Plastic analysis:



Since $A_1 = A_2$, the N.A. is located at the junction of A_1 and A_2, as shown.

$$Z = A_1y_1 + A_2y_2$$

$$= (5)(2.5) + (5)(0.5)$$

$$= 15\ \text{in.}^3$$

$$M_P = S\sigma_y$$

$$= (15\ \text{in.}^3)(36\ \text{kip/in.}^2)$$

$$= 540\ \text{kip}\cdot\text{in.} = 45\ \text{kip}\cdot\text{ft}$$

$$w_p = \frac{8M_p}{L^2} = \frac{8(45\ \text{kip}\cdot\text{ft})}{(20\ \text{ft})^2}$$

$$= 0.9\ \text{kip/ft} = 900\ \text{lb/ft}$$ ◄

15-1

$$V_{max} = \frac{wL}{2} = \frac{(4\ \text{kip/ft})(20\ \text{ft})}{2} = 40\ \text{kips}$$

$$M_{max} = \frac{wL^2}{8} = \frac{(4\ \text{kip/ft})(20\ \text{ft})^2}{8}$$

$$= 200\ \text{kip}\cdot\text{ft} = 2400\ \text{kip}\cdot\text{in.}$$

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{2400\ \text{kip}\cdot\text{in.}}{24\ \text{kip/in.}^2} = 100\ \text{in}^3$$

From Appendix Table A-1(a):

W 18 x 60:	$S = 108\ \text{in.}^3$
W 16 x 89:	$S = 155\ \text{in.}^3$
W 14 x 68:	$S = 103\ \text{in.}^3$

Try W 18 x 60:
$d = 18.24$ in.
$t_w = 0.415$ in.
$S = 108\ \text{in.}^3$

$$\frac{\text{wt of beam}}{\text{Load}} = \frac{60\ \text{lb}}{40\ 000\ \text{lb}} = 0.015$$

$$\frac{\text{Extra S}}{S_{req}} = \frac{108\ \text{in.}^3 - 100\ \text{in.}^3}{100\ \text{in.}^3} = 0.08 > 0.015$$

The section is satisfactory for bending.

$$\tau_{avg} = \frac{V_{max}}{dt_w} = \frac{40\ \text{kips}}{(18.24\ \text{in.})(0.415\ \text{in.})}$$

$$= 5.28\ \text{ksi} < \tau_{allow} = 14.5\ \text{ksi}$$

The section is satisfactory for shear.

Use W 18 x 60 ◄

15-2

$$V_{max} = \frac{wL}{2} = \frac{(60\ \text{kN/m})(6\ \text{m})}{2} = 180\ \text{kN}$$

$$M_{max} = \frac{wL^2}{8} = \frac{(60\ \text{kN/m})(6\ \text{m})^2}{8} = 270\ \text{kN}\cdot\text{m}$$

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{270\ \text{kN}\cdot\text{m}}{165\ 000\ \text{kN/m}^2} = 1.64 \times 10^{-3}\ \text{m}^3$$

From Appendix Table A-1(b):

W 460 x 0.88:	$S = 1.77 \times 10^{-3}\ \text{m}^3$
W 410 x 1.30:	$S = 2.54 \times 10^{-3}\ \text{m}^3$
W 360 x 0.99:	$S = 1.69 \times 10^{-3}\ \text{m}^3$

Try W 460 x 0.88:
$d = 0.463$ m
$t_w = 0.0105$ m
$S = 1.77 \times 10^{-3}\ \text{m}^3$

$$\frac{\text{wt of beam}}{\text{Load}} = \frac{0.88\ \text{kN/m}}{60\ \text{kN/m}} = 0.015$$

$$\frac{\text{Extra S}}{S_{req}} = \frac{1.77 \times 10^{-3}\ \text{m}^3 - 1.64 \times 10^{-3}\ \text{m}^3}{1.64 \times 10^{-3}\ \text{m}^3}$$

$$= 0.08 > 0.015$$

The section is satisfactory for bending.

$$\tau_{avg} = \frac{V_{max}}{dt_w} = \frac{180\ \text{kN}}{(0.463\ \text{m})(0.0105\ \text{m})}$$

$$= 37.0 \times 10^3\ \text{kPa} = 37.0\ \text{MPa} < \tau_{allow} = 100\ \text{MPa}$$

The section is satisfactory for shear.

Use W 460 x 0.88 ◄

15-3

$$V_{max} = \frac{P}{2} = \frac{10\ \text{kips}}{2} = 5\ \text{kips}$$

$$M_{max} = \frac{PL}{4} = \frac{(10\ \text{kips})(15\ \text{ft})}{4}$$

$$= 37.5\ \text{kip}\cdot\text{ft} = 450\ \text{kip}\cdot\text{in.}$$

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{450\ \text{kip}\cdot\text{in.}}{24\ \text{kip/in.}^2} = 18.75\ \text{in.}^3$$

From Appendix Table A-1(a):

W 12 x 22:	$S = 25.4\ \text{in.}^3$
W 10 x 22:	$S = 23.2\ \text{in.}^3$
W 8 x 24:	$S = 20.9\ \text{in.}^3$

Try W 12 x 22:
$d = 12.31$ in.
$t_w = 0.260$ in.
$S = 25.4\ \text{in.}^3$

$$M_w = \frac{wL^2}{8} = \frac{(22\ \text{lb/ft})(15\ \text{ft})^2}{8}$$

$$= 619\ \text{lb}\cdot\text{ft} = 7.43\ \text{kip}\cdot\text{in.}$$

$$\frac{M_w}{M_p} = \frac{7.43\ \text{kip}\cdot\text{in.}}{450\ \text{kip}\cdot\text{in.}} = 0.017$$

$$\frac{\text{Extra S}}{S_{req}} = \frac{25.4\ \text{in.}^3 - 18.75\ \text{in.}^3}{18.75\ \text{in.}^3} = 0.35 > 0.017$$

The section is satisfactory for bending.

(Cont'd)

15-3 (Cont)

$$\tau_{avg} = \frac{V_{max}}{dt_w} = \frac{5 \text{ kips}}{(12.31 \text{ in.})(0.260 \text{ in.})}$$

$= 1.56 \text{ ksi} < \tau_{allow} = 14.5 \text{ ksi}$

The section is satisfactory for shear.

Use W 12 x 22 ◀

15-4

$$V_{max} = \frac{P}{2} = \frac{45 \text{ kN}}{2} = 22.5 \text{ kN}$$

$$M_{max} = \frac{PL}{4} = \frac{(45 \text{ kN})(5 \text{ m})}{4} = 56.25 \text{ kN} \cdot \text{m}$$

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{56.25 \text{ kN} \cdot \text{m}}{165\ 000 \text{ kN/m}^2} = 0.341 \times 10^{-3} \text{ m}^3$$

From Appendix Table A-1(b):

W 300 x 0.32:	$S = 0.416 \times 10^{-3}$ m³
W 250 x 0.32:	$S = 0.380 \times 10^{-3}$ m³
W 200 x 0.35:	$S = 0.343 \times 10^{-3}$ m³

Try W 300 x 0.32:
$d = 0.313$ m
$t_w = 0.0066$ m
$S = 0.416 \times 10^{-3}$ m³

$$M_w = \frac{wL^2}{8} = \frac{(0.32 \text{ kN/m})(5 \text{ m})^2}{8} = 1.00 \text{ kN} \cdot \text{m}$$

$$\frac{M_w}{M_p} = \frac{1.00 \text{ kN} \cdot \text{m}}{56.25 \text{ kN} \cdot \text{m}} = 0.018$$

$$\frac{\text{Extra S}}{S_{req}} = \frac{0.416 \times 10^{-3} \text{ m}^3 - 0.341 \times 10^{-3} \text{ m}^3}{0.341 \times 10^{-3} \text{ m}^3}$$

$= 0.22 > 0.018$

The section is satisfactory for bending.

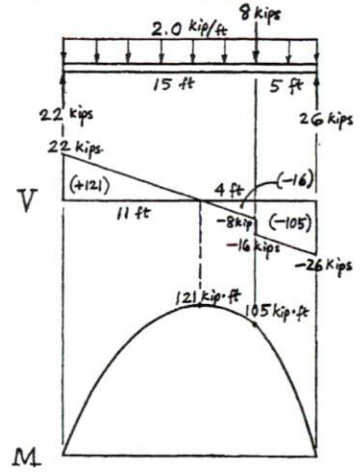
$$\tau_{avg} = \frac{V_{max}}{dt_w} = \frac{22.5 \text{ kN}}{(0.313 \text{ m})(0.0066 \text{ m})}$$

$= 10.9 \times 10^3 \text{ kPa} = 10.9 \text{ MPa} < \tau_{allow} = 100 \text{ MPa}$

The section is satisfactory for shear.

Use W 300 x 0.32 ◀

15-5



$V_{max} = 26$ kips
$M_{max} = 121 \text{ kip} \cdot \text{ft} = 1452 \text{ kip} \cdot \text{in.}$

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{1452 \text{ kip} \cdot \text{in.}}{24 \text{ kip/in.}^2} = 60.5 \text{ in.}^3$$

From Appendix Table A-1(a):

W 18 x 46:	S = 78.8 in.³
W 16 x 50:	S = 81.0 in.³
W 14 x 43:	S = 62.7 in.³

Try W 14 x 43:
d = 13.66 in.
$t_w = 0.305$ in.
S = 62.7 in³

$$M_w = \frac{wL^2}{8} = \frac{(43 \text{ lb/ft})(20 \text{ ft})^2}{8}$$

$= 2150 \text{ lb} \cdot \text{ft} = 25.8 \text{ kip} \cdot \text{in.}$

$$\frac{M_w}{M_p} = \frac{25.8 \text{ kip} \cdot \text{in.}}{1452 \text{ kip} \cdot \text{in.}} = 0.018 = 1.8\%$$

$$\frac{\text{Extra S}}{S_{req}} = \frac{62.7 \text{ in.}^3 - 60.5 \text{ in.}^3}{60.5 \text{ in.}^3}$$

$= 0.036 = 3.6\% > 1.8\%$

The section is satisfactory for bending.

(Cont'd)

15-5 (Cont)

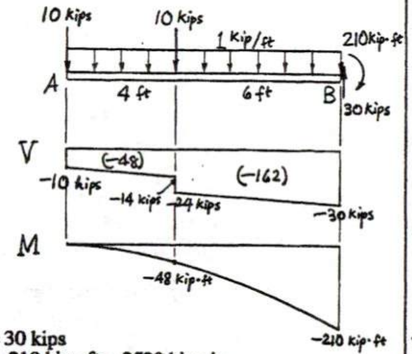
$$\tau_{avg} = \frac{V_{max}}{dt_w} = \frac{26 \text{ kips}}{(13.66 \text{ in.})(0.305 \text{ in.})}$$

$= 6.24 \text{ ksi} < \tau_{allow} = 14.5 \text{ ksi}$

The section is satisfactory for shear.

Use W 14 x 43 ◀

15-6



$V_{max} = 30$ kips
$M_{max} = 210 \text{ kip} \cdot \text{ft} = 2520 \text{ kip} \cdot \text{in.}$

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{2520 \text{ kip} \cdot \text{in.}}{24 \text{ kip/in.}^2} = 105 \text{ in.}^3$$

From Appendix Table A-1(a):

W 21 x 62:	S = 127 in.³
W 18 x 60:	S = 108 in.³
W 16 x 89:	S = 155 in.³
W 14 x 74:	S = 112 in.³

All the above sections have S values greater than S_{req}. They are satisfactory for bending.

Try W 18 x 60:
d = 18.24 in.
$t_w = 0.415$ in.
S = 108 in.³

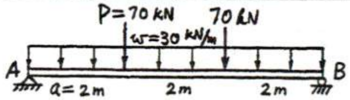
$$\tau_{avg} = \frac{V_{max}}{dt_w} = \frac{30 \text{ kips}}{(18.24 \text{ in.})(0.415 \text{ in.})}$$

$= 3.96 \text{ ksi} < \tau_{allow} = 14.5 \text{ ks}$

The section is satisfactory for Shear.

Use W 18 x 60 ◀

15-7



$$V_{max} = P + \frac{wL}{2} = 70 \text{ kN} + \frac{(30 \text{ kN/m})(6 \text{ m})}{2}$$

$= 160$ kN

$$M_{max} = Pa + \frac{wL^2}{8}$$

$$= (70 \text{ kN})(2 \text{ m}) + \frac{(30 \text{ kN/m})(6 \text{ m})^2}{8}$$

$= 275 \text{ kN} \cdot \text{m}$

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{275 \text{ kN} \cdot \text{m}}{165\ 000 \text{ kN/m}^2} = 1.67 \times 10^{-3} \text{ m}^3$$

From Appendix Table A-1(b):

W 530 x 0.90:	$S = 2.08 \times 10^{-3}$ m³
W 460 x 0.88:	$S = 1.77 \times 10^{-3}$ m³
W 410 x 1.30:	$S = 2.54 \times 10^{-3}$ m³
W 360 x 0.99:	$S = 1.69 \times 10^{-3}$ m³

Try W 460 x 0.88:
d = 0.463 m
$t_w = 0.0105$ m
$S = 1.77 \times 10^{-3}$ m³

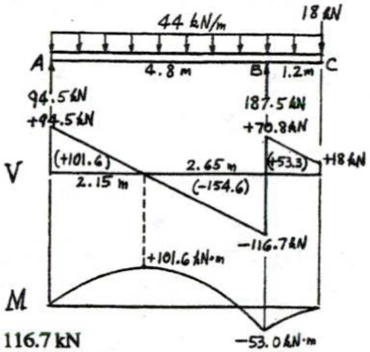
$$\tau_{avg} = \frac{V_{max}}{dt_w} = \frac{160 \text{ kN}}{(0.463 \text{ m})(0.0105 \text{ m})}$$

$= 32.9 \times 10^3 \text{ kPa} = 32.9 \text{ MPa} < \tau_{allow} = 100 \text{ MPa}$

The section is satisfactory for shear.

Use W 460 x 0.88 (W 18 x 60) ◀

15-8



$V_{max} = 116.7$ kN
$M_{max} = 101.6 \text{ kN} \cdot \text{m}$

(Cont'd)

15-8 (Cont)

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{101.6\ \text{kN}\cdot\text{m}}{165\ 000\ \text{kN/m}^2} = 0.616 \times 10^{-3}\ \text{m}^3$$

From Appendix Table A-1(b):

W 410 x 0.38: $S = 0.629 \times 10^{-3}\ \text{m}^3$
W 360 x 0.44: $S = 0.688 \times 10^{-3}\ \text{m}^3$
W 300 x 0.44: $S = 0.633 \times 10^{-3}\ \text{m}^3$

Try W 410 x 0.38:
$d = 0.399$ m
$t_w = 0.00635$ m
$S = 0.629 \times 10^{-3}\ \text{m}^3$

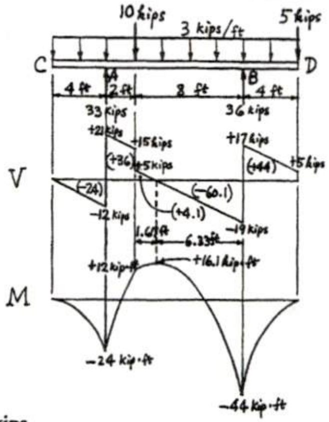
$$\tau_{avg} = \frac{V_{max}}{dt_w} = \frac{116.7\ \text{kN}}{(0.399\ \text{m})(0.00635\ \text{m})}$$

$= 46 \times 10^3$ kPa $= 46$ MPa $< \tau_{allow} = 100$ MPa

The section is satisfactory for shear.

Use W 410 x 0.38 (W 16 x 26) ◀

15-9



$V_{max} = 21$ kips
$M_{max} = 44$ kip·ft

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{44 \times 12\ \text{kip}\cdot\text{in.}}{24\ \text{kip/in.}^2} = 22\ \text{in.}^3$$

From Appendix Table A-1(a):

W 12 x 22: $S = 25.4\ \text{in.}^3$
W 10 x 22: $S = 23.2\ \text{in.}^3$
W 8 x 28: $S = 24.3\ \text{in.}^3$

Try W 12 x 22:
$d = 12.31$ in.
$t_w = 0.260$ in.
$S = 23.2\ \text{in.}^3$

$$\tau_{avg} = \frac{V_{max}}{dt_w} = \frac{21\ \text{kips}}{(12.31\ \text{in.})(0.260\ \text{in.})}$$

$= 6.56$ ksi $< \tau_{allow} = 14.5$ ksi

The section is satisfactory for shear.

Use W 12 x 22 ◀

15-10

(a) Design of Beam

Dead Load $= (150\ \text{lb/ft}^3)\left(\frac{6}{12}\text{ft}\right)(8\ \text{ft}) = 600$ lb/ft

Live Load $= (145\ \text{lb/ft}^2)(8\ \text{ft}) = 1160$ lb/ft

w = 1760 lb/ft
= 1.76 kip/ft

$$V_{max} = \frac{wL}{2} = \frac{(1.76\ \text{kip/ft})(24\ \text{ft})}{2} = 21.1\ \text{kips}$$

$$M_{max} = \frac{wL^2}{8} = \frac{(1.76\ \text{kip/ft})(24\ \text{ft})^2}{8}$$

$= 126.7$ kip·ft $= 1520$ kip·in.

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{1520\ \text{kip}\cdot\text{in.}}{24\ \text{kip/in.}^2} = 63.4\ \text{in.}^3$$

From Appendix Table A-1(a):

W 18 x 46: $S = 78.8\ \text{in.}^3$
W 16 x 50: $S = 81.0\ \text{in.}^3$
W 14 x 53: $S = 77.8\ \text{in.}^3$

Try W 18 x 46:
$d = 18.06$ in.
$t_w = 0.360$ in.
$S = 78.8\ \text{in.}^3$

$$M_{wt} = \frac{(0.046\ \text{kips})(24\ \text{ft})^2}{8} = 3.31\ \text{kip}\cdot\text{ft}$$

$$\frac{M_{wt}}{M_{max}} = \frac{3.31\ \text{kip}\cdot\text{ft}}{126.7\ \text{kip}\cdot\text{ft}} = 0.026$$

$$\frac{\text{Extra S}}{S_{req}} = \frac{78.8\ \text{in.}^3 - 63.4\ \text{in.}^3}{63.4\ \text{in.}^3} = 0.243 > 0.026$$

The beam is satisfactory for bending.

$$\tau_{avg} = \frac{V_{max}}{dt_w} = \frac{21.1\ \text{kips}}{(18.06\ \text{in.})(0.360\ \text{in.})}$$

$= 3.25$ ksi $< \tau_{allow} = 14.5$ ksi

The beam is satisfactory for shear.

Use W 18 x 46 for interior beams ◀

(Cont'd)

15-10 (Cont'd)

(b) Design of Girder

Beam Reaction

$$R = \frac{(1760\ \text{lb/ft} + 46\ \text{lb/ft})(24\ \text{ft})}{2}$$

$= 21\ 700$ lb $= 21.7$ kips

The concentrated force on the girder is

$P = 2R = 43.4$ kips

$$V_{max} = \frac{P}{2} = \frac{43.4\ \text{kips}}{2} = 21.7\ \text{kips}$$

$$M_{max} = \frac{PL}{4} = \frac{(43.4\ \text{kips})(16\ \text{ft})}{4}$$

$= 174$ kip·ft $= 2080$ kip·in.

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{2080\ \text{kip}\cdot\text{in.}}{24\ \text{kip/in.}^2} = 86.7\ \text{in.}^3$$

From Appendix Table A-1(a):

W 21 x 50: $S = 94.5\ \text{in.}^3$
W 18 x 50: $S = 88.9\ \text{in.}^3$
W 16 x 57: $S = 92.2\ \text{in.}^3$

Try W 21 x 50:
$d = 20.83$ in.
$t_w = 0.380$ in.
$S = 94.5\ \text{in.}^3$

$$M_{wt} = \frac{(0.050\ \text{kip/ft})(16\ \text{ft})^2}{8} = 1.6\ \text{kip}\cdot\text{ft}$$

$$\frac{M_{wt}}{M_{max}} = \frac{1.6\ \text{kip}\cdot\text{ft}}{174\ \text{kip}\cdot\text{ft}} = 0.0092$$

$$\frac{\text{Extra S}}{S_{req}} = \frac{94.5\ \text{in}^3 - 86.7\ \text{in}^3}{86.7\ \text{in}^3} = 0.090 > 0.0092$$

The girder is satisfactory for bending.

$$\tau_{avg} = \frac{V_{max}}{dt_w} = \frac{21.7\ \text{kips}}{(20.83\ \text{in.})(0.380\ \text{in.})}$$

$= 2.74$ ksi $< \tau_{allow} = 14.5$ ksi

The girder is satisfactory for shear.

Use W 21 x 50 for the interior Girders ◀

15-11

$$V_{max} = \frac{wL}{2} = \frac{(800\ \text{lb/ft})(16\ \text{ft})}{2} = 6400\ \text{lb}$$

$$M_{max} = \frac{wL^2}{8} = \frac{(800\ \text{lb/ft})(16\ \text{ft})^2}{8}$$

$= 25\ 600$ lb·ft $= 307\ 200$ lb·in.

For an Eastern Hemlock beam:

$\sigma_{allow} = 1350$ psi, $\tau_{allow} = 80$ psi

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{307\ 200\ \text{lb}\cdot\text{in.}}{1350\ \text{lb/in.}^2} = 228\ \text{in.}^3$$

$$A_{req} = \frac{1.5\,V_{max}}{\tau_{allow}} = \frac{1.5(6400\ \text{lb})}{80\ \text{lb/in.}^2} = 120\ \text{in.}^2$$

From Appendix Table A-6(a):

8 x 18: $A = 131\ \text{in.}^2$, $S = 383\ \text{in.}^3$, wt = 36.5 lb/ft
10 x 14: $A = 128\ \text{in.}^2$, $S = 289\ \text{in.}^3$, wt = 35.6 lb/ft

Try the lightest section, 10 x 14:

$$\frac{\text{wt of beam}}{\text{Load}} = \frac{35.6\ \text{lb/ft}}{800\ \text{lb/ft}} = 0.045 = 4.5\%$$

$$\frac{\text{Extra S}}{S_{req}} = \frac{289\ \text{in.}^3 - 228\ \text{in.}^3}{228\ \text{in.}^3}$$

$= 0.268 = 26.8\% > 4.5\%$

The section is satisfactory for bending.

$$\frac{\text{Extra A}}{A_{req}} = \frac{128\ \text{in.}^2 - 120\ \text{in.}^2}{120\ \text{in.}^2}$$

$= 0.063 = 6.3\% > 4.5\%$

The section is satisfactory for shear.

Use 10 x 14 rectangular section ◀

15-12

$$V_{max} = \frac{wL}{2} = \frac{(12\ \text{kN/m})(5\ \text{m})}{2} = 30\ \text{kN}$$

$$M_{max} = \frac{wL^2}{8} = \frac{(12\ \text{kN/m})(5\ \text{m})^2}{8} = 37.5\ \text{kN}\cdot\text{m}$$

For an Eastern Hemlock beam:

$\sigma_{allow} = 9310\ \text{kN/m}^2$, $\tau_{allow} = 550\ \text{kN/m}^2$

(Cont'd)

15-12 (Cont)

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{37.5\ kN\cdot m}{9310\ kN/m^2} = 4.03 \times 10^{-3}\ m^3$$

$$A_{req} = \frac{1.5\,V_{max}}{\tau_{allow}} = \frac{1.5(30\ kN)}{550\ kN/m^2} = 81.8 \times 10^{-3}\ m^2$$

From Appendix Table A-6(b):

200 x 460:
$A = 84.5 \times 10^{-3}\ m^2$
$S = 6.28 \times 10^{-3}\ m^3$
$wt = 0.533$ kN/m

200 x 510:
$A = 94.2 \times 10^{-3}\ m^2$
$S = 7.79 \times 10^{-3}\ m^3$
$wt = 0.592$ kN/m

250 x 360:
$A = 82.6 \times 10^{-3}\ m^2$
$S = 4.74 \times 10^{-3}\ m^3$
$wt = 0.519$ kN/m

250 x 410:
$A = 94.8 \times 10^{-3}\ m^2$
$S = 6.23 \times 10^{-3}\ m^3$
$wt = 0.597$ kN/m

Try 250 x 360:

$$\frac{\text{wt of beam}}{\text{Load}} = \frac{0.519\ kN/m}{12\ kN/m} = 0.043 = 4.3\%$$

$$\frac{\text{Extra S}}{S_{req}} = \frac{4.74 \times 10^{-3}\ m^3 - 4.03 \times 10^{-3}\ m^3}{4.03 \times 10^{-3}\ m^3} = 0.18 = 18\% > 4\%$$

The section is satisfactory for bending.

$$\frac{\text{Extra A}}{A_{req}} = \frac{82.6 \times 10^{-3}\ m^2 - 81.8 \times 10^{-3}\ m^2}{81.8 \times 10^{-3}\ m^2} = 0.01 = 1\% < 4.3\%$$

The section is not satisfactory for shear.

Try 200 x 460:

$$\frac{\text{wt of beam}}{\text{Load}} = \frac{0.533\ kN/m}{12\ kN/m} = 0.044 = 4.4\%$$

$$\frac{\text{Extra S}}{S_{req}} = \frac{6.28 \times 10^{-3}\ m^3 - 4.03 \times 10^{-3}\ m^3}{4.03 \times 10^{-3}\ m^3} = 0.56 = 56\% > 4.4\%$$

The section is satisfactory for bending.

$$\frac{\text{Extra A}}{A_{req}} = \frac{84.5 \times 10^{-3}\ m^2 - 81.8 \times 10^{-3}\ m^2}{81.8 \times 10^{-3}\ m^2} = 0.033 = 3.3\% < 4.4\%$$

The section is not satisfactory for shear.

Try 200 x 510:

$$\frac{\text{wt of beam}}{\text{Load}} = \frac{0.592\ kN/m}{12\ kN/m} = 0.049 = 4.9\%$$

$$\frac{\text{Extra S}}{S_{req}} = \frac{7.79 \times 10^{-3}\ m^3 - 4.03 \times 10^{-3}\ m^3}{4.03 \times 10^{-3}\ m^3} = 0.93 = 93\% > 4.9\%$$

The section is satisfactory for bending.

$$\frac{\text{Extra A}}{A_{req}} = \frac{94.2 \times 10^{-3}\ m^2 - 81.8 \times 10^{-3}\ m^2}{81.8 \times 10^{-3}\ m^2} = 0.15 = 15\% > 4.9\%$$

The section is satisfactory for shear.

Use 200 x 510 rectangular section ◀

15-13

$$V_{max} = \frac{P}{2} = \frac{45\ kN}{2} = 22.5\ kN$$

$$M_{max} = \frac{PL}{4} = \frac{(45\ kN)(4\ m)}{4} = 45\ kN\cdot m$$

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{45\ kN\cdot m}{13\,000\ kN/m^2} = 3.46 \times 10^{-3}\ m^3$$

$$A_{req} = \frac{1.5\,V_{max}}{\tau_{allow}} = \frac{1.5(22.5\ kN)}{1000\ kN/m^2} = 33.8 \times 10^{-3}\ m^2$$

From Appendix Table A-6(b):

150 x 410:
$A = 55.0 \times 10^{-3}\ m^2$
$S = 3.61 \times 10^{-3}\ m^3$
$wt = 0.346$ kN/m

200 x 360:
$A = 65.2 \times 10^{-3}\ m^2$
$S = 3.74 \times 10^{-3}\ m^3$
$wt = 0.410$ kN/m

250 x 360:
$A = 82.6 \times 10^{-3}\ m^2$,
$S = 4.74 \times 10^{-3}\ m^3$,
$wt = 0.519$ kN/m

Try 150 x 410:

$$M_{wt} = \frac{(0.346\ kM/N)(4\ m)^2}{8} = 0.692\ kN\cdot m$$

$$\frac{M_{wt}}{M_{max}} = \frac{0.692\ kN\cdot m}{45\ kN\cdot m} = 0.015 = 1.5\%$$

$$\frac{\text{Extra S}}{S_{req}} = \frac{3.61 \times 10^{-3}\ m^3 - 3.46 \times 10^{-3}\ m^3}{3.46 \times 10^{-3}\ m^3} = 0.043 = 4.3\% > 1.5\%$$

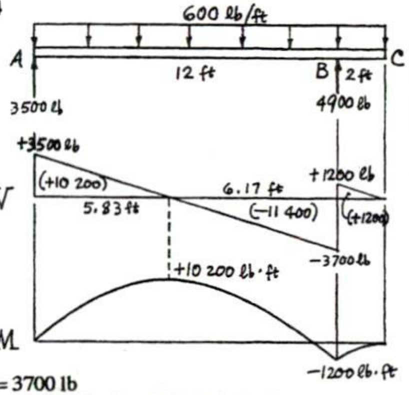
The section is satisfactory for bending.

$$\frac{\text{Extra A}}{A_{req}} = \frac{55.0 \times 10^{-3}\ m^2 - 33.8 \times 10^{-3}\ m^2}{33.8 \times 10^{-3}\ m^2} = 0.63 = 63\% > 1.5\%$$

The section is satisfactory for shear.

Use 150 x 410 rectangular section ◀

15-14



$V_{max} = 3700$ lb
$M_{max} = 10\,200$ lb·ft $= 1.22 \times 10^5$ lb·in.

For a California Redwood beam:

$\sigma_{allow} = 1350$ psi, $\tau_{allow} = 100$ psi

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{1.22 \times 10^5\ lb\cdot in.}{1350\ lb/in.^2} = 90.7\ in.^3$$

$$A_{req} = \frac{1.5\,V_{max}}{\tau_{allow}} = \frac{1.5(3700\ lb)}{100\ lb/in.^2} = 55.5\ in.^2$$

From Appendix Table A-6(a):

6 x 12: $A = 63.3\ in.^2$, $S = 121\ in.^3$, $wt = 17.6$ lb/ft
8 x 10: $A = 71.3\ in.^2$, $S = 113\ in.^3$, $wt = 19.8$ lb/ft
10 x 10: $A = 90.3\ in.^2$, $S = 143\ in.^3$, $wt = 25.1$ lb/ft

Try 6 x 12:

$$\frac{\text{wt of beam}}{\text{Load}} = \frac{17.6\ lb/ft}{600\ lb/ft} = 0.029 = 2.9\%$$

$$\frac{\text{Extra S}}{S_{req}} = \frac{121\ in.^3 - 90.7\ in.^3}{90.7\ in.^3} = 0.334 = 33.4\% > 2.9\%$$

The section is unsatisfactory for bending.

$$\frac{\text{Extra A}}{A_{req}} = \frac{63.3\ in.^2 - 55.5\ in.^2}{55.5\ in.^2} = 0.141 = 14.1\% > 2.9\%$$

The section is unsatisfactory for shear.

Use 6 x 12 rectangular section ◀

15-15

$V_{max} = wL = (300\ lb/ft)(8\ ft) = 2400$ lb

$$M_{max} = \frac{wL^2}{2} = \frac{(300\ lb/ft)(8\ ft)^2}{2} = 9600\ lb\cdot ft = 115\,200\ lb\cdot in.$$

For Douglas Fir:

$\sigma_{allow} = 1450$ psi, $\tau_{allow} = 95$ psi

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{115\,200\ lb\cdot in.}{1450\ lb/in.^2} = 79.4\ in.^3$$

$$A_{req} = \frac{1.5V_{max}}{\tau_{allow}} = \frac{1.5\,(2400\ lb)}{95\ lb/in.^2} = 37.9\ in^2$$

Section	A (in.²)	S (in.³)	w (lb/ft)
6 x 10	52.3	82.7	14.5
6 x 12	63.3	121	17.6
8 x 10	71.3	113	19.8

Try the lightest section 6 x 10:

$$\frac{\text{wt of beam}}{\text{Load}} = \frac{14.5\ lb/ft}{300\ lb/ft} = 0.048 = 4.8\%$$

$$\frac{\text{Extra S}}{S_{req}} = \frac{82.7\ in.^3 - 79.4\ in.^3}{79.4\ in.^3} = 0.042 = 4.2\% < 4.8\%$$

The section is unsatisfactory for bending.
Try the heavier section 6 x 12:

$$\frac{\text{wt of beam}}{\text{Load}} = \frac{17.6\ lb/ft}{300\ lb/ft} = 0.059 = 5.9\%$$

$$\frac{\text{Extra S}}{S_{req}} = \frac{121\ in.^3 - 79.6\ in.^3}{79.6\ in.^3} = 0.52 = 52\% > 4.8\%$$

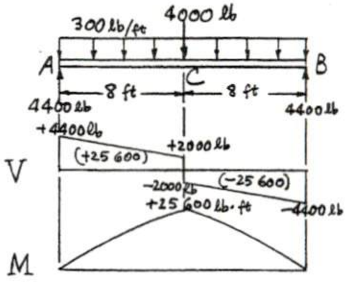
The section is satisfactory for bending.

$$\frac{\text{Extra A}}{A_{req}} = \frac{63.3\ in.^2 - 37.9\ in.^2}{37.9\ in.^2} = 0.67 = 67\% > 4.8\%$$

The section is satisfactory for bending.

Use 6 x 12 rectangular section ◀

15-16



$V_{max} = 4400$ lb
$M_{max} = 25\,600$ lb • ft = 307 200 lb• in.

From Table 15-2, for Southern Pine:
$\sigma_{allow} = 1600$ psi, $\tau_{allow} = 90$ psi

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{307\,200 \text{ lb• in.}}{1600 \text{ lb / in.}^2} = 192 \text{ in.}^3$$

$$A_{req} = \frac{1.5V_{max}}{\tau_{allow}} = \frac{1.5\,(4400 \text{ lb})}{90 \text{ lb / in.}^2} = 73.3 \text{ in.}^2$$

From Appendix Table A-6(a):

Section	A (in.²)	S (in.³)	w (lb/ft)
6 x 16	85.3	220	23.7
6 x 18	96.3	281	26.7
8 x 14	101	228	28.1

Try the lightest section 6 x 16:

$$M_{wt} = \frac{(23.7 \text{ lb / ft})(16 \text{ ft})^2}{8} = 758 \text{ lb • ft}$$

$$\frac{M_{wt}}{M_{max}} = \frac{758 \text{ lb• ft}}{25\,600 \text{ lb• ft}} = 0.03 = 3\%$$

$$\frac{\text{Extra S}}{S_{req}} = \frac{220 \text{ in.}^3 - 192 \text{ in.}^3}{192 \text{ in.}^3}$$
$= 0.146 = 14.6\% > 3\%$

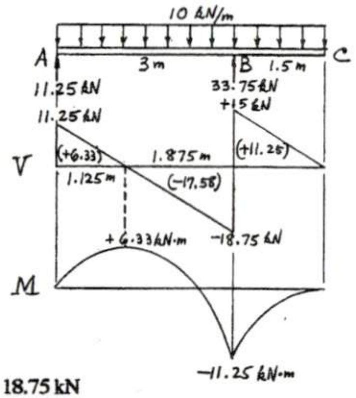
The section is satisfactory for bending.

$$\frac{\text{Extra A}}{A_{req}} = \frac{85.3 \text{ in.}^2 - 73.3 \text{ in.}^2}{73.3 \text{ in.}^2}$$
$= 0.164 = 16.4\%$

The section is satisfactory for shear.

Use 6 x 16 rectangular section ◀

15-17



$V_{max} = 18.75$ kN
$M_{max} = 11.25$ kN • m

From Table 15-2, for California Redwood:
$\sigma_{allow} = 9310$ kPa, $\tau_{allow} = 690$ kPa

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{11.25 \text{ kN• m}}{9310 \text{ kN / m}^2} = 1.21 \times 10^{-3} \text{ m}^3$$

$$A_{req} = \frac{1.5V_{max}}{\tau_{allow}} = \frac{1.5\,(18.75 \text{ kN})}{690 \text{ kN / m}^2} = 40.8 \times 10^{-3} \text{ m}^2$$

Section	A (x 10⁻³ m²)	S (x 10⁻³ m²)	w (kN/m)
150 x 360	47.9	2.74	0.301
200 x 250	46.0	1.85	0.289
250 x 250	58.3	2.34	0.366

Try the lightest section 200 x 250:

$$\frac{\text{wt of beam}}{\text{Load}} = \frac{0.289 \text{ kN / m}}{10 \text{ kN / m}} = 0.029 = 2.9\%$$

$$\frac{\text{Extra S}}{S_{req}} = \frac{1.85 \times 10^{-3} \text{ m}^3 - 1.21 \times 10^{-3} \text{ m}^3}{1.21 \times 10^{-3} \text{ m}^3}$$
$= 0.53 = 53\% > 2.9\%$

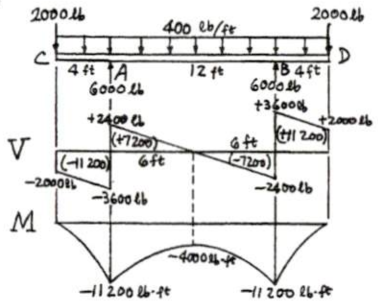
The section is satisfactory for bending.

$$\frac{\text{Extra A}}{A_{req}} = \frac{46.0 \times 10^{-3} \text{ m}^2 - 40.8 \times 10^{-3} \text{ m}^2}{40.8 \times 10^{-3} \text{ m}^3}$$
$= 0.127 = 12.7\% > 2.9\%$

The section is satisfactory for shear.

Use 200 x 250 rectangular section ◀

15-18



$V_{max} = 3600$ lb
$M_{max} = 11\,200$ lb • ft = 134 400 lb• in.

From Table 15-2, for Southern Pine:
$\sigma_{allow} = 1600$ psi, $\tau_{allow} = 90$ psi

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{134\,400 \text{ lb• in.}}{1600 \text{ lb / in.}^2} = 84 \text{ in.}^3$$

$$A_{req} = \frac{1.5V_{max}}{\tau_{allow}} = \frac{1.5\,(3600 \text{ lb})}{90 \text{ lb / in.}^2} = 60 \text{ in.}^2$$

From Appendix Table A-6(a):

Section	A (in.²)	S (in.³)	w (lb/ft)
6 x 12	63.3	121	17.6
8 x 10	71.3	113	19.8
10 x 10	90.3	143	25.1

The weight of the beam is already included in the uniform load. All the above sections would satisfy the requirements on strength. However, the section 6 x 12 is the lightest.

Use 6 x 12 rectangular section ◀

15-19

Dead Load = $(25 \text{ lb/ft}^2)(1.5 \text{ ft})$ = 37.5 lb/ft
Live Load = $(145 \text{ lb/ft}^2)(1.5 \text{ ft})$ = 217.5 lb/ft
Total Design Load = 255 lb/ft

$$V_{max} = \frac{wL}{2} = \frac{(255 \text{ lb / ft})(18 \text{ ft})}{2} = 2295 \text{ lb}$$

$$M_{max} = \frac{wL^2}{8} = \frac{(255 \text{ lb / ft})(18 \text{ ft})^2}{8}$$
= 10 330 lb• ft = 123 900 lb• in.

From Table 15-2, for Southern Pine:
$\sigma_{allow} = 1600$ psi, $\tau_{allow} = 90$ psi

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{123\,900 \text{ lb• in.}}{1600 \text{ lb / in.}^2} = 77.4 \text{ in.}^3$$

$$A_{req} = \frac{1.5V_{max}}{\tau_{allow}} = \frac{1.5\,(2295 \text{ lb})}{90 \text{ lb / in.}^2} = 38.3 \text{ in.}^2$$

From Appendix Table A-6(a):

Section	A (in.²)	S (in.³)	w (lb/ft)
4 x 14	46.4	102	12.9
6 x 10	52.3	82.7	14.5
8 x 10	71.3	113	19.8

Try the lightest section 4 x 14:

$$\frac{\text{wt of beam}}{\text{Load}} = \frac{12.9 \text{ lb / ft}}{255 \text{ lb / ft}} = 0.051 = 5.1\%$$

$$\frac{\text{Extra S}}{S_{req}} = \frac{102 \text{ in.}^3 - 77.4 \text{ in.}^3}{77.4 \text{ in.}^3}$$
$= 0.318 = 31.8\% > 5.1\%$

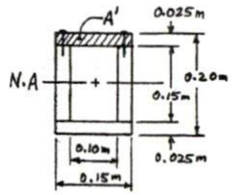
The section is satisfactory for bending.

$$\frac{\text{Extra A}}{A_{req}} = \frac{46.4 \text{ in.}^2 - 38.3 \text{ in.}^2}{38.3 \text{ in.}^2}$$
$= 0.211 = 21.1\% > 5.1\%$

The section is satisfactory for shear.

Use 4 x 14 Southern Pine section ◀

15-20



$$I = \frac{(0.15 \text{ m})(0.2 \text{ m})^3}{12} - \frac{(0.1 \text{ m})(0.15 \text{ m})^3}{12}$$
$= 7.19 \times 10^{-5} \text{ m}^4$

$$Q = A'\,\bar{y}' = (0.15 \text{ m} \times 0.025 \text{ m})\left(0.1 \text{ m} - \frac{0.025 \text{ m}}{2}\right)$$
$= 3.28 \times 10^{-4} \text{ m}^3$

(Cont'd)

15-20 (Cont)

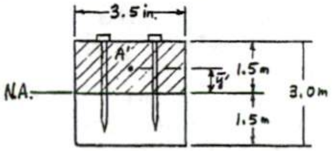
$$q = \frac{VQ}{I} = \frac{(4500\ \text{N})(3.28 \times 10^{-4}\ \text{m}^3)}{7.19 \times 10^{-5}\ \text{m}^3} = 20\ 500\ \text{N/m}$$

$$p_{max} = \frac{2(F_s)_{allow}}{q} = \frac{2(800\ \text{N})}{20\ 500\ \text{N/m}}$$

$$= 0.0779\ \text{m} = 77.9\ \text{mm}$$

Use a 75 mm pitch ◀

15-21



$$I = \frac{bh^3}{12} = \frac{(3.5\ \text{in.})(3.0\ \text{in.})^3}{12} = 7.875\ \text{in.}^4$$

$$S = \frac{I}{c} = \frac{7.875\ \text{in.}^4}{1.5\ \text{in.}} = 5.25\ \text{in.}^3$$

$$Q = A'\bar{y}' = (3.5\ \text{in.} \times 1.5\ \text{in.})(0.75\ \text{in.}) = 3.94\ \text{in.}^3$$

$$(F_s)_{allow} = A\tau_{allow} = \frac{\pi}{4}(0.192\ \text{in.})^2(12\ 000\ \text{lb/in.}^2)$$

$$= 347\ \text{lb/nail}$$

$$q_{allow} = \frac{2(F_s)_{allow}}{p} = \frac{2(347\ \text{lb})}{2.5\ \text{in.}} = 278\ \text{lb/in.}$$

$$V_{allow} = \frac{I\,q_{allow}}{Q} = \frac{(7.875\ \text{in.}^4)(278\ \text{lb/in.})}{3.94\ \text{in.}^3}$$

$$= 556\ \text{lb} = \frac{wL}{2}$$

$$w = \frac{2V_{allow}}{L} = \frac{2(556\ \text{lb})}{4\ \text{ft}} = 278\ \text{lb/ft}$$

For Southern Pine:

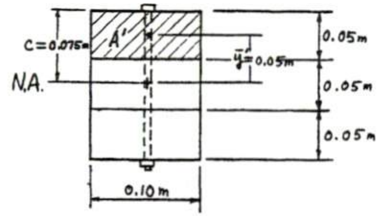
$\sigma_{allow} = 1600$ psi

$$M_{allow} = S\,\sigma_{allow} = (5.25\ \text{in.}^3)(1600\ \text{lb/in.}^2)$$
$$= 8400\ \text{lb}\cdot\text{in.} = 700\ \text{lb}\cdot\text{ft}$$

$$w = \frac{8M_{allow}}{L^2} = \frac{8(700\ \text{lb/ft})}{(4\ \text{ft})^2} = 350\ \text{lb/ft}$$

$w_{allow} = 278$ lb/ft ◀

15-22



$$I = \frac{bh^3}{12} = \frac{(0.10\ \text{m})(0.15\ \text{m})^3}{12} = 2.81 \times 10^{-5}\ \text{m}^4$$

$$S = \frac{I}{c} = \frac{2.81 \times 10^{-5}\ \text{m}^4}{0.075\ \text{m}} = 3.75 \times 10^{-4}\ \text{m}^3$$

$$Q = A'\bar{y}' = (0.05\ \text{m} \times 0.10\ \text{m})(0.05\ \text{m})$$
$$= 2.50 \times 10^{-4}\ \text{m}^3$$

For bolt in single shear:

$$(F_s)_{allow} = A\tau_{allow} = \frac{\pi}{4}(0.005\ \text{m})^2(100\ 000\ \text{kN/m}^2)$$

$$= 1.96\ \text{kN}$$

$$q_{allow} = \frac{(F_s)_{allow}}{p} = \frac{1.96\ \text{kN}}{0.090\ \text{m}} = 21.8\ \text{kN/m}$$

$$V_{allow} = \frac{I\,q_{allow}}{Q} = \frac{(2.81 \times 10^{-5}\ \text{m}^4)(21.8\ \text{kN/m})}{2.50 \times 10^{-4}\ \text{m}^3}$$

$$= 2.45\ \text{kN} = \frac{P}{2}$$

$$P = 2(2.45\ \text{kN}) = 4.9\ \text{kN}$$

For Douglas Fir planks:

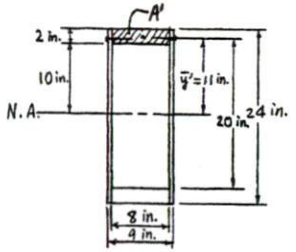
$\sigma_{allow} = 10\ 000$ kPa

$$M_{allow} = S\,\sigma_{allow} = (3.75 \times 10^{-4}\ \text{m}^3)(10\ 000\ \text{kN/m}^2)$$
$$= 3.75\ \text{kN}\cdot\text{m} = \frac{PL}{4}$$

$$P = \frac{4M_{allow}}{L} = \frac{4(3.75\ \text{kN}\cdot\text{m})}{3\ \text{m}} = 5.0\ \text{kN}$$

$P_{allow} = 4.9$ kN ◀

15-23



$$I = \frac{(9\ \text{in.})(24\ \text{in.})^3}{12} - \frac{(8\ \text{in.})(20\ \text{in.})^3}{12}$$

$$= 5035\ \text{in.}^4$$

$$Q = A'\bar{y}' = (2\ \text{in.} \times 8\ \text{in.})(11\ \text{in.})$$

$$= 176\ \text{in.}^3$$

Allowable shear force per nail is

$$(F_s)_{allow} = A\tau_{allow} = \frac{\pi}{4}\left(\frac{1}{8}\ \text{in.}\right)^2(8000\ \text{lb/in.}^2)$$

$$= 98.2\ \text{lb}$$

For the 12-ft simple span with a uniform load w = 1200 lb/ft,

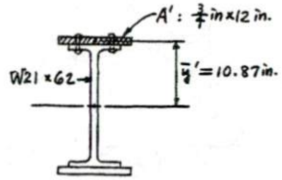
$$V_{max} = \frac{wL}{2} = \frac{(1200\ \text{lb/ft})(12\ \text{ft})}{2} = 7200\ \text{lb}$$

$$q = \frac{VQ}{I} = \frac{(7200\ \text{lb})(176\ \text{in.}^3)}{5035\ \text{in.}^4} = 252\ \text{lb/in.}$$

$$p = \frac{2(F_s)_{allow}}{q} = \frac{2(98.2\ \text{lb})}{252\ \text{lb/in.}} = 0.78\ \text{in.}$$

Use $\frac{3}{4}$ in. pitch ◀

15-24



For W 21 x 62, I = 1330 in.^{4}

$$I = 1330 + 2\left[\frac{(12)(0.75)^3}{12} + (12 \times 0.75)(10.87)^2\right]$$

$$= 3458\ \text{in.}^4$$

$$Q = A'\bar{y}' = (12\ \text{in.} \times 0.75\ \text{in.})(10.87\ \text{in.})$$
$$= 97.8\ \text{in.}^3$$

$$q = \frac{VQ}{I} = \frac{(120\ \text{kips})(97.8\ \text{in.}^3)}{3458\ \text{in.}^4} = 3.40\ \text{kip/in.}$$

The allowable shear force per rivet is

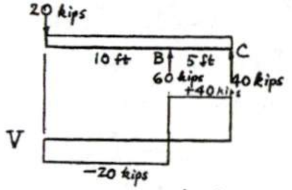
$$(F_s)_{allow} = A\tau_{allow} = \frac{\pi}{4}(0.75\ \text{in.})^2(15\ \text{kip/in.}^2)$$

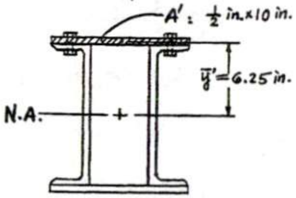
$$= 6.63\ \text{kips}$$

$$p_{max} = \frac{2(F_s)_{allow}}{q} = \frac{2(6.63\ \text{kips})}{3.40\ \text{kip/in.}} = 3.90\ \text{in.}$$

Use $3\frac{1}{2}$ in. pitch ◀

15-25





For C 12 x 30, I = 162 in.^{4}

$$I = 2\left[162 + \frac{(10)(0.5)^3}{12} + (10 \times 0.5)(6.25)^2\right]$$

$$= 715\ \text{in.}^4$$

$$Q = A'\bar{y}' = (10\ \text{in.} \times 0.5\ \text{in.})(6.25\ \text{in.})$$
$$= 31.25\ \text{in.}^3$$

(Cont'd)

15-25 (Cont)

The allowable load for an $\frac{1}{2}$ in. diameter bolt is

$$(F_s)_{allow} = A\tau_{allow} = \frac{\pi}{4}\left(\frac{1}{2}\text{ in.}\right)^2(15\text{ kip/in.}^2) = 2.95\text{ kips}$$

$$q_{AB} = \frac{V_{AB}Q}{I} = \frac{(20\text{ kips})(31.25\text{ in.}^3)}{715\text{ in.}^4} = 0.874\text{ kip/in.}$$

$$(p_{AB})_{req} = \frac{2(F_s)_{allow}}{q_{AB}} = \frac{2(2.95\text{ kips})}{0.875\text{ kip/in.}} = 6.75\text{ in.}$$

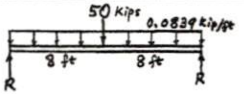
Use 6 in. pitch for AB part ◄

$$q_{BC} = \frac{V_{BC}Q}{I} = \frac{(40\text{ kips})(31.25\text{ in.}^3)}{715\text{ in.}^4} = 1.75\text{ kip/in.}$$

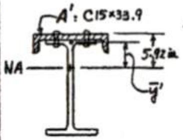
$$(p_{BC})_{req} = \frac{2(F_s)_{allow}}{q_{BC}} = \frac{2(2.95\text{ kips})}{1.75\text{ kip/in.}} = 3.38\text{ in.}$$

Use 3 in. pitch for BC part ◄

15-26



$$V_{max} = R = \frac{1}{2}(50 + 0.0839 \times 16) = 25.7\text{ kips}$$



$A' = 9.96\text{ in.}^2$

$\bar{y}' = 5.92 - 0.787 = 5.13\text{ in.}$

$Q = A'\bar{y}' = (9.96\text{ in.}^2)(5.13\text{ in.}) = 51.1\text{ in.}^3$

$$q = \frac{VQ}{I} = \frac{(25.7\text{ kips})(51.1\text{ in.}^3)}{1250\text{ in.}^4} = 1.05\text{ kip/in.}$$

$$(F_s)_{allow} = 2A\tau_{allow} = 2\left[\frac{\pi(1)^2}{4}\right](15\text{ kip/in.}^2) = 23.6\text{ kips}$$

$$p = \frac{(F_s)_{allow}}{q} = \frac{23.6\text{ kips}}{1.05\text{ kip/in.}} = 22.5\text{ in.}$$

Use 20 in. pitch ◄

15-27

$w_u = 1.2(DL) + 1.6(LL)$
$= 1.2(1\text{ kip/ft}) + 1.6(2\text{ kip/ft})$
$= 4.4\text{ kip/ft}$

$$V_u = \frac{w_u L}{2} = \frac{(4.4\text{ kip/ft})(20\text{ ft})}{2} = 44\text{ kips}$$

$$M_u = \frac{w_u L^2}{8} = \frac{(4.4\text{ kip/ft})(20\text{ ft})^2}{8} = 220\text{ kip}\cdot\text{ft} = 2640\text{ kip}\cdot\text{in.}$$

$$Z_{req} = \frac{M_u}{0.90\,\sigma_y} = \frac{2640\text{ kip}\cdot\text{in.}}{0.90\,(36\text{ kip/in.}^2)} = 81.5\text{ in.}^3$$

From Appendix Table A-1(a):

W 18 x 46:	$Z_x = 90.7\text{ in.}^3$
W 16 x 50:	$Z_x = 92.0\text{ in.}^3$
W 14 x 53:	$Z_x = 87.1\text{ in.}^3$

Try the lightest section W 18 x 46:
$d = 18.06$ in.
$t_w = 0.360$ in.

$V_n = 0.60\,\sigma_y A_w = 0.60(36\text{ ksi})(18.06\text{ in.} \times 0.360\text{ in.}) = 140.4\text{ kips}$

$\phi_v V_n = 0.90(140.4\text{ kips}) = 126.4\text{ kips} > V_u = 44\text{ kips}$

The section is satisfactory for Shear

Use W 18 x 46 ◄

15-28

$w_u = 1.2(DL) + 1.6(LL)$
$= 1.2(20\text{ kN/m}) + 1.6(0)$
$= 24\text{ kN/m}$

$P_u = 1.2(DL) + 1.6(LL)$
$= 1.2(15\text{ kN}) + 1.6(30\text{ kN})$
$= 66\text{ kN}$

$$V_u = \frac{w_u L}{2} + \frac{P_u}{2} = \frac{(24\text{ kN/m})(6\text{ m})}{2} + \frac{66\text{ kN}}{2} = 105\text{ kN}$$

$$M_u = \frac{w_u L^2}{8} + \frac{P_u L}{4} = \frac{(24\text{ kN/m})(6\text{ m})^2}{8} + \frac{(66\text{ kN})(6\text{ m})}{4} = 207\text{ kN}\cdot\text{m}$$

(Cont'd)

15-28 (Cont)

$$Z_{req} = \frac{M_u}{0.90\,\sigma_y} = \frac{207\text{ kN}\cdot\text{m}}{0.90\,(250\,000\text{ kN/m}^2)} = 0.92 \times 10^{-3}\text{ m}^3$$

From Appendix Table A-1(b):

W 460 x 0.51:	$Z_x = 1.09 \times 10^{-3}\text{ m}^3$
W 410 x 0.53:	$Z_x = 1.05 \times 10^{-3}\text{ m}^3$
W 360 x 0.55:	$Z_x = 1.01 \times 10^{-3}\text{ m}^3$

Try the lightest section W 460 x 0.51:
$d = 0.450$ m
$t_w = 0.00792$ m

$V_n = 0.60\,\sigma_y A_w$
$= 0.60(250\,000\text{ kN/m}^2)(0.450\text{ m} \times 0.00792\text{ m})$
$= 534.6\text{ kN}$

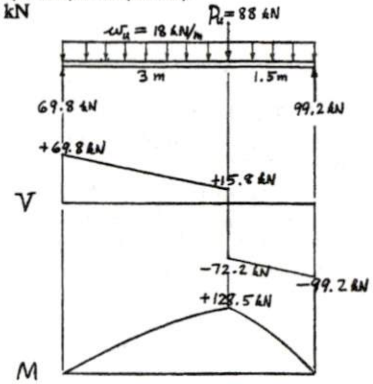
$\phi_v V_n = 0.90(534.6\text{ kN}) = 481\text{ kN} > V_u = 105\text{ kN}$

The section is satisfactory for Shear.

Use W 460 x 0.51 ◄

15-29

$w_u = 1.2(DL) = 1.2(15\text{ kN/m}) = 18\text{ kN/m}$
$P_u = 1.2(DL) + 1.6(LL)$
$= 1.2(20\text{ kN}) + 1.6(40\text{ kN})$
$= 88\text{ kN}$



$V_u = 99.2\text{ kN}$
$M_u = 128.5\text{ kN}\cdot\text{m}$

$$Z_{req} = \frac{M_u}{0.90\,\sigma_y} = \frac{128.5\text{ kN}\cdot\text{m}}{0.90(250\,000\text{ kN/m}^2)} = 0.57 \times 10^{-3}\text{ m}^3$$

From Appendix Table A-1(b):

W 410 x 0.38:	$Z_x = 0.724 \times 10^{-3}\text{ m}^3$
W 360 x 0.44:	$Z_x = 0.775 \times 10^{-3}\text{ m}^3$
W 300 x 0.44:	$Z_x = 0.706 \times 10^{-3}\text{ m}^3$

Try the lightest section W 410 x 0.38:
$d = 0.399$ m
$t_w = 0.00635$ m

$V_n = 0.60\,\sigma_y A_w$
$= 0.60(250\,000\text{ kN/m}^2)(0.399\text{ m} \times 0.00625\text{ m})$
$= 374\text{ kN}$

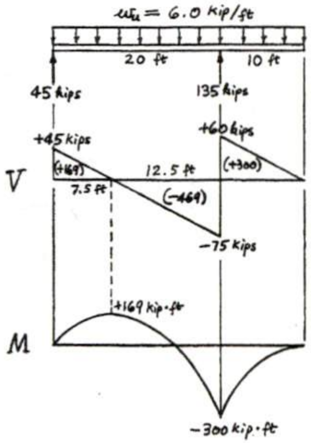
$\phi_v V_n = 0.90(374\text{ kN}) = 337\text{ kN} > V_u = 99.2\text{ kN}$

The section is satisfactory for Shear.

Use W 410 x 0.38 ◄

15-30

$w_u = 1.2(DL) + 1.6(LL)$
$= 1.2(1\text{ kip/ft}) + 1.6(3\text{ kip/ft})$
$= 6.0\text{ kip/ft}$



$V_u = 75\text{ kips}$
$M_u = 300\text{ kip}\cdot\text{ft} = 3600\text{ kip}\cdot\text{in.}$

(Cont'd)

15-30 (Cont)

$$Z_{req} = \frac{M_u}{0.90\ \sigma_y} = \frac{3600\ \text{kip}\cdot\text{in.}}{0.90\ (36\ \text{kip / in.}^2)} = 111\ \text{in.}^3$$

From Appendix Table A-1(a):

W 21 x 62: $Z_x = 144$ in.³
W 18 x 60: $Z_x = 123$ in.³
W 16 x 89: $Z_x = 175$ in.³

Try the lightest section W 18 x 60:

$d = 18.24$ in.
$t_w = 0.415$ in.

$V_n = 0.60\ \sigma_y A_w = 0.60(36\ \text{ksi})(18.24\ \text{in.} \times 0.415\ \text{in.}) = 163.5$ kips

$\phi_v V_n = 0.90(163.5\ \text{kips}) = 147.2\ \text{kips} > V_u = 75$ kips

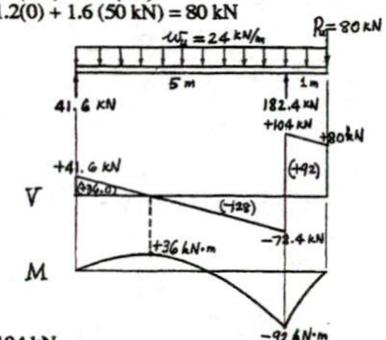
The section is satisfactory for shear.

Use W 18 x 60 ◀

15-31

$w_u = 1.2(DL) + 1.6\ (LL)$
$= 1.2\ (20\ \text{kN/m}) + 1.6(0) = 24$ kN/m

$P_u = 1.2(DL) + 1.6\ (LL)$
$= 1.2(0) + 1.6\ (50\ \text{kN}) = 80$ kN



$V_u = 104$ kN
$M_u = 92$ kN· m

$$Z_{req} = \frac{M_u}{0.90\ \sigma_y} = \frac{92\ \text{kN}\cdot\text{m}}{0.90\ (250\ 000\ \text{kN / m}^2)}$$

$= 0.409 \times 10^{-3}$ m³

From Appendix Table A-1(b):

W 300 x 0.32: $Z_x = 0.480 \times 10^{-3}$ m³
W 250 x 0.32: $Z_x = 0.426 \times 10^{-3}$ m³
W 200 x 0.41: $Z_x = 0.446 \times 10^{-3}$ m³

Try the lightest section W 300 x 0.32:

$d = 0.313$ m
$t_w = 0.0066$ m

$V_n = 0.60\ \sigma_y A_w$
$= 0.60(250\ 000\ \text{kN/m}^2)(0.313\ \text{m} \times 0.0066\ \text{m})$
$= 310$ kN

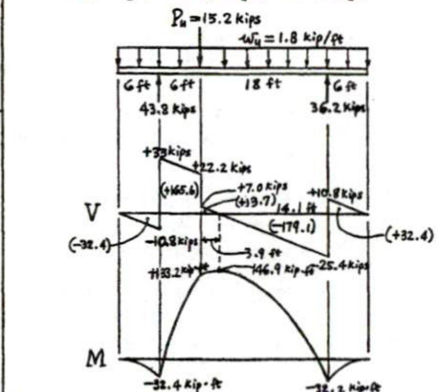
$\phi_v V_n = 0.90(310\ \text{kN}) = 279\ \text{kN} > V_u = 104$ kN

The section is satisfactory for shear.

Use W 300 x 0.32 ◀

15-32

$w_u = 1.2(DL) = 1.2\ (1.5\ \text{kip/ft}) = 1.8$ kip/ft
$P_u = 1.2(DL) + 1.6\ (LL)$
$= 1.2\ (2\ \text{kips}) + 1.6(8\ \text{kips}) = 15.2$ kips



$V_u = 33$ kips
$M_u = 146.9$ kip· ft = 1763 kip· in.

$$Z_{req} = \frac{M_u}{0.90\ \sigma_y} = \frac{1763\ \text{kip}\cdot\text{in.}}{0.90\ (36\ \text{kip / in.}^2)} = 54.4\ \text{in.}^3$$

From Appendix Table A-1(a):

W 18 x 35: $Z_x = 66.5$ in.³
W 16 x 36: $Z_x = 64.0$ in.³
W 14 x 36: $Z_x = 54.6$ in.³

Try the lightest section W 18 x 35

$d = 17.70$ in.
$t_w = 0.300$ in.

$V_n = 0.60\ \sigma_y A_w = 0.60(36\ \text{ksi})(17.70\ \text{in.} \times 0.300\ \text{in.})$
$= 115$ kips

$\phi_v V_n = 0.90(115\ \text{kips}) = 103\ \text{kips} > V_u = 33$ kips

The section is satisfactory for shear.

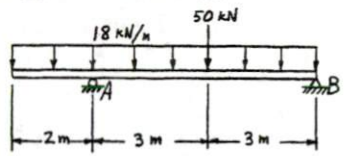
Use W 18 x 35 ◀

Test Problems for Chapter 15

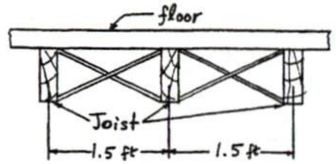
The following problems may either be given to students for them to practice their problem solving skills or be used as test problems. The answers are provided on the next page.

(1) A simply supported girder has a span length of 24 ft. The girder is subjected to two concentrated loads of 5 kips each at the third points along the span, and a uniform load of 1 kip/ft which includes the weight of the girder. The girder is braced laterally throughout its length. Select the lightest W shape to carry the loads for A36 steel which has allowable stresses of σ_{allow} = 24 ksi and τ_{allow} = 14.5 ksi.

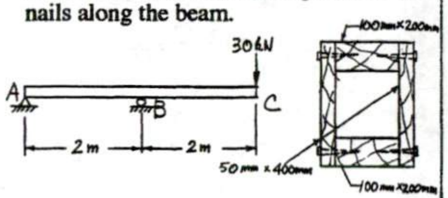
(2) The overhanging beam is subjected to the loads shown. The beam is braced laterally throughout its length. Select the lightest W shape to carry the loads for A36 steel which has allowable stresses of σ_{allow} = 165 MPa, τ_{allow} = 100 MPa.



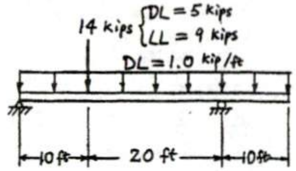
(3) The joist supporting the floor in an office building have a span of 20 ft and a spacing of 1.5 ft on centers, as shown. The floor supports a live load of 50 psf and a dead load of the floor system of 20 psf. Select the lightest section of southern pine, which has allowable stresses of σ_{allow} = 1600 psi and τ_{allow} = 90 psi.



(4) An overhanging beam is subjected to a concentrated load shown. The beam is made up of four full sized boards, two 100 mm x 200 mm and two 50 mm x 400 mm, nail together to form a box section as shown. Each nail is capable of resisting 500 N of shear force. Determine the maximum pitch of the nails along the beam.



(5) The beam subjected to the service loads shown. The weight of the beam is included in the dead load. Assume that the compression flange is fully braces and that localized buckling will not occur. Select the lightest W shape using A36 steel and the LRFD method.



Solutions to Test Problems for Chapter 15

(1)

$$V_{max} = \frac{wL}{2} + P = \frac{(1\text{ kip/ft})(24\text{ ft})}{2} + 5\text{ kip} = 17\text{ kips}$$

$$M_{max} = \frac{wL^2}{8} + \frac{PL}{3}$$

$$= \frac{(1\text{ kip/ft})(24\text{ ft})^2}{8} + \frac{(5\text{ kip})(24\text{ ft})}{3}$$

$$= 112\text{ kip}\cdot\text{ft} = 1344\text{ kip}\cdot\text{in.}$$

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{1344\text{ kip}\cdot\text{in.}}{24\text{ kip/in.}^2} = 56\text{ in.}^3$$

From Appendix Table A-1(a):

W 18 x 35:	$S = 57.6\text{ in.}^3$
W 16 x 36:	$S = 56.5\text{ in.}^3$
W 14 x 43:	$S = 62.7\text{ in.}^3$

Try W 18 x 35:
$d = 17.7$ in.
$t_w = 0.300$ in.
$S = 57.6\text{ in.}^3$

Check the average shear stress:

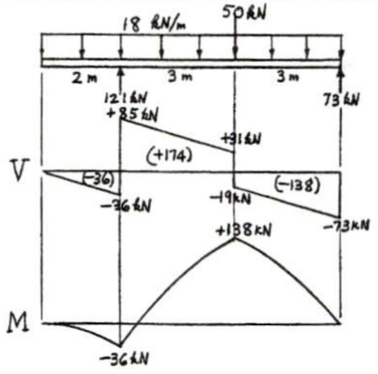
$$\tau_{avg} = \frac{V_{max}}{dt_w} = \frac{17\text{ kips}}{(17.7\text{ in.})(0.300\text{ in.})}$$

$$= 3.20\text{ ksi} < \tau_{allow} = 14.5\text{ ksi}$$

The section is satisfactory for shear.

Use W 18 x 35 ◀

(2)



$V_{max} = 85$ kN
$M_{max} = 138\text{ kN}\cdot\text{m}$

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{138\text{ kN}\cdot\text{m}}{165\,000\text{ kN/m}^2} = 0.836 \times 10^{-3}\text{ m}^3$$

From Appendix Table A-1(b):

W 460 x 0.51:	$S = 0.944 \times 10^{-3}\text{ m}^3$
W 460 x 0.53:	$S = 0.926 \times 10^{-3}\text{ m}^3$
W 360 x 0.55:	$S = 0.895 \times 10^{-3}\text{ m}^3$

Try W 460 x 0.51:
$d = 0.450$ m
$t_w = 0.00762$ m
$S = 0.944 \times 10^{-3}\text{ m}^3$

Check shear stress:

$$\tau_{avg} = \frac{V_{max}}{dt_w} = \frac{85\text{ kN}}{(0.450\text{ m})(0.00762\text{ m})}$$

$$= 24.8 \times 10^3\text{ kPa} = 24.8\text{ MPa} < \tau_{allow} = 100\text{ MPa}$$

The section is satisfactory for shear.

Use W 460 x 0.51 (W 18 x 35) ◀

(3)

Dead Load =	(20 lb/ft²)(1.5 ft)	=	30 lb/ft
Live Load =	(50 lb/ft²)(1.5 ft)	=	75 lb/ft
Total Design Load		w =	105 lb/ft

$$V_{max} = \frac{wL}{2} = \frac{(105\text{ lb/ft})(20\text{ ft})}{2} = 1050\text{ lb}$$

$$M_{max} = \frac{wL^2}{8} = \frac{(105\text{ lb/ft})(20\text{ ft})^2}{8}$$

$$= 5250\text{ lb}\cdot\text{ft} = 63\,000\text{ lb}\cdot\text{in.}$$

The allowable stresses for Southern pine are

$\sigma_{allow} = 1600$ psi, $\tau_{allow} = 90$ psi

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{63\,000\text{ lb}\cdot\text{in.}}{1600\text{ lb/in.}^2} = 39.4\text{ in.}^3$$

$$A_{req} = \frac{1.5V_{max}}{\tau_{allow}} = \frac{1.5\,(1050\text{ lb})}{90\text{ lb/in.}^2} = 17.5\text{ in.}^2$$

From Appendix Table A-6(a):

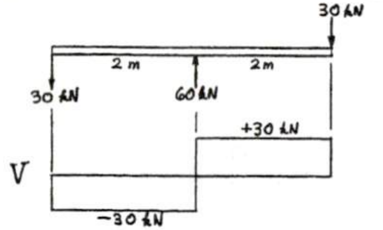
Section	A (in.²)	S (in.³)	w (lb/ft)
4 x 14	46.4	102	12.9
6 x 10	52.3	82.7	14.5
8 x 10	71.3	113	19.8

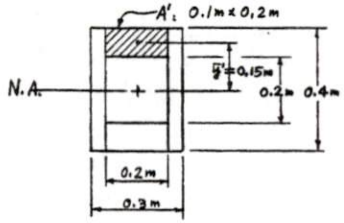
For closely spaced joists, the aspect ratio of 4 is alright. Thus,

Use 3 x 12 Southern pine section ◀

Solutions to Test Problems for Chapter 15 (Cont'd)

(4)





$$I = \frac{(0.3\text{ m})(0.4\text{ m})^3}{12} - \frac{(0.2\text{ m})(0.2\text{ m})^3}{12}$$

$$= 1.47 \times 10^{-3}\text{ m}^4$$

$$Q = A'\bar{y}' = (0.1\text{ m} \times 0.2\text{ m})(0.15\text{ m})$$

$$= 0.003\text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(30\text{ kN})(0.003\text{ m}^3)}{1.47 \times 10^{-3}\text{ m}^4} = 61.2\text{ kN/m}$$

$$p_{allow} = \frac{2(F_s)_{allow}}{q} = \frac{2(0.500\text{ kN})}{61.2\text{ kN/m}}$$

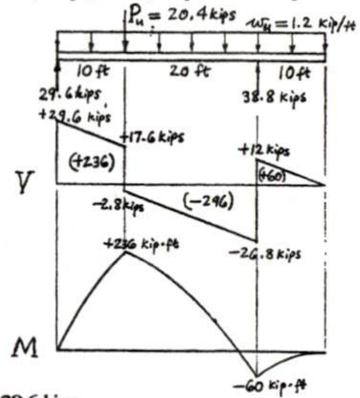
$$= 0.0163\text{ m} = 16.3\text{ mm}$$

Use 16 mm pitch ◀

(5)

$$w_u = 1.2(DL) = 1.2\,(1.0\text{ kip/ft}) = 1.2\text{ kip/ft}$$

$$P_u = 1.2(DL) + 1.6\,(LL)$$
$$= 1.2\,(5\text{ kips}) + 1.6(9\text{ kips}) = 20.4\text{ kips}$$



$V_u = 29.6$ kips
$M_u = 236\text{ kip}\cdot\text{ft} = 2832\text{ kip}\cdot\text{in.}$

$$Z_{req} = \frac{M_u}{0.90\,\sigma_y} = \frac{2832\text{ kip}\cdot\text{in.}}{0.90\,(36\text{ kip/in.}^2)} = 87.4\text{ in.}^3$$

From Appendix Table A-1(a):

W 18 x 46:	$Z_x = 90.7\text{ in.}^3$
W 16 x 50:	$Z_x = 92.0\text{ in.}^3$
W 14 x 61:	$Z_x = 102\text{ in.}^3$

Try the lightest section W 18 x 46
$d = 18.06$ in.
$t_w = 0.36$ in.

$$V_n = 0.60\,\sigma_y A_w = 0.60(36\text{ ksi})(18.06\text{ in.} \times 0.36\text{ in.})$$
$$= 140\text{ kips}$$

$$\phi_v V_n = 0.90(140\text{ kips}) = 126\text{ kips} > V_u = 29.6\text{ kips}$$

The section is satisfactory for shear.

Use W 18 x 46 ◀

16-1

$$\sigma_{max} = \frac{Ec}{\rho} = \frac{(30 \times 10^3 \text{ ksi})\left(\frac{1}{16}\text{ in.}\right)}{5 \times 12 \text{ in.}} = 31.25 \text{ ksi}$$ ◀

16-2

From Appendix Table A-7(a), for copper:
$E = 120 \text{ GPa} = 120 \times 10^3 \text{ MPa}$

$$\sigma_{max} = \frac{Ec}{\rho} = \frac{(120 \times 10^3 \text{ MPa})(0.0015 \text{ m})}{1.5 \text{ m}}$$

$= 120$ MPa ◀

16-3

$$\sigma_{max} = \frac{Ec}{\rho} = \frac{(30\,000 \text{ ksi})\left(\frac{1}{64}\text{ in.}\right)}{1.5 \times 12 \text{ in.}} = 26.0 \text{ ksi}$$ ◀

16-4

From Appendix Table A-7(b), for copper :
$E = 120 \text{ GPa} = 120 \times 10^3 \text{ MPa}$

$$\rho = \frac{Ec}{\sigma_{max}} = \frac{(120 \times 10^3 \text{ MPa})(0.001 \text{ m})}{60 \text{ MPa}} = 2.00 \text{ m}$$ ◀

16-5

$$\rho = \frac{Ec}{\sigma_{max}} = \frac{(30\,000 \text{ ksi})\left(\frac{1}{32}\text{ in.}\right)}{24 \text{ ksi}}$$

$= 39.1 \text{ in.} = 3.26 \text{ ft}$ ◀

16-6

$$\rho = \frac{EI}{M} = \frac{(210 \times 10^9 \text{ N/m}^2)\left[\frac{\pi(0.010 \text{ m})^4}{64}\right]}{20 \text{ N}\cdot\text{m}}$$

$= 5.15$ m ◀

16-7

From Appendix Table A-1(a), for W 16 x 36:
$I = 448 \text{ in.}^4$

$EI = (30 \times 10^3 \text{ kip/in.}^2)(448 \text{ in.}^4)$
$= 1.34 \times 10^7 \text{ kip}\cdot\text{in.}^2 = 9.33 \times 10^4 \text{ kip}\cdot\text{ft}^2$

From Table 16-1 case 5

$$\delta_{max} = \frac{PL^3}{48\,EI} = \frac{(12 \text{ kips})(30 \text{ ft})^3}{48(9.33 \times 10^4 \text{ kip}\cdot\text{ft}^2)}$$

$= 0.0723 \text{ ft} = 0.868 \text{ in.}$ ◀

16-8

From the Solution to Prob 16-7,
$EI = 9.33 \times 10^4 \text{ kip-ft}^2$

From Table 16-1 case 7

$$\delta_{max} = \frac{5wL^4}{384\,EI} = \frac{5(0.4 \text{ kip/ft})(30 \text{ ft})^4}{384(9.33 \times 10^4 \text{ kip}\cdot\text{ft}^2)}$$

$= 0.0452 \text{ ft} = 0.543 \text{ in.}$ ◀

16-9

From Appendix Table A-6(b), for 250 x 360 timber section,
$I = 811 \times 10^{-6} \text{ m}^4$
From Appendix Table A-7(b), for California Redwood,
$E = 9.0$ GPa
$EI = (9.0 \times 10^6 \text{ kN/m}^2)(811 \times 10^6 \text{ m}^4) = 7299 \text{ kN}\cdot\text{m}^2$

From Table 16-1 case 7

$$\delta_{max} = \frac{5wL^4}{384\,EI} = \frac{5(4 \text{ kN/m})(5 \text{ m})^4}{384(7299 \text{ kN}\cdot\text{m}^2)}$$

$= 0.00446 \text{ m} = 4.46 \text{ mm}$ ◀

16-10

From the solution to Prob. 16-9, $EI = 7299 \text{ kN}\cdot\text{m}^2$

From Table 16-1 case 5

$$\delta_{max} = \frac{PL^3}{48\,EI} = \frac{(20 \text{ kN})(5 \text{ m})^3}{48(7299 \text{ kN}\cdot\text{m}^2)}$$

$= 0.00714 \text{ m} = 7.14 \text{ mm}$ ◀

16-11

From Appendix Table A-6(b), for 150 x 360 section,
$I = 469 \times 10^{-6} \text{ m}^4$

From Appendix Table A-7(b), for Southern Pine,
$E = 12$ GPa

$EI = (12 \times 10^6 \text{ kN/m}^2)(469 \times 10^{-6} \text{ m}^4) = 5628 \text{ kN}\cdot\text{m}^2$

From Table 16-1 case 3

$$\delta_{max} = \frac{wL^4}{8\,EI} = \frac{(15 \text{ kN/m})(3 \text{ m})^4}{8(5628 \text{ kN}\cdot\text{m}^2)}$$

$= 0.0270 \text{ m} = 27.0 \text{ mm}$ ◀

$$\theta_{max} = \frac{wL^3}{6\,EI} = \frac{(15 \text{ kN/m})(3 \text{ m})^3}{6(5628 \text{ kN}\cdot\text{m}^2)}$$

$= 0.0120 \text{ rad} = 0.688°$ ◀

16-12

From Appendix Table A-6(a), for 4 x 10 section,
$I = 231 \text{ in.}^4$

From Appendix Table A-7(a), for Southern Pine,
$E = 1.8 \times 10^3$ ksi

$EI = (1.8 \times 10^3 \text{ ksi})(231 \text{ in.}^4) = 4.16 \times 10^5 \text{ kip}\cdot\text{in.}^2$
$= 2890 \text{ kip}\cdot\text{ft}^2$

From Table 16-1 case 3

$$\delta_{max} = \frac{wL^4}{8\,EI} = \frac{(0.3 \text{ kip/ft})(10 \text{ ft})^4}{8(2890 \text{ kip}\cdot\text{ft}^2)}$$

$= 0.1298 \text{ ft} = 1.56 \text{ in.}$ ◀

$$\theta_{max} = \frac{wL^3}{6\,EI} = \frac{(0.3 \text{ kip/ft})(10 \text{ ft})^3}{6(2890 \text{ kip}\cdot\text{ft}^2)}$$

$= 0.0173 \text{ rad} = 0.991°$ ◀

16-13

From Appendix Table A-1(a), for W 18 x 60 section,
$I = 984 \text{ in.}^4$ $S = 108 \text{ in.}^3$
$d = 18.24$ in. $t_w = 0.415$ in.

$EI = (30 \times 10^3 \text{ kip/in.}^2)(984 \text{ in.}^4)$
$= 2.95 \times 10^7 \text{ kip}\cdot\text{in.}^2 = 2.05 \times 10^5 \text{ kip}\cdot\text{ft}^2$
$M_{allow} = S\sigma_{allow} = (108 \text{ in.}^3)(15 \text{ kip/in.}^2)$
$= 1620 \text{ kip}\cdot\text{in.} = 135 \text{ kip}\cdot\text{ft}$

$$M_{max} = \frac{wL^2}{8} = M_{allow}$$

$$w = \frac{8\,M_{allow}}{L^2} = \frac{8\,(135 \text{ kip}\cdot\text{ft})}{(25 \text{ ft})^2} = 1.73 \text{ kip/ft}$$

$$\delta_{allow} = \frac{L}{360} = \frac{25 \text{ ft}}{360} = 0.0694 \text{ ft}$$

$$\delta_{max} = \frac{5wL^4}{384\,EI} = \delta_{allow}$$

$$w = \frac{384\,\delta_{allow}\,EI}{5\,L^4}$$

$$= \frac{384\,(0.0694 \text{ ft})(2.05 \times 10^5 \text{ kip}\cdot\text{ft}^2)}{5(25 \text{ ft})^4}$$

$= 2.80$ kip/ft

Check Shear:

$$V_{max} = \frac{wL}{2} = \frac{(1.73 \text{ kip/ft})(25 \text{ ft})}{2} = 21.6 \text{ kips}$$

$$\tau_{avg} = \frac{V_{max}}{d\,t_w} = \frac{21.6 \text{ kips}}{(18.24 \text{ in.})(0.415 \text{ in.})}$$

$= 2.86 \text{ ksi} < 15 \text{ ksi}$

$w_{allow} = 1.73$ kip/ft ◀

16-14

From Appendix Table A-1(a), for W 410 x 0.83 section,
$I = 315 \times 10^{-6} \text{ m}^4$ $S = 1.51 \times 10^3 \text{ m}^3$
$d = 0.417$ m $t_w = 0.0109$ m

$EI = (210 \times 10^6 \text{ kN/m}^2)(315 \times 10^{-6} \text{ m}^4)$
$= 6.62 \times 10^4 \text{ kN}\cdot\text{m}^2$
$M_{allow} = S\sigma_{allow} = (1.51 \times 10^{-3} \text{ m}^3)(165\,000 \text{ kN/m}^2)$
$= 249 \text{ kN}\cdot\text{m}$

$$M_{max} = \frac{wL^2}{8} = M_{allow}$$

$$w = \frac{8\,M_{allow}}{L^2} = \frac{8\,(249 \text{ kN}\cdot\text{m})}{(10 \text{ m})^2}$$

$= 19.92$ kN/m

(Cont'd)

16-14 (Cont)

$$\delta_{allow} = \frac{L}{360} = \frac{10\text{ m}}{360} = 0.0278\text{ m}$$

$$\delta_{max} = \frac{5wL^4}{384\,EI} = \delta_{allow}$$

$$w = \frac{384\,EI\,\delta_{allow}}{5L^4}$$

$$= \frac{384(6.62 \times 10^4\text{ kN}\cdot\text{m}^2)(0.0278\text{ m})}{5(10\text{ m})^4}$$

$$= 14.13\text{ kN/m}$$

Check Shear:

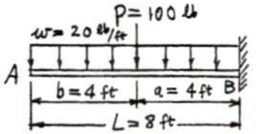
$$V_{max} = \frac{wL}{2} = \frac{(14.13\text{ kN/m})(10\text{ m})}{2} = 70.65\text{ kN}$$

$$\tau_{avg} = \frac{V_{max}}{d\,t_w} = \frac{70.65\text{ kN}}{(0.417\text{ m})(0.0109\text{ m})}$$

$$= 15\,540\text{ kPa} = 15.5\text{ MPa} < \tau_{allow} = 100\text{ MPa}$$

$$w_{allow} = 14.13\text{ kN/m}$$ ◀

16-15



From Appendix Table A-6(a), for 3 x 6 section,
$I = 34.7\text{ in.}^4$

From Appendix Table A-7(a), for Southern Pine
$E = 1.8 \times 10^3$ ksi

$EI = (1.8 \times 10^3\text{ kip/in.}^2)(34.7\text{ in.}^4)$
$= 6.25 \times 10^4\text{ kip}\cdot\text{in.}^2 = 4.34 \times 10^5\text{ lb}\cdot\text{ft}^2$

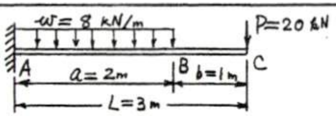
$$\delta_A = \left[\delta_{max}\right]_{case\,3}^{due\ to\ w} + \left[\delta_{max}\right]_{case\,2}^{due\ to\ F}$$

$$= \frac{wL^4}{8EI} + \frac{Pa^2}{6EI}(3L - a)$$

$$= \frac{(20\text{ lb/ft})(8\text{ ft})^4}{8(4.34 \times 10^5\text{ lb}\cdot\text{ft}^2)} + \frac{(100\text{ lb})(4\text{ ft})^2}{6(4.34 \times 10^5\text{ lb}\cdot\text{ft}^2)}(3 \times 8\text{ ft} - 4\text{ ft})$$

$$= 0.0359\text{ ft} = 0.431\text{ in.} \downarrow$$ ◀

16-16



From Appendix Table A-1(a), for W410 x 1.30 section,
$I = 541 \times 10^{-6}\text{ m}^4$
$EI = (210 \times 10^6\text{ kN/m}^2)(541 \times 10^{-6}\text{ m}^4)$
$= 1.14 \times 10^5\text{ kN}\cdot\text{m}^2$

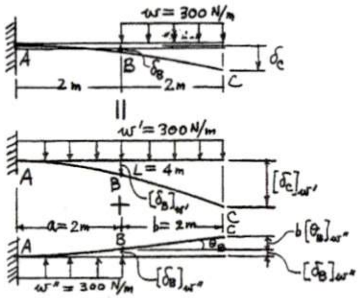
$$\delta_C = \left[\delta_{max}\right]_{case\,1}^{due\ to\ P} + \left[\delta_B\right]_{case\,3}^{due\ to\ w} + b\left[\theta_B\right]_{case\,3}^{due\ to\ w}$$

$$= \frac{PL^3}{3EI} + \frac{wa^4}{8EI} + b\frac{wa^3}{6EI}$$

$$= \frac{(20)(3)^3}{3(1.14\times10^5)} + \frac{(8)(4)^4}{8(1.14\times10^5)} + (1)\frac{(8)(2)^3}{6(1.14\times10^5)}$$

$$= +0.00181\text{ m} = 1.81\text{ mm}$$ ◀

16-17



From Appendix Table A-6(b), for 80 x 250 section,
$I = 68.7 \times 10^{-6}\text{ m}^4$
From Appendix Table A-7(b), for White Oak,
$E = 12$ GPa
$EI = (12 \times 10^9\text{ N/m}^2)(68.7 \times 10^{-6}\text{ m}^4)$
$= 8.24 \times 10^5\text{ N}\cdot\text{m}^2$

$$\delta_B = \left[\delta_B\right]_{case\,3}^{due\ to\ w'} - \left[\delta_B\right]_{case\,3}^{due\ to\ w''}$$

$$= \frac{w'x^2}{24EI}\left[x^2 - 4Lx + 6L^2\right] - \frac{w''a^4}{8EI}$$

$$= \frac{(300)(2)^2}{24(8.24\times10^5)}\left[(2)^2 - 4(4)(2) + 6(4)^2\right] - \frac{(300)(2)^4}{8(8.24 \times 10^5)}$$

$$= +0.00340\text{ m} = 3.40\text{ mm} \downarrow$$ ◀

(Cont'd)

16-17 (Cont)

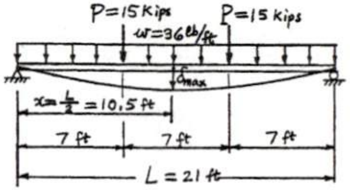
$$\delta_C = \left[\delta_C\right]_{case\,3}^{due\ to\ w'} - \left[\delta_B\right]_{case\,3}^{due\ to\ w''} - b\left[\theta_B\right]_{case\,3}^{due\ to\ w''}$$

$$= \frac{w'L^4}{8EI} - \frac{w''a^4}{8EI} - b\frac{w''a^3}{6EI}$$

$$= \frac{(300\text{ N/m})(4\text{ m})^4}{8(8.24 \times 10^5\text{ N}\cdot\text{m}^2)} - \frac{(300\text{ N/m})(2\text{ m})^4}{8(8.24 \times 10^5\text{ N}\cdot\text{m}^2)} - (2)\frac{(300\text{ N/m})(2\text{ m})^3}{6(8.24 \times 10^5\text{ N}\cdot\text{m}^2)}$$

$$= 0.00995\text{ m} = 9.95\text{ mm} \downarrow$$ ◀

16-18



From Appendix Table A-1(a), for W 16 x 36,
$I = 448\text{ in.}^4$

$EI = (30 \times 10^3\text{ kip/in.}^2)(448\text{ in.}^4)$
$= 1.344 \times 10^7\text{ kip}\cdot\text{in}^2 = 9.33 \times 10^4\text{ kip}\cdot\text{ft}^2$
$w = 36\text{ lb/ft} = 0.036\text{ kip/ft}$

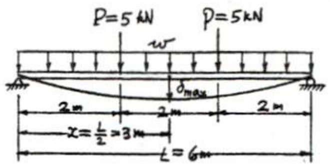
$$\delta_{max} = 2\left[\delta_{L/2}\right]_P + \left[\delta_{max}\right]_w$$

$$= 2\left\{\frac{Pb\,x}{6EIL}\left[L^2 - x^2 - b^2\right]\right\} + \frac{5wL^4}{384EI}$$

$$= 2\left\{\frac{(15)(7)(10.5)}{6(9.33 \times 10^4)(21)}\left[(21)^2 - (10.5)^2 - (7)^2\right]\right\} + \frac{5(0.036)(21)^4}{384(9.33 \times 10^4)}$$

$$= +0.0538\text{ ft} = 0.646\text{ in.} \downarrow$$ ◀

16-19



From Appendix Table A-6(b), for 100 x 360 section,
$I = 282 \times 10^{-6}\text{ m}^4$

From Appendix Table A-7(b), for Douglas Fir,
$E = 13$ GPa

$EI = (13 \times 10^6\text{ kN/m}^2)(282 \times 10^{-6}\text{ m}^4)$
$= 3670\text{ kN}\cdot\text{m}^2$

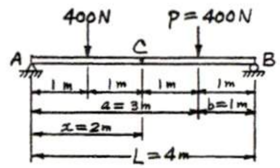
$$\delta_{max} = 2\left[\delta_{L/2}\right]_P + \left[\delta_{max}\right]_w$$

$$= 2\left\{\frac{Pb\,x}{6EIL}\left[L^2 - x^2 - b^2\right]\right\} + \frac{5wL^4}{384EI}$$

$$= 2\left\{\frac{(5)(2)(3)}{6(3670)(6)}\left[(6)^2 - (3)^2 - (2)^2\right]\right\} + \frac{5(0.188)(6)^4}{384(3670)}$$

$$= +0.0113\text{ m} = 11.3\text{ mm} \downarrow$$ ◀

16-20



$EI = 2.1 \times 10^4\text{ N}\cdot\text{m}^2$

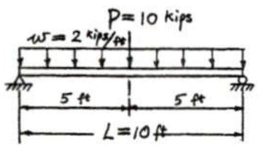
$$\delta_C = 2\left[\delta_C\right]_{case\,6}^{due\ to\ P}$$

$$= 2\left[\frac{Pb\,x}{6EIL}(L^2 - x^2 - b^2)\right]$$

$$= 2\left[\frac{(400)(1)(2)}{6(2.1 \times 10^4)(4)}(4^2 - 2^2 - 1^2)\right]$$

$$= 0.0349\text{ m} = 34.9\text{ mm} \downarrow$$ ◀

16–21



$EI = 2.6 \times 10^4\text{ kip}\cdot\text{ft}^2$

(Cont'd)

16-21 (Cont)

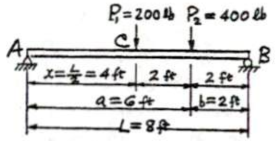
$$\delta_C = \left[\delta_{max}\right]_{case\ 5}^{due\ to\ P} + \left[\delta_{max}\right]_{case\ 7}^{due\ to\ w}$$

$$= \frac{PL^3}{48EI} + \frac{5wL^4}{384EI}$$

$$= \frac{(10)(10)^3}{48(2.6 \times 10^4)} + \frac{5(2)(10)^4}{384(2.6 \times 10^4)}$$

$$= 0.01803 \text{ ft} = 0.216 \text{ in.} \downarrow$$ ◄

16-22



$EI = 1.9 \times 10^5 \text{ lb} \cdot \text{ft}^2$

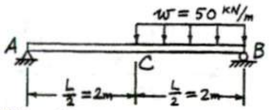
$$\delta_C = \left[\delta_{max}\right]_{case\ 5}^{due\ to\ P_1} + \left[\delta_C\right]_{case\ 6}^{due\ to\ P_2}$$

$$= \frac{P_1L^3}{48EI} + \frac{P_2 b x}{6EIL}(L^2 - x^2 - b^2)$$

$$= \frac{(200)(8)^3}{48(1.9 \times 10^5)} + \frac{(400)(2)(4)}{6(1.9 \times 10^5)(8)}(8^2 - 4^2 - 2^2)$$

$$= 0.0267 \text{ ft} = 0.320 \text{ in.} \downarrow$$ ◄

16-23



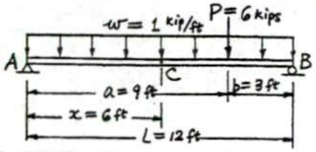
$EI = 8900 \text{ kN} \cdot \text{m}^2$

$$\delta_C = \frac{1}{2}\left[\delta_{max}\right]_{case\ 7}^{due\ to\ w\ over\ entire\ span}$$

$$= \frac{1}{2}\left(\frac{5wL^4}{384EI}\right) = \frac{5(50)(4)^4}{2(384)(8900)}$$

$$= 0.00936 \text{ m} = 9.36 \text{ mm} \downarrow$$ ◄

16-24



$EI = 2.7 \times 10^4 \text{ kip} \cdot \text{ft}^2$

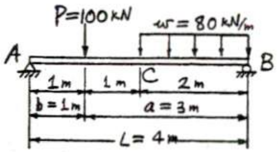
$$\delta_C = \left[\delta_C\right]_P + \left[\delta_{max}\right]_w$$

$$= \frac{Pb\,x}{6EIL}(L^2 - x^2 - b^2) + \frac{5wL^4}{384EI}$$

$$= \frac{(6)(3)(6)}{6(2.7 \times 10^4)(12)}(12^2 - 6^2 - 3^2) + \frac{5(1)(12)^4}{384(2.7 \times 10^4)}$$

$$= 0.0155 \text{ ft} = 0.186 \text{ in.} \downarrow$$ ◄

16–25



$EI = 3.4 \times 10^4 \text{ kN} \cdot \text{m}^2$

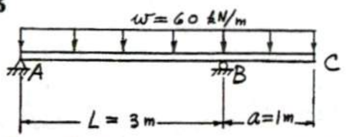
$$\delta_C = \left[\delta_C\right]_{case\ 6}^{due\ to\ P} + \frac{1}{2}\left[\delta_{max}\right]_{case\ 7}^{due\ to\ w\ over\ entire\ span}$$

$$= \frac{Pb\,x}{6EIL}(L^2 - x^2 - b^2) + \frac{1}{2}\left(\frac{5wL^4}{384EI}\right)$$

$$= \frac{(100)(1)(2)}{6(3.4 \times 10^4)(4)}(4^2 - 2^2 - 1^2) + \frac{1}{2}\left(\frac{5(80)(4)^4}{384(3.4 \times 10^4)}\right)$$

$$= 0.00662 \text{ m} = 6.62 \text{ mm} \downarrow$$ ◄

16-26

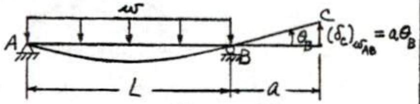


From Appendix Table A-1(b), for W 300 x 0.51 section,

$I = 119 \times 10^{-6} \text{ m}^4$

$EI = (210 \times 10^6 \text{ kN/m}^2)(119 \times 10^{-6} \text{ m}^4)$

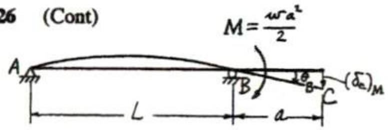
$= 2.50 \times 10^4 \text{ kN} \cdot \text{m}^2$



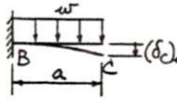
$$(\delta_C)_{w_{AB}} = a\,\theta_B = a\frac{wL^3}{24EI} = \frac{waL^3}{24EI} \uparrow$$

(Cont'd)

16-26 (Cont)



$$(\delta_C)_M = a\,\theta_B = a\frac{ML}{3EI} = \frac{wa^3L}{6EI} \downarrow$$



$$(\delta_C)_{w_{BC}} = \frac{wa^4}{8EI} \downarrow$$

$$\delta_C = (\delta_C)_{w_{AB}} + (\delta_C)_M + (\delta_C)_{w_{BC}}$$

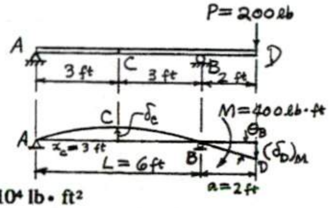
$$= \frac{waL^3}{24EI} - \frac{wa^3L}{6EI} - \frac{wa^4}{8EI}$$

$$= \frac{wa}{2EI}\left(\frac{L^3}{12} - \frac{a^2L}{3} - \frac{a^3}{4}\right)$$

$$= \frac{60(3)}{2(2.50 \times 10^4)}\left(\frac{3^3}{12} - \frac{(1)^2(3)}{3} - \frac{1^3}{4}\right)$$

$$= +0.0036 \text{ m} = 3.6 \text{ mm} \uparrow$$ ◄

16-27



$EI = 7.6 \times 10^4 \text{ lb} \cdot \text{ft}^2$

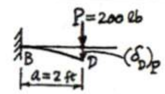
$$\delta_C = \frac{Mx_C}{6EIL}(L^2 - x_C^2) = \frac{400(3)}{6(7.6 \times 10^4)(6)}(6^2 - 3^2)$$

$$= 0.0118 \text{ ft} = 0.142 \text{ in.} \uparrow$$ ◄

$$(\delta_D)_M = a\,\theta_B = a\frac{ML}{3EI}$$

$$= \frac{(2)(400)(6)}{3(7.6 \times 10^4)}$$

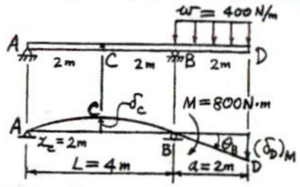
$$= 0.0211 \text{ ft}$$



$$(\delta_D)_P = \frac{Pa^3}{3EI} = \frac{(200)(2)^3}{3(7.6 \times 10^4)} = 0.00712 \text{ ft}$$

$$\delta_D = (\delta_D)_M + (\delta_D)_P = 0.0281 \text{ ft} = 0.337 \text{ in.} \downarrow$$ ◄

16-28

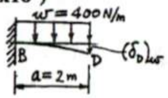


$EI = 2.2 \times 10^5 \text{ N} \cdot \text{m}^2$

$$\delta_C = \frac{Mx_C}{6EIL}(L^2 - x_C^2) = \frac{800(2)}{6(2.2 \times 10^5)(4)}(4^2 - 2^2)$$

$$= 0.00364 \text{ m} = 3.64 \text{ mm} \uparrow$$ ◄

$$(\delta_D)_M = a\theta_B = a\frac{ML}{3EI} = (2)\frac{(800)(4)}{3(2.2 \times 10^5)} = 0.00970 \text{ m} \downarrow$$

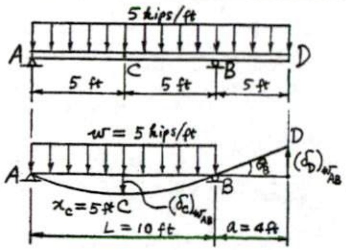


$$(\delta_D)_w = \frac{wa^4}{8EI} = \frac{(400)(2)^4}{8(2.2 \times 10^5)} = 0.00364 \text{ m} \downarrow$$

$$\delta_D = (\delta_D)_M + (\delta_D)_w = 0.00970 + 0.00364$$

$$= 0.0133 \text{ m} = 13.3 \text{ mm} \downarrow$$ ◄

16-29



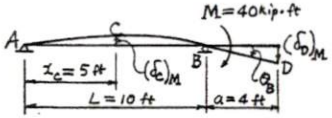
$EI = 2.3 \times 10^4 \text{ kip} \cdot \text{ft}^2$

(Cont'd)

$$(\delta_C)_{w_{AB}} = \frac{5wL^4}{384EI} = \frac{5(5)(10)^4}{384(2.3 \times 10^4)} = 0.0283 \text{ ft} \downarrow$$

$$(\delta_D)_{w_{AB}} = a\theta_B = a\frac{wL^3}{24EI} = (4)\frac{(5)(10)^3}{24(2.3 \times 10^4)}$$

$$= 0.0362 \text{ ft} \uparrow$$

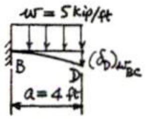


$$(\delta_C)_M = \frac{Mx_C}{6EIL}\left[L^2 - x_C^2\right]$$

$$= \frac{(40)(5)}{6(2.3 \times 10^4)(10)}(10^2 - 5^2) = 0.0109 \text{ ft} \uparrow$$

$$(\delta_D)_M = a\theta_B = a\frac{ML}{3EI} = (4)\frac{(40)(10)}{3(2.3 \times 10^4)}$$

$$= 0.0232 \text{ ft} \downarrow$$



$$(\delta_D)_{w_{BD}} = \frac{wa^4}{8EI} = \frac{(5)(4)^4}{8(2.3 \times 10^4)} = 0.00696 \text{ ft} \downarrow$$

$$\delta_C = (\delta_C)_{w_{AB}} + (\delta_C)_M$$

$$= 0.0283 - 0.0109$$

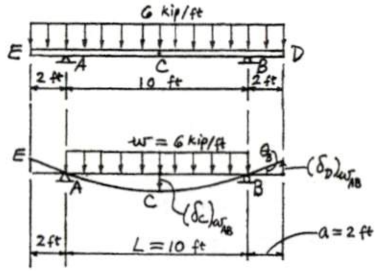
$$= 0.0174 \text{ ft} = 0.209 \text{ in.} \downarrow \quad ◀$$

$$\delta_D = (\delta_D)_{w_{AB}} + (\delta_D)_M + (\delta_D)_{w_{BD}}$$

$$= 0.0362 - 0.0232 - 0.00696$$

$$= 0.00604 \text{ ft} = 0.0725 \text{ in.} \uparrow \quad ◀$$

16-30

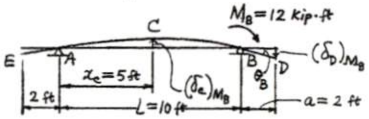


EI = 4.7 x 10⁴ kip • ft²

$$(\delta_C)_{w_{AB}} = \frac{5wL^4}{384EI} = \frac{5(6)(10)^4}{384(4.7 \times 10^4)} = 0.0166 \text{ ft} \downarrow$$

$$(\delta_D)_{w_{AB}} = a\theta_B = a\frac{wL^3}{24EI} = (2)\frac{(6)(10)^3}{24(4.7 \times 10^4)}$$

$$= 0.0106 \text{ ft} \uparrow$$

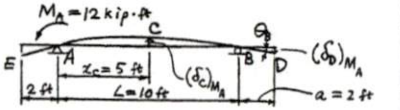


$$(\delta_C)_{M_B} = \frac{Mx_C}{6EIL}\left[L^2 - x_C^2\right]$$

$$= \frac{(12)(5)}{6(4.7 \times 10^4)(10)}(10^2 - 5^2) = 0.0016 \text{ ft} \uparrow$$

$$(\delta_D)_{M_B} = a\theta_B = a\frac{M_BL}{3EI} = (2)\frac{(12)(10)}{3(4.7 \times 10^4)}$$

$$= 0.0017 \text{ ft} \downarrow$$



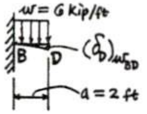
$$(\delta_C)_{M_A} = (\delta_C)_{M_B} = 0.0016 \text{ ft} \uparrow$$

$$(\delta_D)_{M_A} = a\theta_B = a\frac{M_AL}{6EI} = (2)\frac{(12)(10)}{6(4.7 \times 10^4)}$$

$$= 0.00085 \text{ ft} \downarrow$$

(Cont'd)

16-30 (Cont)



$$(\delta_D)_{w_{BD}} = \frac{wa^4}{8EI} = \frac{(6)(2)^4}{8(4.7 \times 10^4)} = 0.00026 \text{ ft} \downarrow$$

$$\delta_C = (\delta_C)_{w_{AB}} + (\delta_C)_{M_B} + (\delta_C)_{M_A}$$

$$= 0.0166 - 0.0016 - 0.0016 = +0.0134 \text{ ft}$$

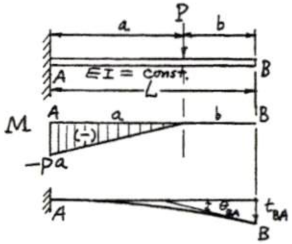
$$= +0.0134 \text{ ft} = 0.161 \text{ in.} \downarrow \quad ◀$$

$$\delta_D = (\delta_D)_{w_{AB}} + (\delta_D)_{M_B} + (\delta_D)_{M_A} + (\delta_D)_{w_{BD}}$$

$$= +0.0106 - 0.0017 - 0.00085 - 0.00026$$

$$= +0.00779 \text{ ft} = 0.0935 \text{ in.} \uparrow \quad ◀$$

16-31



$$\theta_{BA} = \frac{1}{EI}A_{AB} = \frac{1}{EI}\left[\frac{1}{2}(a)(-Pa)\right] = -\frac{Pa^2}{2EI}$$

$$\theta_{max} = |\theta_{BA}| = \frac{Pa^2}{2EI} \quad ◀$$

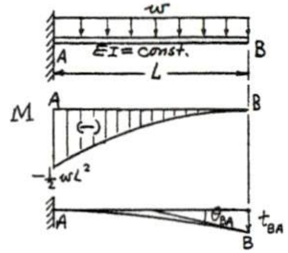
$$t_{BA} = \frac{1}{EI}A_{AB}\,\bar{x}_B = \frac{1}{EI}\left[\frac{1}{2}(a)(-Pa)\right]\left(\frac{2a}{3} + b\right)$$

$$= -\frac{Pa^2}{2EI}(2a + 3b)$$

$$2a + 3b = 3(a + b) - a = 3L - a$$

$$\delta_{max} = |t_{BA}| = \frac{Pa^2}{2EI}(3L - a) \downarrow \quad ◀$$

16-32



$$\theta_{BA} = \frac{1}{EI}A_{AB} = \frac{1}{EI}\left[\frac{1}{3}(L)\left(-\frac{1}{2}wL^2\right)\right] = -\frac{wL^3}{6EI}$$

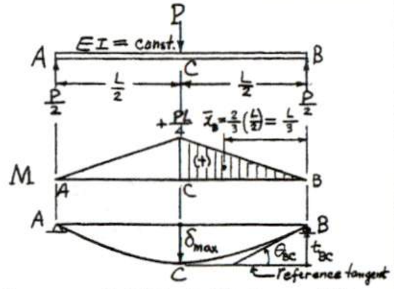
$$\theta_{max} = |\theta_{BA}| = \frac{wL^3}{6EI} \quad ◀$$

$$t_{BA} = \frac{1}{EI}A_{AB}\,\bar{x}_B = \frac{1}{EI}\left[\frac{1}{3}(L)\left(-\frac{1}{2}wL^2\right)\right]\left(\frac{3L}{4}\right)$$

$$= -\frac{wL^4}{8EI}$$

$$\delta_{max} = |t_{BA}| = \frac{wL^4}{8EI} \downarrow \quad ◀$$

16-33



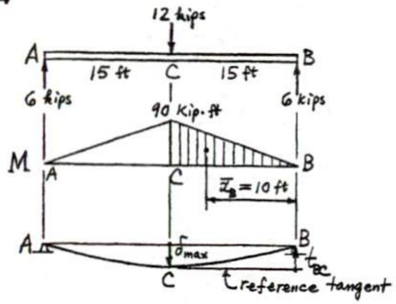
$$\theta_{BC} = \frac{1}{EI}A_{BC} = \frac{1}{EI}\left[\frac{1}{2}\left(\frac{L}{2}\right)\left(+\frac{PL}{4}\right)\right] = +\frac{PL^2}{16EI}$$

$$\theta_{max} = \theta_{BC} = \frac{PL^2}{16EI} \quad ◀$$

$$t_{BC} = \frac{1}{EI}A_{BC}\,\bar{x}_B = \frac{1}{EI}\left[\frac{1}{2}\left(\frac{L}{2}\right)\left(\frac{PL}{4}\right)\right]\left(\frac{L}{3}\right) = \frac{PL^3}{48EI}$$

$$\delta_{max} = t_{BC} = \frac{PL^3}{48EI} \downarrow \quad ◀$$

16-34



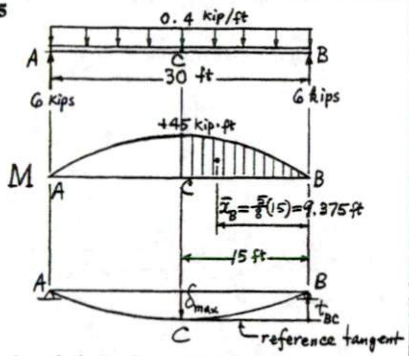
From Appendix Table A-1(a), for W 16 x 36,
I = 448 in.^{4}
EI = (30 x 10^3 kip/in.^{2})(448 in.^{4})
= 1.34 x 10^7 kip • in.^{2} = 9.33 x 10^4 kip • ft^2

$$\delta_{max} = t_{BC} = \frac{1}{EI} A_{BC}\,\bar{x}_B$$

$$= \frac{1}{9.33 \times 10^4}\left[\left(\frac{1}{2} \times 90 \times 15\right)(10)\right]$$

$$= 0.0723 \text{ ft} = 0.868 \text{ in.}$$ ◄

16-35



From the solution to Prob. 16-34,
EI = 9.33 x 10^4 kip • ft^2

$$\delta_{max} = t_{BC} = \frac{1}{EI} A_{BC}\,\bar{x}_B$$

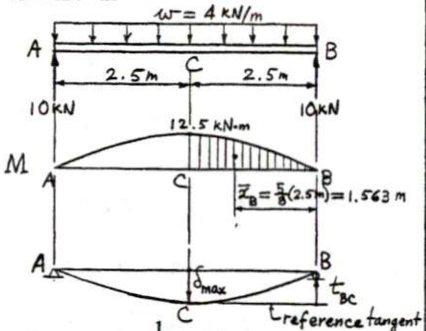
$$= \frac{1}{9.33 \times 10^4}\left[\left(\frac{2}{3}\right)(15)(45)\right](9.375)$$

$$= 0.0452 \text{ ft} = 0.543 \text{ in.}$$ ◄

16-36

From Appendix Table A-6(b), for 250 x 360 timber section,
I = 811 x 10^{-6} m^4
From Appendix Table A-7(b), for California Redwood,
E = 9.0 GPa

EI = (9.0 x 10^6 kN/m^2)(811 x 10^{-6} m^4)
= 7299 kN • m^2

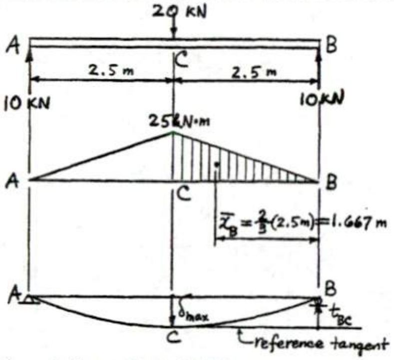


$$\delta_{max} = t_{BC} = \frac{1}{EI} A_{BC}\,\bar{x}_B$$

$$= \frac{1}{7299}\left[\left(\frac{2}{3}\right)(2.5)(12.5)\right](1.563)$$

$$= 0.00446 \text{ m} = 4.46 \text{ mm}$$ ◄

16-37



From the solution to Prob. 16-36,
EI = 7299 kN • m^2

(Cont'd)

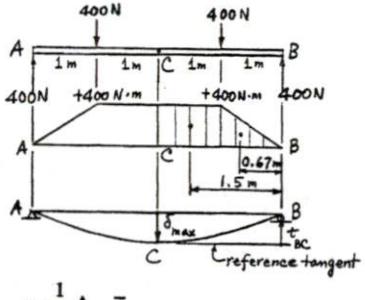
16-37 (Cont)

$$\delta_{max} = t_{BC} = \frac{1}{EI} A_{BC}\,\bar{x}_B$$

$$= \frac{1}{7299}\left[\left(\frac{1}{2}\right)(2.5)(25)\right](1.667)$$

$$= 0.00714 \text{ m} = 7.14 \text{ mm}$$ ◄

16-38

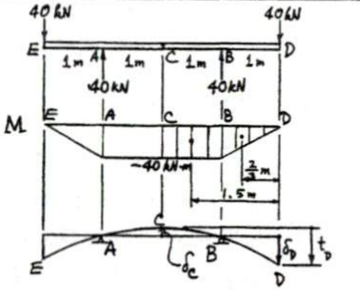


$$\delta_{max} = t_{BC} = \frac{1}{EI} A_{BC}\,\bar{x}_B$$

$$= \frac{1}{2.1 \times 10^4}\left[(1)(+400)(1.5) + \frac{1}{2}(1)(+400)(0.67)\right]$$

$$= +0.0349 \text{ m} = +34.9 \text{ mm} \downarrow$$ ◄

16-39



EI = 5100 kN • m^2

$$t_{DC} = \frac{1}{EI} A_{CD}\,\bar{x}_D$$

$$= \frac{1}{5100}\left[(1)(-40)(1.5) + \frac{1}{2}(1)(-40)\left(\frac{2}{3}\right)\right]$$

$$= -0.0144 \text{ m}$$

$$t_{BC} = \frac{1}{EI} A_{BC}\,\bar{x}_B$$

$$= \frac{1}{5100}\left[(1)(-40)(0.5)\right] = -0.00392 \text{ m}$$

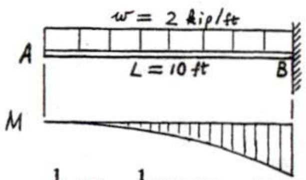
$\delta_C = |t_{BC}| = 0.00392$ m = 3.92 mm ↑ ◄

$\delta_D = |t_{DC}| - |t_{BC}|$

= 0.0144 - 0.0039

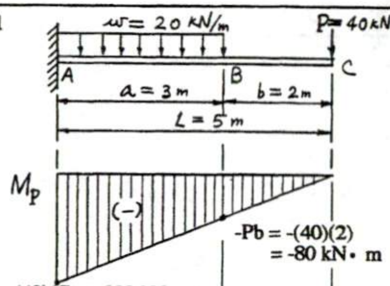
= 0.0105 m = 10.5 mm ↓ ◄

16-40



$$-\frac{1}{2}wL^2 = -\frac{1}{2}(2)(10)^2 = -100 \text{ kip} \cdot \text{ft}$$

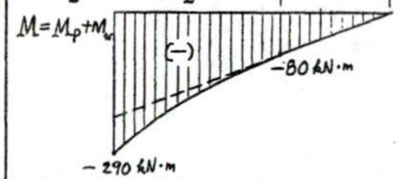
16-41



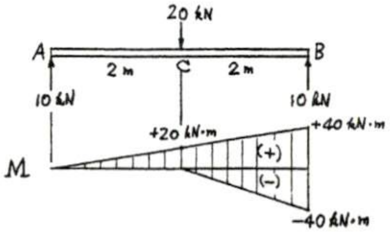
-Pb = -(40)(2)
= -80 kN • m

-PL = -(40)(5) = -200 kN • m

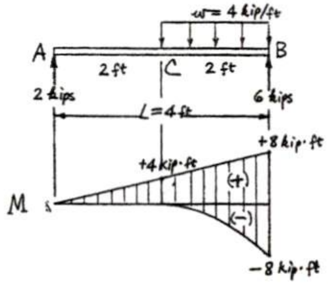
$$-\frac{wa^2}{2} = -\frac{(20)(3)^2}{2} = -90 \text{ kN} \cdot \text{m}$$



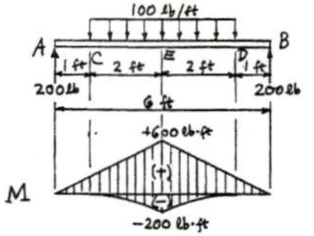
16-42



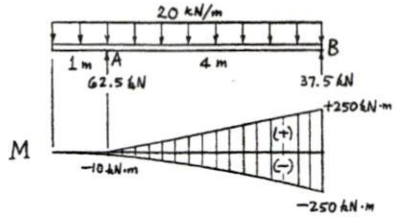
16-43



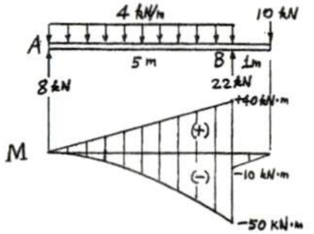
16-44



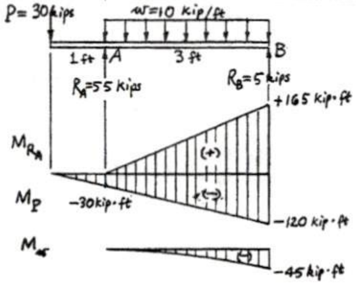
16-45



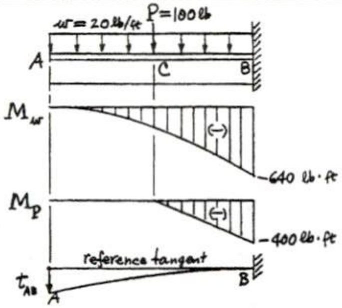
16-46



16-47



16-48



$EI = 2.7 \times 10^5$ lb • ft²

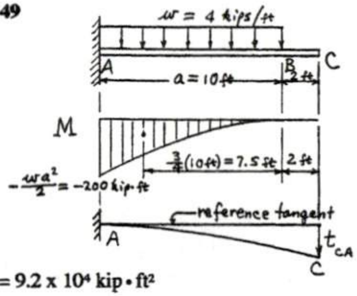
$$t_{AB} = \frac{1}{EI} A_{AB}\,\bar{x}_A$$

$$= \frac{1}{2.7\times10^5}\left[\frac{1}{3}(8)(-640)\left(\frac{3}{4}\times8\right) + \frac{1}{2}(4)(-400)\left(4 + \frac{2}{3}\times4\right)\right]$$

$$= -0.0577 \text{ ft} = -0.692 \text{ in.}$$

$\delta_{max} = |t_{AB}| = 0.692$ in. ↓ ◀

16-49



$EI = 9.2 \times 10^4$ kip • ft²

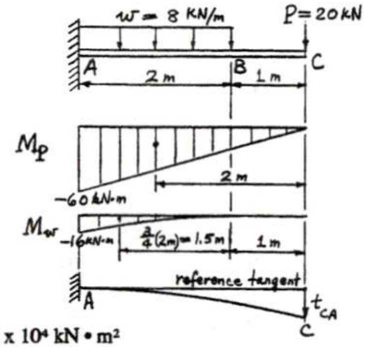
$$t_{CA} = \frac{1}{EI} A_{AC}\,\bar{x}_C$$

$$= \frac{1}{9.2\times10^4}\left[\frac{1}{3}(10)(-200)\right](7.5 + 2)$$

$$= -0.0688 \text{ ft} = -0.826 \text{ in.}$$

$\delta_{max} = |t_{CA}| = 0.826$ in. ↓ ◀

16-50



$EI = 1.2 \times 10^4$ kN • m²

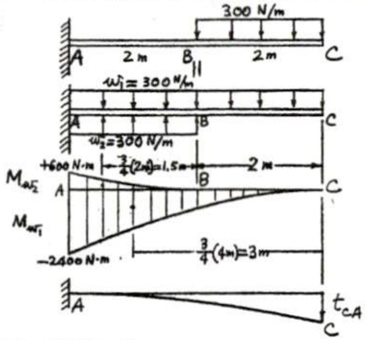
$$t_{CA} = \frac{1}{EI} A_{AC}\,\bar{x}_C$$

$$= \frac{1}{1.2\times10^4}\left[\frac{1}{2}(3)(-60)(2) + \frac{1}{3}(2)(-16)(1.5+1)\right]$$

$$= -0.0172 \text{ m} = -17.2 \text{ mm}$$

$\delta_{max} = |t_{CA}| = 17.2$ mm ↓ ◀

16-51



$EI = 1.6 \times 10^5$ N • m²

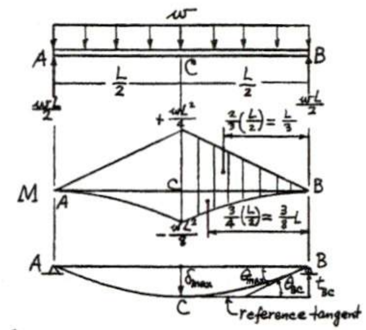
$$t_{CA} = \frac{1}{EI} A_{AC}\,\bar{x}_C$$

$$= \frac{1}{9.6\times10^5}\left[\frac{1}{3}(2)(+600)(1.5+2) + \frac{1}{3}(4)(-2400)(3)\right]$$

$$= -0.00854 \text{ m} = -8.54 \text{ mm}$$

$\delta_{max} = |t_{CA}| = 8.54$ mm ↓ ◀

16-52



$$\theta_{BC} = \frac{1}{EI} A_{BC}$$

$$= \frac{1}{EI}\left[\frac{1}{2}\left(\frac{L}{2}\right)\left(+\frac{wL^2}{4}\right) + \frac{1}{3}\left(\frac{L}{2}\right)\left(-\frac{wL^2}{8}\right)\right]$$

$$= +\frac{wL^3}{24EI}$$

$$\theta_{max} = |\theta_{BC}| = \frac{wL^3}{24EI}$$ ◀

(Cont'd)

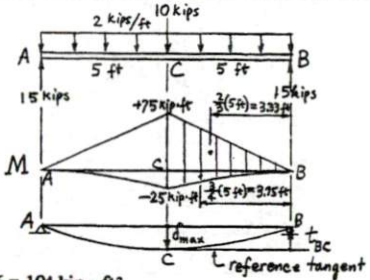
16-52 (Cont)

$$t_{BC} = \frac{1}{EI} A_{BC}\,\bar{x}_B$$

$$= \frac{1}{EI}\left[\frac{1}{2}\left(\frac{L}{2}\right)\left(+\frac{wL^2}{4}\right)\left(\frac{L}{3}\right) + \frac{1}{2}\left(\frac{L}{2}\right)\left(-\frac{wL^2}{8}\right)\left(\frac{3}{8}L\right)\right] = -\frac{5wL^2}{384EI}$$

$$\delta_{max} = |\,t_{BC}\,| = \frac{5wL^2}{384EI}\ \downarrow$$ ◀

16-53



$EI = 2.6 \times 10^4$ kip • ft²

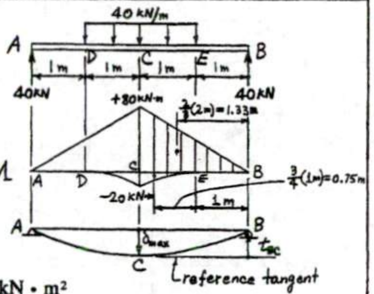
$$t_{BC} = \frac{1}{EI} A_{BC}\,\bar{x}_B$$

$$= \frac{1}{2.6 \times 10^4}\left[\frac{1}{2}(5)(+75)(3.33) + \frac{1}{3}(5)(-25)(3.75)\right]$$

$$= +0.01803 \text{ ft} = +0.216 \text{ in.}$$

$$\delta_{max} = t_{BC} = 0.216 \text{ in.}\ \downarrow$$ ◀

16-54



$EI = 9500$ kN • m²

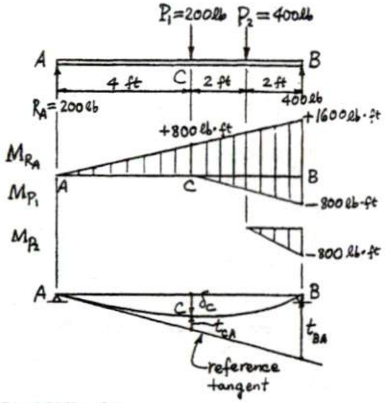
$$t_{BC} = \frac{1}{EI} A_{BC}\,\bar{x}_B$$

$$= \frac{1}{9500}\left[\frac{1}{2}(2)(+80)(1.33) + \frac{1}{3}(1)(-20)(1.75)\right]$$

$$= +0.010 \text{ m} = +10.0 \text{ mm}$$

$$\delta_{max} = t_{BC} = 10.0 \text{ mm}\ \downarrow$$ ◀

16-55



$EI = 1.9 \times 10^5$ lb • ft²

$$t_{BA} = \frac{1}{EI} A_{AB}\,\bar{x}_B$$

$$= \frac{1}{1.9 \times 10^5}\left[\frac{1}{2}(8)(+1600)\left(\frac{8}{3}\right) + \frac{1}{2}(4)(-800)\left(\frac{4}{3}\right) + \frac{1}{2}(2)(-800)\left(\frac{2}{3}\right)\right]$$

$= 0.0758$ ft

$$t_{CA} = \frac{1}{EI} A_{AC}\,\bar{x}_C$$

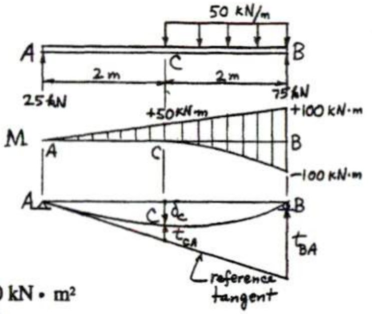
$$= \frac{1}{1.9 \times 10^5}\left[\frac{1}{2}(4)(+800)\left(\frac{4}{3}\right)\right]$$

$= 0.0112$ ft

$$\delta_C = \frac{1}{2}t_{BA} - t_{CA} = \frac{1}{2}(0.0758) - 0.0112$$

$$= +0.0267 \text{ ft} = 0.320 \text{ in.}\ \downarrow$$ ◀

16-56



$EI = 8900$ kN • m²

$$t_{BA} = \frac{1}{EI} A_{AB}\,\bar{x}_B$$

$$= \frac{1}{8900}\left[\frac{1}{2}(4)(+100)\left(\frac{4}{3}\right) + \frac{1}{3}(2)(-100)\left(\frac{2}{4}\right)\right]$$

$= 0.0262$ m

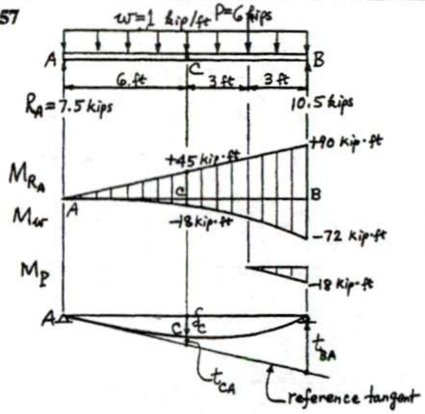
$$t_{CA} = \frac{1}{EI} A_{AC}\,\bar{x}_C = \frac{1}{8900}\left[\frac{1}{2}(2)(50)\left(\frac{2}{3}\right)\right]$$

$= 0.00375$ m

$$\delta_C = \frac{1}{2}t_{BA} - t_{CA} = \frac{1}{2}(0.0262) - 0.00375$$

$$= 0.00935 \text{ m} = 9.35 \text{ mm}\ \downarrow$$ ◀

16-57



$EI = 2.7 \times 10^4$ kip • ft²

$$t_{BA} = \frac{1}{EI} A_{AB}\,\bar{x}_B$$

$$= \frac{1}{2.7 \times 10^4}\left[\frac{1}{2}(12)(90)\left(\frac{12}{3}\right) + \frac{1}{3}(12)(-72)\left(\frac{12}{4}\right) + \frac{1}{2}(3)(-18)\left(\frac{3}{3}\right)\right]$$

$= 0.0470$ ft

$$t_{CA} = \frac{1}{EI} A_{AC}\,\bar{x}_C$$

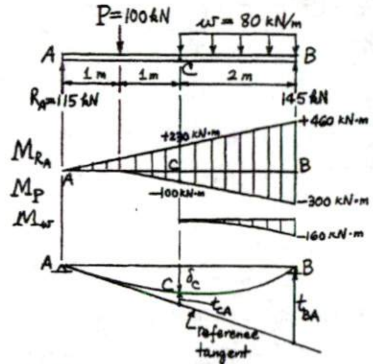
$$= \frac{1}{2.7 \times 10^4}\left[\frac{1}{2}(6)(+45)\left(\frac{6}{3}\right) + \frac{1}{3}(6)(-18)\left(\frac{6}{4}\right)\right]$$

$= 0.008$ ft

$$\delta_C = \frac{1}{2}t_{BA} - t_{CA} = \frac{1}{2}(0.0470) - 0.008$$

$$= +0.0155 \text{ ft} = +0.186 \text{ in.}\ \downarrow$$ ◀

16-58



$EI = 3.4 \times 10^4$ kN • m²

$$t_{BA} = \frac{1}{EI} A_{AB}\,\bar{x}_B$$

$$= \frac{1}{3.4 \times 10^4}\left[\frac{1}{2}(4)(+460)\left(\frac{4}{3}\right) + \frac{1}{2}(3)(-300)\left(\frac{3}{3}\right) + \frac{1}{3}(2)(-160)\left(\frac{2}{4}\right)\right]$$

$= 0.0213$ m

(Cont'd)

16-58 (Cont)

$$t_{CA} = \frac{1}{EI}A_{AC}\,\bar{x}_C$$

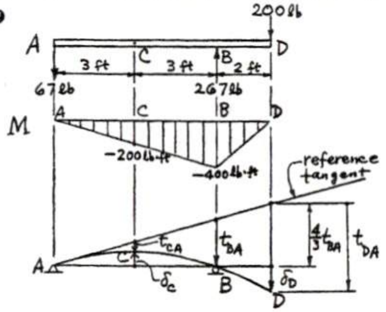
$$= \frac{1}{3.4 \times 10^4}\left[\frac{1}{2}(2)(+230)\left(\frac{2}{3}\right) + \frac{1}{2}(1)(-100)\left(\frac{1}{3}\right)\right]$$

$$= 0.00402 \text{ m}$$

$$\delta_C = \frac{1}{2}t_{BA} - t_{CA} = \frac{1}{2}(0.0213) - 0.00402$$

$$= 0.00663 \text{ m} = 6.63 \text{ mm} \downarrow$$ ◀

16-59



$$EI = 7.6 \times 10^4 \text{ lb} \cdot \text{ft}^2$$

$$t_{CA} = \frac{1}{EI}A_{AC}\,\bar{x}_C$$

$$= \frac{1}{7.6 \times 10^4}\left[\frac{1}{2}(3)(-200)(1)\right] = -0.00395 \text{ ft}$$

$$t_{BA} = \frac{1}{EI}A_{AB}\,\bar{x}_B$$

$$= \frac{1}{7.6 \times 10^4}\left[\frac{1}{2}(6)(-400)(2)\right] = -0.0316 \text{ ft}$$

$$t_{DA} = \frac{1}{EI}A_{AD}\,\bar{x}_D$$

$$= \frac{1}{7.6 \times 10^4}\left[\frac{1}{2}(6)(-400)\left(\frac{1}{3}6+2\right) + \frac{1}{2}(2)(-400)\left(\frac{2}{3}x2\right)\right]$$

$$= -0.0702 \text{ ft}$$

$$\delta_C = \frac{1}{2}|t_{BA}| - |t_{CA}|$$

$$= \frac{1}{2}(0.0316) - 0.00395$$

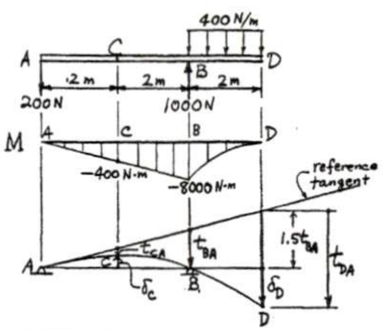
$$= 0.0119 \text{ ft} = 0.142 \text{ in.} \uparrow$$ ◀

$$\delta_D = |t_{DA}| - \frac{4}{3}|t_{BA}|$$

$$= 0.0702 \text{ ft} - \frac{4}{3}(0.0316 \text{ ft})$$

$$= 0.0281 \text{ ft} = 0.337 \text{ in.} \downarrow$$ ◀

16-60



$$EI = 2.2 \times 10^5 \text{ N} \cdot \text{m}^2$$

$$t_{CA} = \frac{1}{EI}A_{AC}\,\bar{x}_C$$

$$= \frac{1}{2.2 \times 10^5}\left[\frac{1}{2}(2)(-400)\left(\frac{1}{3} \times 2\right)\right]$$

$$= -0.00121 \text{ m}$$

$$t_{BA} = \frac{1}{EI}A_{AB}\,\bar{x}_B$$

$$= \frac{1}{2.2 \times 10^5}\left[\frac{1}{2}(4)(-800)\left(\frac{1}{3} \times 4\right)\right]$$

$$= -0.00970 \text{ m}$$

$$t_{DA} = \frac{1}{EI}A_{AD}\,\bar{x}_D$$

$$= \frac{1}{2.2 \times 10^5}\left[\frac{1}{2}(4)(-800)\left(\frac{4}{3}+2\right) + \frac{1}{3}(2)(-800)\left(\frac{3}{4}x2\right)\right]$$

$$= -0.02788 \text{ m}$$

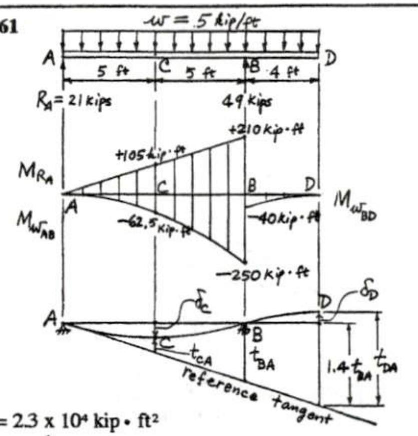
$$\delta_C = \frac{1}{2}|t_{BA}| - |t_{CA}| = \frac{1}{2}(0.00970) - 0.00121$$

$$= 0.00364 \text{ m} = 3.64 \text{ mm} \uparrow$$ ◀

$$\delta_D = |t_{DA}| - 1.5|t_{BA}| = 0.02788 - 1.5(0.00970)$$

$$= 0.0133 \text{ m} = 13.3 \text{ mm} \downarrow$$ ◀

16-61



$$EI = 2.3 \times 10^4 \text{ kip} \cdot \text{ft}^2$$

$$t_{CA} = \frac{1}{EI}A_{AC}\,\bar{x}_C$$

$$= \frac{1}{2.3 \times 10^4}\left[\frac{1}{2}(5)(+105)\left(\frac{5}{3}\right) + \frac{1}{3}(5)(-62.5)\left(\frac{5}{4}\right)\right]$$

$$= 0.0134 \text{ ft}$$

$$t_{BA} = \frac{1}{EI}A_{AB}\,\bar{x}_B$$

$$= \frac{1}{2.3 \times 10^4}\left[\frac{1}{2}(10)(+210)\left(\frac{10}{3}\right) + \frac{1}{3}(10)(-250)\left(\frac{10}{4}\right)\right]$$

$$= 0.0616 \text{ ft}$$

$$t_{DA} = \frac{1}{EI}A_{AD}\,\bar{x}_D$$

$$= \frac{1}{2.3 \times 10^4}\left[\frac{1}{2}(10)(+210)\left(\frac{10}{3}+4\right) + \frac{1}{3}(10)(-250)\left(\frac{10}{4}+4\right) + \frac{1}{3}(4)(-40)\left(\frac{3}{4}x4\right)\right]$$

$$= 0.09232 \text{ ft}$$

$$\delta_C = \frac{1}{2}t_{BA} - t_{CA} = \frac{1}{2}(0.0616) - 0.0134$$

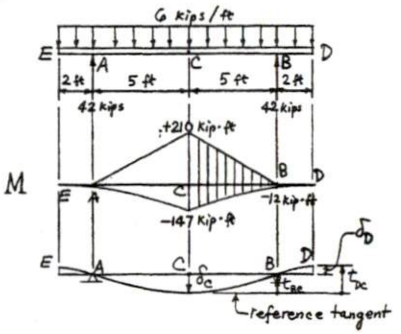
$$= 0.0174 \text{ ft} = 0.209 \text{ in.} \downarrow$$ ◀

$$\delta_D = t_{DA} - 1.4\,t_{BA}$$

$$= 0.09232 - 1.4(0.0616)$$

$$= 0.00608 \text{ ft} = 0.0730 \text{ in.} \uparrow$$ ◀

16-62



$$EI = 4.7 \times 10^4 \text{ kip} \cdot \text{ft}^2$$

$$t_{DC} = \frac{1}{EI}A_{CD}\,\bar{x}_D$$

$$= \frac{1}{4.7 \times 10^4}\left[\frac{1}{2}(5)(210)\left(\frac{2}{3}5+2\right) + \frac{1}{3}(7)(-147)\left(\frac{3}{4}x7\right)\right]$$

$$= 0.02126 \text{ ft}$$

$$t_{BC} = \frac{1}{EI}A_{BC}\,\bar{x}_B$$

$$= \frac{1}{4.7 \times 10^4}\left[\frac{1}{2}(5)(210)\left(\frac{2}{5} \times 5\right) + \frac{1}{3}(7)(-147)\left(\frac{3}{4} \times 7 - 2\right) - \frac{1}{3}(2)(-12)\left(-\frac{2}{4}\right)\right]$$

$$= 0.01343 \text{ ft}$$

$$\delta_C = t_{BC} = 0.01343 \text{ ft} = 0.161 \text{ in.} \downarrow$$ ◀

$$\delta_D = |t_{DC}| - |t_{BC}|$$

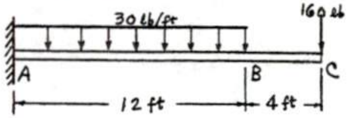
$$= 0.02126 \text{ ft} - 0.01343 \text{ ft}$$

$$= 0.00783 \text{ ft} = 0.0940 \text{ in.} \uparrow$$ ◀

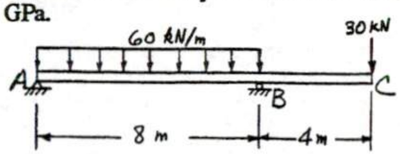
Test Problems for Chapter 16

The following problems may either be given to students for them to practice their problem solving skills or be used as test problems. The answers are provided on the next page.

(1) The 10-ft span cantilever beam of 3 x 10 southern pine section is subjected to the loads shown. Compute the deflections at the free-end C by using the deflection formulas. The modulus of elasticity of southern pine is $E = 1800$ ksi.



(2) The overhanging beam of W410 x 1.30 section is subjected to the loads shown. Compute the deflection at the free-end C by using the deflection formulas. The modulus of elasticity of steel is $E = 210$ GPa.



(3) Rework Prob. (1) by using the moment-area method.

(4) Rework Prob. (2) by using the moment-area method.

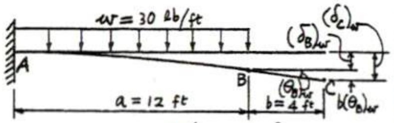
Solutions to Test Problems for Chapter 16

(1)

From Appendix Table A-6(a), for 3 x 10 section,
$I = 165 \text{ in.}^4$

$EI = (1800 \text{ kip/in.}^2)(165 \text{ in.}^4) = 2.97 \times 10^5 \text{ kip}\cdot\text{in.}^2 = 2.06 \times 10^6 \text{ lb}\cdot\text{ft}^2$

Due to the uniform load:

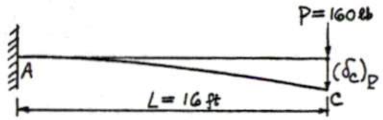


$$(\delta_C)_w = (\delta_B)_w + b(\theta_B)_w = \frac{wa^4}{8EI} + b\frac{wa^3}{6EI}$$

$$= \frac{(30 \text{ lb/ft})(12 \text{ ft})^4}{8(2.06 \times 10^6 \text{ lb}\cdot\text{ft}^2)} + (4 \text{ ft})\frac{(30 \text{ lb/ft})(12 \text{ ft})^3}{6(2.06 \times 10^6 \text{ lb}\cdot\text{ft}^2)}$$

$$= 0.0545 \text{ ft}$$

Due to the concentrated load:



$$(\delta_C)_P = \frac{PL^3}{3EI} = \frac{(160 \text{ lb})(16 \text{ ft})^3}{3(2.06 \times 10^6 \text{ lb}\cdot\text{ft}^2)} = 0.1060 \text{ ft}$$

Total deflection:

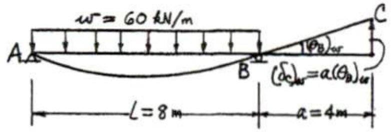
$$\delta_C = (\delta_C)_w + (\delta_C)_P = 0.0545 \text{ ft} + 0.1060 \text{ ft} = 0.1605 \text{ ft} = 1.93 \text{ in.}$$ ◀

(2)

From Appendix Table A-1(b), for W 540 x 1.30 section,
$I = 541 \times 10^{-6} \text{ m}^4$

$EI = (210 \times 10^6 \text{ kN/m}^2)(540 \times 10^{-6} \text{ m}^4) = 1.14 \times 10^5 \text{ kN}\cdot\text{m}^2$

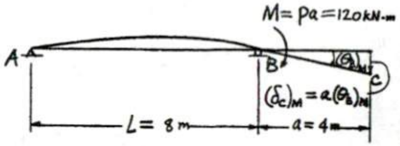
Due to the uniform load between AB:



$$(\delta_C)_w = a(\theta_B)_w = a\frac{wL^3}{24EI}$$

$$= (4 \text{ m})\frac{(60 \text{ kN/m})(8 \text{ m})^3}{24(1.14 \times 10^5 \text{ kN}\cdot\text{m}^2)} = 0.0449 \text{ m} \uparrow$$

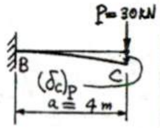
Due to the couple moment M at B:



$$(\delta_C)_M = a(\theta_B)_M = a\frac{ML}{3EI}$$

$$= (4 \text{ m})\frac{(120 \text{ kN}\cdot\text{m})(8 \text{ m})}{3(1.14 \times 10^5 \text{ kN}\cdot\text{m}^2)} = 0.0112 \text{ m} \downarrow$$

Due to P on overhang:



$$(\delta_C)_P = \frac{Pa^3}{3EI} = \frac{(30 \text{ kN})(4 \text{ m})^3}{3(1.14 \times 10^5 \text{ kN}\cdot\text{m}^2)} = 0.0056 \text{ m} \downarrow$$

Total Deflection:

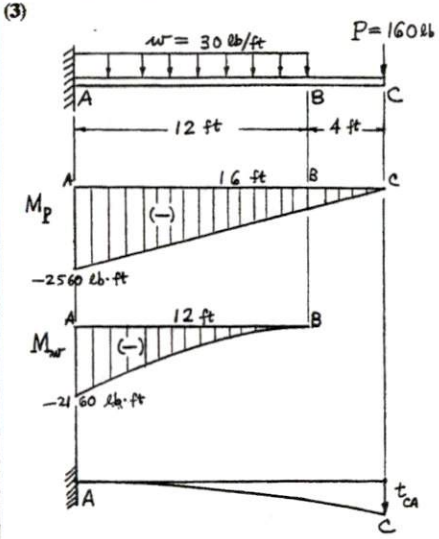
$$\delta_C = (\delta_C)_w + (\delta_C)_M + (\delta_C)_P$$

$$= 0.0449 - 0.0112 - 0.0056 = 0.0281 \text{ m}$$

$$= 28.1 \text{ mm} \downarrow$$ ◀

Solutions to Test Problems for Chapter 16 (Cont'd)

(3)



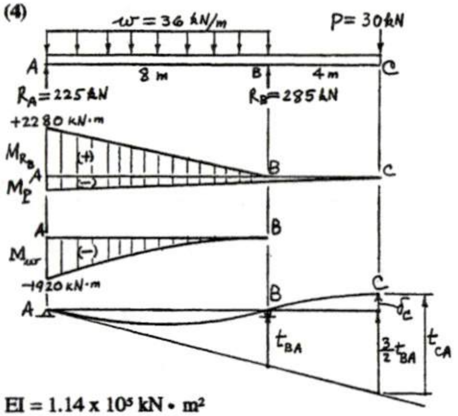
$EI = 2.06 \times 10^6$ lb • ft²

$$\delta_C = t_{CA} = \frac{1}{EI} A_{AC}\, \bar{x}_C$$

$$= \frac{1}{2.06\times10^6}\left[\frac{1}{2}(16)(-2560)\left(\frac{2}{3}\times16\right) + \frac{1}{3}(12)(-2160)\left(\frac{3}{4}\times12+4\right)\right]$$

$$= 0.1606 \text{ ft} = 1.93 \text{ in.}$$ ◀

(4)



$EI = 1.14 \times 10^5$ kN • m²

$$t_{BA} = \frac{1}{EI} A_{AB}\, \bar{x}_B$$

$$= \frac{1}{1.14\times10^5}\left[\frac{1}{2}(8)(2280)\left(\frac{2}{3}\times8\right) + (8)(-120)(4) + \frac{1}{2}(8)(-240)\left(\frac{2}{3}\times8\right) + \frac{1}{3}(8)(-1920)\left(\frac{3}{4}\times8\right)\right]$$

$$= 0.0786 \text{ m}$$

$$t_{CA} = \frac{1}{EI} A_{AC}\, \bar{x}_C$$

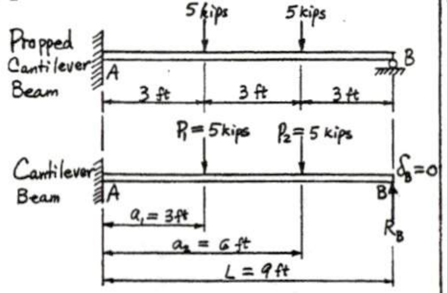
$$= \frac{1}{1.14\times10^5}\left[\frac{1}{2}(8)(2280)\left(\frac{2}{3}\times8+4\right) + \frac{1}{2}(12)(-360)\left(\frac{2}{3}\times12\right) + \frac{1}{3}(8)(-1920)\left(\frac{3}{4}\times8+4\right)\right]$$

$$= 0.146 \text{ m}$$

$$\delta_C = t_{CA} - \frac{3}{2}t_{BA} = 0.146 \text{ m} - \frac{3}{2}(0.0786 \text{ m})$$

$$= 0.0281 \text{ m} = 28.1 \text{ mm}$$ ◀

17-1



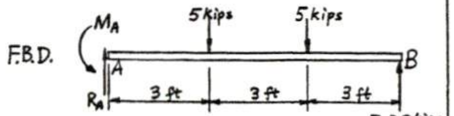
$$\delta_B(+\uparrow) = (\delta_B)_{R_B} - (\delta_B)_{P_1} - (\delta_B)_{P_2}$$

$$= \frac{R_B L^3}{3EI} - \frac{P_1 a_1^2}{6EI}(3L - a_1) - \frac{P_2 a_2^2}{6EI}(3L - a_2)$$

$$= \frac{R_B (9)^3}{3EI} - \frac{5(3)^2}{6EI}(3\times9-3) - \frac{5(6)^2}{6EI}(3\times9-6)$$

$$= \frac{1}{EI}[243 R_B - 180 - 630] = 0$$

$$R_B = \frac{180 + 630}{240} = 3.38 \text{ kips} \uparrow$$ ◀



$$\sum M_A = M_A + 3.38(9) - 5(3) - 5(6) = 0$$

$$M_A = 14.63 \text{ kip}\cdot\text{ft} \curvearrowleft$$ ◀

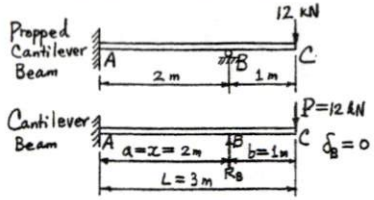
$$\sum M_B = -R_A(9) + 14.63 + 5(6) + 5(3) = 0$$

$$R_A = 6.63 \text{ kip} \uparrow$$ ◀

Check:

$$\sum F_y = 6.63 + 3.38 - 5 - 5 = 0$$ (Checks)

17-2



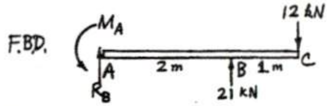
$$\delta_B(+\uparrow) = (\delta_B)_{R_B} - (\delta_B)_P$$

$$= \frac{R_B a^3}{3EI} - \frac{Px^2}{6EI}(3L - x)$$

$$= \frac{R_B (2)^3}{3EI} - \frac{(12)(2)^2}{6EI}[3(3) - 2]$$

$$= \frac{1}{EI}[2.667 R_B - 56] = 0$$

$$R_B = 21 \text{ kN} \uparrow$$ ◀



$$\sum M_A = M_A + 21(2) - 12(3) = 0$$

$$M_A = -6 \text{ kN}\cdot\text{m} \curvearrowright$$ ◀

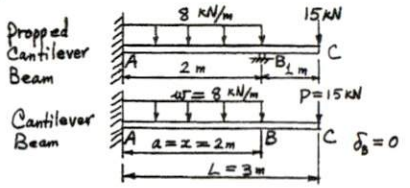
$$\sum M_B = -R_B(2) - 6 - 12(1) = 0$$

$$R_B = -9 \text{ kN} \downarrow$$ ◀

Check:

$$\sum F_y = -9 + 21 - 12 = 0$$ (Check)

17-3



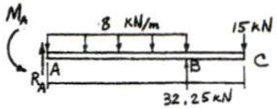
$$\delta_B(+\uparrow) = (\delta_B)_{R_B} - (\delta_B)_w - (\delta_B)_P$$

$$= \frac{R_B a^3}{3EI} - \frac{wa^4}{8EI} - \frac{Px^2}{6EI}(3L - x)$$

$$= \frac{R_B (2)^3}{3EI} - \frac{(8)(2)^4}{8EI} - \frac{(15)(2)^2}{6EI}[3(3) - 2]$$

$$= \frac{1}{EI}\left(\frac{8}{3}R_B - 16 - 70\right) = 0$$

$$R_B = 32.25 \text{ kN} \uparrow$$ ◀



(Cont'd)

17-3 (Cont)

$\sum M_A = M_A - (8x2)(1) - (15)(3) + (32.25)(2) = 0$

$M_A = -3.5\ \text{kN}\cdot\text{m}$ ↻ ◀

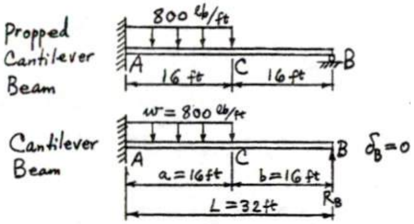
$\sum M_B = -R_A(2) - 3.5 + (8x2)(1) - (15)(1) = 0$

$R_A = -1.25\ \text{kN}$ ↓ ◀

Check:

$\sum F_y = -1.25 - 8(2) + 32.25 - 15 = 0$ (Checks)

17-4



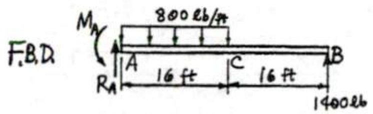
$\delta_B(+\uparrow) = (\delta_B)_{R_B} - (\delta_C)_w - b(\theta_C)_w$

$= \frac{R_B L^3}{3EI} - \frac{wa^4}{8EI} - b\frac{wa^3}{6EI}$

$= \frac{R_B(32)^3}{3EI} - \frac{(800)(16)^4}{8EI} - (16)\frac{(800)(16)^3}{6EI}$

$= \frac{1}{EI}\left(10923R_B - 6.55x10^6 - 8.74x10^6\right) = 0$

$R_B = 1400\ \text{lb}$ ↑ ◀



$\sum M_A = M_A - (800x16)(8) + (1400)(32) = 0$

$M_A = 57\,600\ \text{lb}\cdot\text{ft}$ ↺ ◀

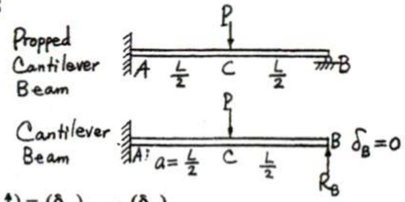
$\sum M_B = -R_A(32) + 57\,600 + (800 \times 16)(8 + 16) = 0$

$R_A = 11\,400\ \text{lb}$ ↑ ◀

Check:

$\sum F_y = 11\,400 - 800 \times 16 + 1400 = 0$ (Checks)

17-5

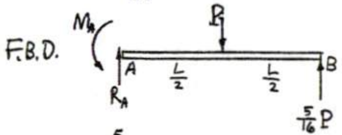


$\delta_B(+\uparrow) = (\delta_B)_{R_B} - (\delta_B)_P$

$= \frac{R_B L^3}{3EI} - \frac{P\left(\frac{L}{2}\right)^2}{6EI}\left[3L - \left(\frac{L}{2}\right)\right]$

$= \frac{L^3}{EI}\left[\frac{R_B}{3} - \frac{5}{48}P\right] = 0$

$R_B = \frac{5}{16}P$ ↑ ◀

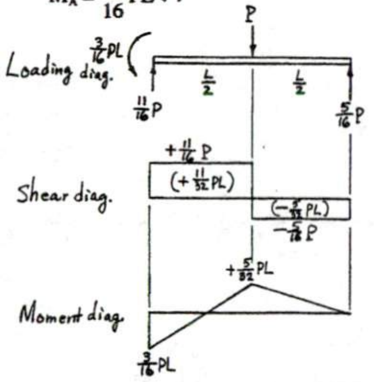


$\sum F_y = R_A - P + \frac{5}{16}P = 0$

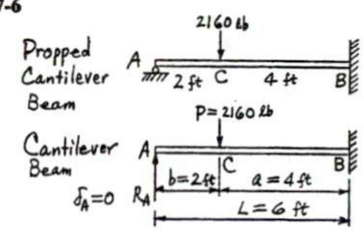
$R_A = \frac{11}{16}P$ ↑ ◀

$\sum M_A = M_A - P(L/2) + \frac{5}{16}P(L) = 0$

$M_A = \frac{3}{16}PL$ ↺ ◀



17-6



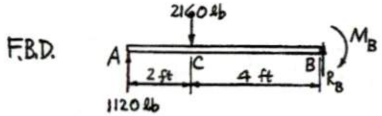
$\delta_A(+\uparrow) = (\delta_A)_{R_A} - (\delta_A)_P$

$= \frac{R_A L^3}{3EI} - \frac{Pa^2}{6EI}(3L - a)$

$= \frac{R_A(6)^3}{3EI} - \frac{(2160)(4)^2}{6EI}[3(6) - 4]$

$= \frac{1}{EI}[72\,R_A - 80\,640] = 0$

$R_A = 1120\ \text{lb}$ ↑ ◀

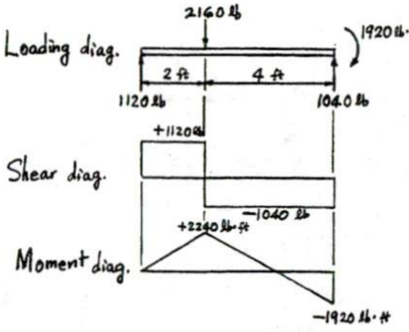


$\sum M_B = -M_B - (1120)(6) + (2160)(4) = 0$

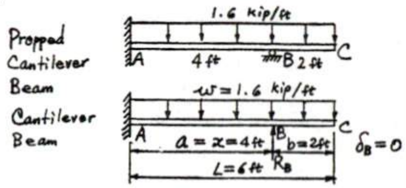
$M_B = +1920\ \text{lb}\cdot\text{ft}$ ↺ ◀

$\sum F_y = R_B + 1120 - 2160 = 0$

$R_B = +1040\ \text{lb}$ ↑ ◀



17-7



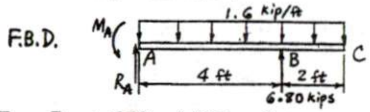
$\delta_B(+\uparrow) = (\delta_B)_{R_B} - (\delta_B)_w$

$= \frac{R_B a^3}{3EI} - \frac{wx^2}{24EI}(x^2 + 6L^2 - 4Lx)$

$= \frac{R_B(4)^3}{3EI} - \frac{(1.6)(4)^2}{24EI}[4^2 + 6(6)^2 - 4(6)(4)]$

$= \frac{1}{EI}(21.33\,R_B - 145.1) = 0$

$R_B = 6.80\ \text{kips}$ ↑ ◀

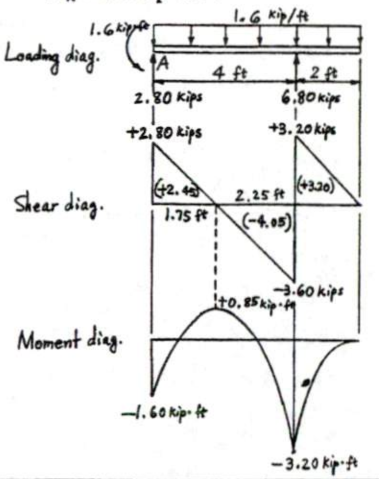


$\sum F_y = R_A + 6.80 - 1.6(6) = 0$

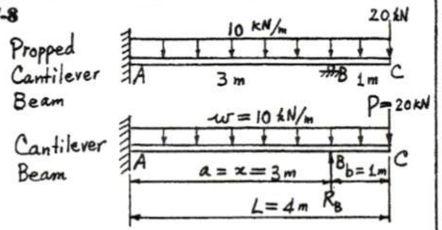
$R_A = 2.80\ \text{kips}$ ↑ ◀

$\sum M_A = M_A + (6.80)(4) - (1.6x6)(3) = 0$

$M_A = 1.60\ \text{kip}\cdot\text{ft}$ ↺ ◀



17-8



$\delta_B(+\uparrow) = (\delta_B)_{R_B} - (\delta_B)_w - (\delta_B)_P$

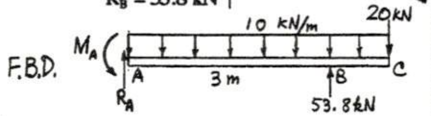
$= \dfrac{R_B a^3}{3EI} - \dfrac{wx^2}{24EI}(x^2 - 4Lx + 6L^2) - \dfrac{Px^2}{6EI}[3L - x]$

$= \dfrac{R_B(3)^3}{3EI} - \dfrac{(10)(3)^2}{24EI}[3^2 + 4(4)(3) + (6)(4)^2]$

$- \dfrac{(20)(3)^2}{6EI}[3(4) - 3]$

$= \dfrac{1}{EI}(9\,R_B - 214 - 270) = 0$

$R_B = 53.8$ kN ↑ ◀

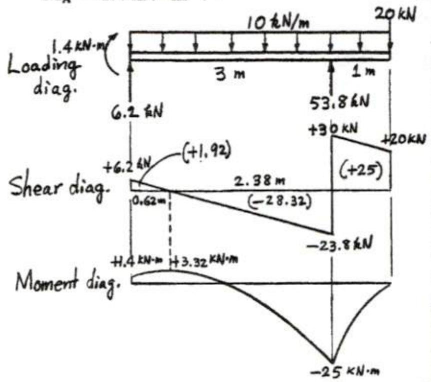


$\sum F_y = R_A - (10)(4) - 20 + 53.8 = 0$

$R_A = 6.2$ kN ↑ ◀

$\sum M_A = M_A - (10x4)(2) - 20(4) + 53.8(3) = 0$

$M_A = -1.4$ kN•m ↺ ◀



17-9

From Example 17-1,

$V_{max} = \dfrac{5}{8}wL = \dfrac{5}{8}(2.5 \text{ kip/ft})(20 \text{ ft}) = 31.25$ kips

$|M_{max}| = \dfrac{wL^2}{8} = \dfrac{(2.5 \text{ kip/ft})(20 \text{ ft})^2}{8}$

$= 125$ kip• ft $= 1500$ kip• in.

$S_{req} = \dfrac{M_{max}}{\sigma_{allow}} = \dfrac{1500 \text{ kip} \cdot \text{in.}}{2400 \text{ kip/in.}^2} = 62.5 \text{ in.}^3$

From Appendix Table A-1(a):
W 18 x 46 (S = 78.8 in.^{3})
W 16 x 50 (S = 81.0 in.^{3})
W 14 x 43 (S = 62.7 in.^{3})

Try W 14 x 43 (d = 13.66 in., t_w = 0.305 in.)

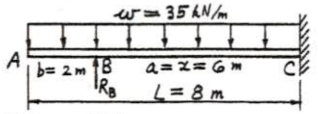
$\tau_{avg} = \dfrac{V_{max}}{dt_w} = \dfrac{31.25 \text{ kips}}{(13.66 \text{ in.})(0.305 \text{ in.})}$

$= 7.50 \text{ ksi} < \tau_{allow} = 14.5$ ksi

The section is satisfactory for shear.

Use W 14 x 43 ◀

17-10



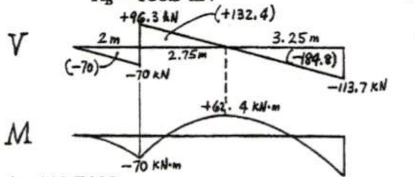
$\delta_B(+\uparrow) = (\delta_B)_{R_B} - (\delta_B)_w$

$= \dfrac{R_B a^3}{3EI} - \dfrac{wx^2}{24EI}(x^2 - 4Lx + 6L^2)$

$= \dfrac{R_B(6)^3}{3EI} - \dfrac{(35)(6)^2}{24EI}[6^2 - 4(8)(6) - (6)(8)^2]$

$= \dfrac{1}{EI}(72\,R_B - 11\,970) = 0$

$R_B = 166.3$ kN ◀



$|V_{max}| = 113.7$ kN

$|M_{max}| = 122.4$ kN•m

$S_{req} = \dfrac{M_{max}}{\sigma_{allow}} = \dfrac{122.4 \text{ kN} \cdot \text{m}}{165\,000 \text{ kN/m}^2} = 0.742 \times 10^{-3} \text{ m}^3$

(Cont'd)

17-10 (Cont)

From Appendix Table A-1(b):
W 410 x 0.53 (S = 0.742 x 10^{-3} m^3)
W 360 x 0.50 (S = 0.797 x 10^{-3} m^3)
W 300 x 0.51 (S = 0.747 x 10^{-3} m^3)

Try W 360 x 0.50 (d = 0.355 m, t_w = 0.00724 m)

$\tau_{avg} = \dfrac{V_{max}}{dt_w} = \dfrac{113.7 \text{ kN}}{(0.355 \text{ m})(0.00724 \text{ m})}$

$= 44\,200 \text{ kPa} = 44.2 \text{ MPa} < \tau_{allow} = 100$ MPa

The section is satisfactory for shear.

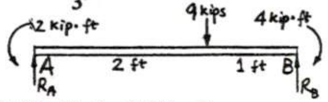
Use W 360 x 0.50 ◀

17-11

From Example 17-5, the end moments are

$M_A = \dfrac{Pab^2}{L^2} = \dfrac{9(2)(1)^2}{3^2} = 2$ kip • ft ◀

$M_B = \dfrac{Pa^2b}{L^2} = \dfrac{9(2)^2(1)}{3^2} = 4$ kip • ft ◀



$\sum M_B = -R_A(3) + 2 - 4 + (9)(1) = 0$

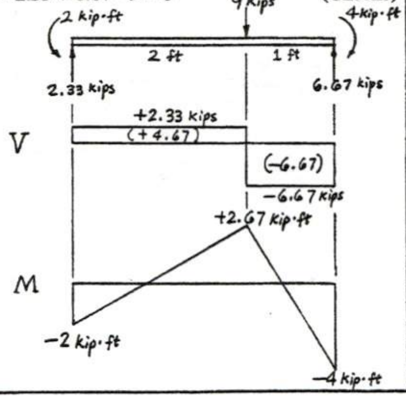
$R_A = 2.33$ kips ↑ ◀

$\sum M_A = R_B(3) + 2 - 4 - (9)(2) = 0$

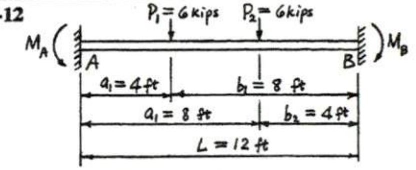
$R_B = 6.67$ kips ↑ ◀

Check:

$\sum F_y = 2.33 + 6.67 - 9 = 0$ (Checks)



17-12



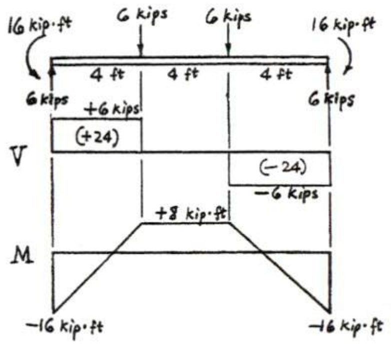
From Example 17-5, the end moments are

$M_A = \dfrac{P_1 a_1 b_1^2}{L^2} + \dfrac{P_2 a_2 b_2^2}{L^2} = \dfrac{6(4)(8)^2}{12^2} + \dfrac{6(8)(4)^2}{12^2}$

$= 16$ kip• ft ◀

From symmetry, $M_B = M_A = 16$ kip• ft ◀

$R_A = R_B = P = 6$ kips ◀



17-13

From Examples 17-3 and 17-5:

$R_A = R_B = \dfrac{wL}{2} + \dfrac{P}{2}$

$= \dfrac{(2 \text{ kN/m})(6 \text{ m})}{2} + \dfrac{10 \text{ kN}}{2}$

$= 11$ kN ↑ ◀

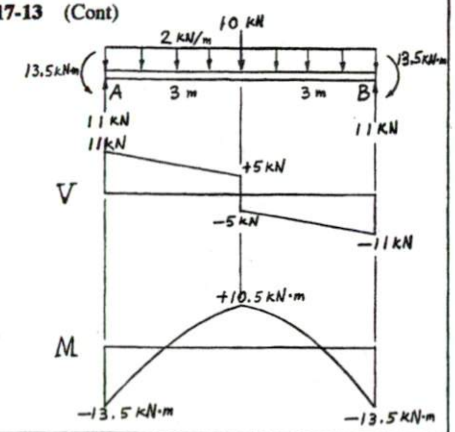
$M_A = M_B = \dfrac{wL^2}{12} + \dfrac{PL}{8}$

$= \dfrac{(2 \text{ kN/m})(6 \text{ m})^2}{12} + \dfrac{(10 \text{ kN})(6 \text{ m})}{8}$

$= 13.5$ kN• m ◀

(Cont'd)

17-13 (Cont)



17-14

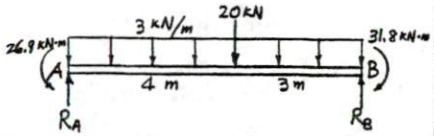
From Examples 17-3 and 17-5, the fixed end moments are

$$M_A = -\frac{wL^2}{12} - \frac{Pab^2}{L^2} = -\frac{(3)(7)^2}{12} - \frac{(20)(4)(3)^2}{7^2}$$

$= -26.9$ kN• m ◀

$$M_B = -\frac{wL^2}{12} - \frac{Pa^2b}{L^2} = -\frac{(3)(7)^2}{12} - \frac{(20)(4)^2(3)}{7^2}$$

$= -31.8$ kN• m ◀



$\sum M_B = -R_A(7) + 26.9 - 31.8 + (3x7)(3.5) + 20(3) = 0$

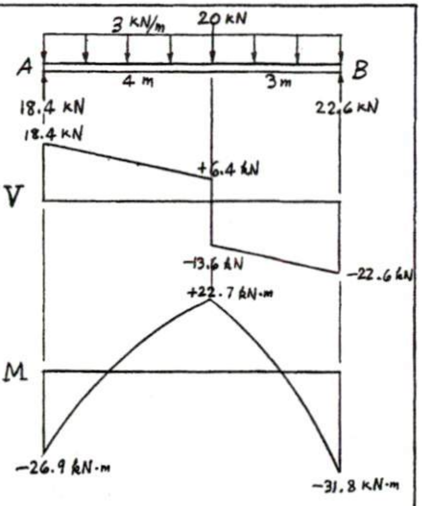
$R_A = 18.4$ kN ◀

$\sum M_A = -R_B(7) + 26.9 - 31.8 - (3x7)(3.5) - 20(4) = 0$

$R_B = 22.6$ kN ◀

Check:

$\sum F_y = 18.4 + 22.6 - 3x7 - 20 = 0$ (Checks)



17-15

From Example 17-5,

$$V_{max} = \frac{P}{2} = \frac{45 \text{ kN}}{2} = 22.5 \text{ kN}$$

$$M_{max} = \frac{PL}{8} = \frac{(45 \text{ kN})(8 \text{ m})}{8} = 45 \text{ kN• m}$$

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{45 \text{ kN•m}}{165\ 000 \text{ kN/m}^2} = 0.273 \times 10^{-3} \text{ m}^3$$

From Appendix Table A-1(b):
W 250 x 0.32 ($S = 0.380 \times 10^{-3}$ m³)
W 200 x 0.31 ($S = 0.298 \times 10^{-3}$ m³)

Try W 200 x 0.31 (d = 0.210 m, $t_w = 0.00635$ m)

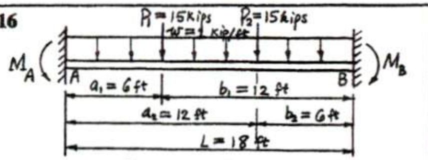
$$\tau_{avg} = \frac{V_{max}}{dt_w} = \frac{22.5 \text{ kN}}{(0.210 \text{ m})(0.00635 \text{ m})}$$

$= 16\ 900$ kPa $= 16.9$ MPa $< \tau_{allow} = 100$ MPa

The section is satisfactory for shear.

Use W 200 x 0.31 ◀

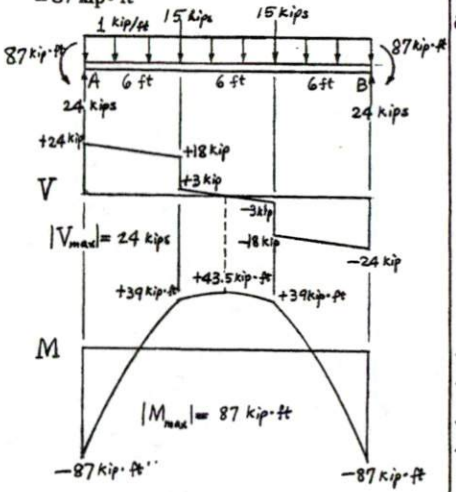
17-16



$$|M_A| = |M_B| = \frac{wL^2}{12} + \frac{P_1a_1b_1^2}{L^2} + \frac{P_2a_2b_2^2}{L^2}$$

$$= \frac{(1)(18)^2}{12} + \frac{15(6)(12)^2}{18^2} + \frac{15(12)(6)^2}{18^2}$$

$= 87$ kip• ft



$$S_{req} = \frac{87 \times 12}{24} = 43.5 \text{ in.}^3$$

From Appendix Table A-1(a):
W 16 x 36 (S = 56.5 in.³)
W 14 x 34 (S = 48.6 in.³)
W 12 x 35 (S = 45.6 in.³)

Try W 14 x 34 (d = 13.98 in., $t_w = 0.285$ in.)

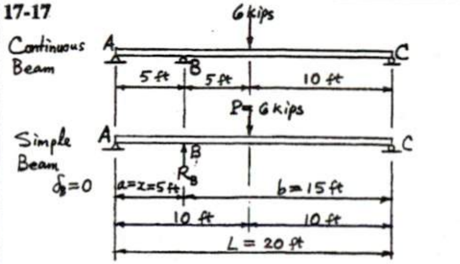
$$\tau_{avg} = \frac{V_{max}}{dt_w} = \frac{24 \text{ kips}}{(13.98 \text{ in.})(0.285 \text{ in.})}$$

$= 6.02$ ksi $< \tau_{allow} = 14.5$ ksi

The section is satisfactory for shear.

Use W 14 x 34 ◀

17-17



$\delta_B(+\uparrow) = (\delta_B)_{R_B} - (\delta_B)_P$

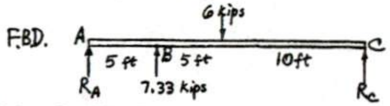
$$= \frac{R_Bbx}{6EIL}(L^2 - x^2 - b^2) - \frac{Px}{12EI}\left(\frac{3L^2}{4} - x^2\right)$$

$$= \frac{R_B(15)(5)}{6EI(20)}(20^2 - 5^2 - 15^2) - \frac{6(5)}{12EI}\left(\frac{3x20^2}{4} - 5^2\right)$$

$$= \frac{1}{EI}(93.75 R_B - 687.5) = 0$$

$R_B = 7.33$ kips ↑ ◀

F.B.D.



$\sum M_C = -R_A(20) - 7.33(15) + 6(10) = 0$

$R_A = -2.50$ kips ↓ ◀

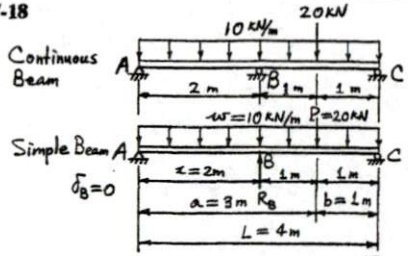
$\sum M_A = R_C(20) + 7.33(5) - 6(10) = 0$

$R_C = +1.17$ kips ↑ ◀

Check:

$\sum F_y = -2.50 + 7.33 + 1.17 - 6 = 0$ (Checks)

17-18



(Cont'd)

17-18 (Cont)

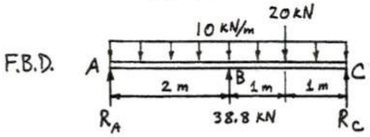
$\delta_B\,(+\uparrow) = (\delta_B)_{R_B} - (\delta_B)_w - (\delta_B)_P$

$$= \frac{R_B L^3}{48EI} - \frac{5wL^4}{384EI} - \frac{Pbx}{6EIL}(L^2 - x^2 - b^2)$$

$$= \frac{R_B(4)^3}{48EI} - \frac{5(10)(4)^4}{384EI} - \frac{(20)(1)(2)}{6EI(4)}(4^2 - 2^2 - 1^2)$$

$$= \frac{1}{EI}(1.333\,R_B - 51.67) = 0$$

$R_B = 38.8\text{ kN}\uparrow$ ◀



$\sum M_C = -R_A(4) + (10\text{x}4)(2) - (38.8)(2) + (20)(1) = 0$

$R_A = +5.6\text{ kN}\uparrow$ ◀

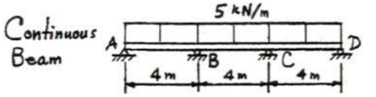
$\sum M_A = R_C(4) - (10\text{x}4)(2) + (38.8)(2) - (20)(3) = 0$

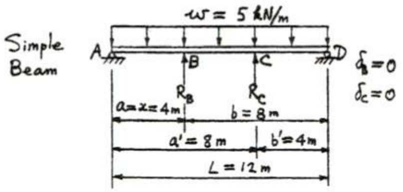
$R_C = +15.6\text{ kN}\uparrow$ ◀

Check:

$\sum F_y = 5.6 + 38.8 + 15.6 - 10(4) - 20 = 0$ (Checks)

17-19





$\delta_B\,(+\uparrow) = (\delta_B)_{R_B} + (\delta_B)_{R_C} - (\delta_B)_w$

$$= \frac{R_B bx}{6EIL}(L^2 - x^2 - b^2) - \frac{R_C b'x}{6EIL}(L^2 - x^2 - b'^2)$$

$$- \frac{wx}{24EI}(L^3 + x^3 - 2Lx^2)$$

$$= \frac{R_B(8)(4)}{6EI(12)}[(12)^2 - (4)^2 - (8)^2]$$

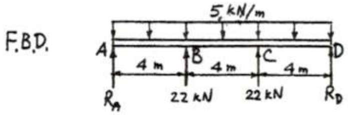
$$+ \frac{R_C(4)(4)}{6EI(12)}[(12)^2 - (4)^2 - (4)^2]$$

$$- \frac{(5)(4)}{24EI}[(12)^3 + (4)^3 - 2(12)(4)^2]$$

$$= \frac{1}{EI}(28.44\,R_B + 24.89\,R_C - 1173) = 0$$

Due to Symmetry, $R_B = R_C$

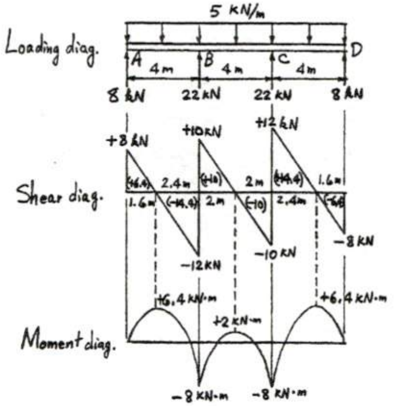
$R_B = R_C = 22.0\text{ kN}\uparrow$ ◀



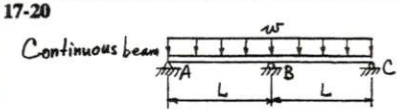
$\sum F_y = R_A + R_D + 22 + 22 - 5(12) = 0$

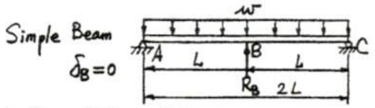
Due to Symmetry:

$R_A = R_D = 8\text{ kN}\uparrow$ ◀



17-20



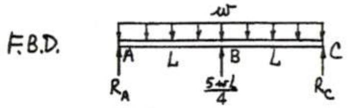


$\delta_B\,(+\uparrow) = (\delta_B)_{R_B} - (\delta_B)_w$

$$= \frac{R_B(2L)^3}{48EI} - \frac{5w(2L)^4}{384EI}$$

$$= \frac{L^3}{48EI}(8\,R_B - 10\,wL) = 0$$

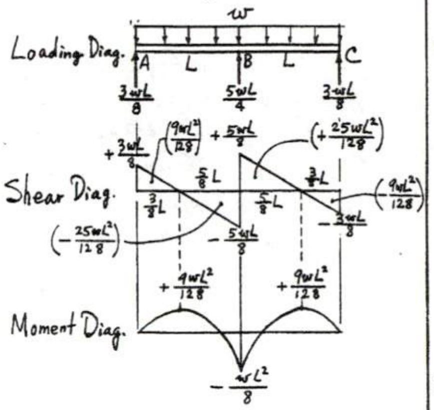
$R_B = \frac{5wL}{4}\uparrow$ ◀



$\sum F_y = R_A + R_C + \frac{5wL}{4} - 2wL = 0$

From Symmetry: $R_A = R_C$

$R_A = R_C = \frac{3wL}{8}\uparrow$ ◀



17-21

From the results in Prob 17-20:

$$V_{max} = \frac{5wL}{8} = \frac{5(2.5\text{ kip/ft})(12\text{ ft})}{8} = 18.75\text{ kips}$$

$$|M_{max}| = \frac{wL^2}{8} = \frac{(2.5\text{ kip/ft})(12\text{ ft})^2}{8}$$

$= 45\text{ kip}\cdot\text{ft} = 540\text{ kip}\cdot\text{in.}$

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{540\text{ kip}\cdot\text{in.}}{24\text{ kip/in.}^2} = 22.5\text{ in.}^3$$

From Appendix Table A-1(a):
W 12 x 22 ($S = 25.4\text{ in}^3$)
W 10 x 22 ($S = 23.2\text{ in}^3$)
W 8 x 28 ($S = 24.3\text{ in}^3$)

Try W 12 x 22 ($d = 12.31$ in., $t_w = 0.260$ in.)

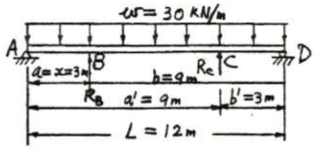
$$\tau_{avg} = \frac{V_{max}}{dt_w} = \frac{18.75\text{ kips}}{(12.31\text{ in.})(0.260\text{ in.})}$$

$= 5.86\text{ ksi} < \tau_{allow} = 14.5\text{ ksi}$

The section is satisfactory for shear.

Use W 12 x 22 ◀

17-22



$\delta_B\,(+\uparrow) = (\delta_B)_{R_B} + (\delta_B)_{R_C} - (\delta_B)_w$

$$= \frac{R_B bx}{6EIL}(L^2 - x^2 - b^2) + \frac{R_C b'x}{6EIL}(L^2 - x^2 - b'^2)$$

$$- \frac{wx}{24EI}(L^3 + x^3 - 2Lx^2)$$

$$= \frac{R_B(9)(3)}{6EI(12)}[(12)^2 - (3)^2 - (9)^2]$$

$$+ \frac{R_C(3)(3)}{6EI(12)}[(12)^2 - (3)^2 - (3)^2]$$

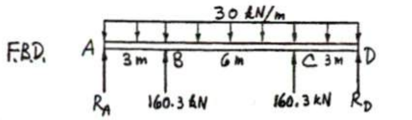
$$- \frac{(30)(3)}{24EI}[(12)^3 + (3)^3 - 2(12)(3)^2]$$

$$= \frac{1}{EI}(20.25\,R_B + 15.75\,R_C - 5771) = 0$$

Due to Symmetry, $R_B = R_C = 160.3$ kN

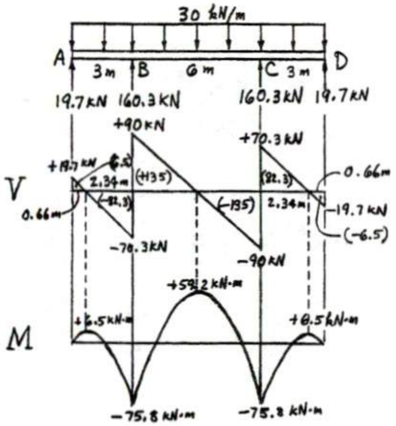
(Cont'd)

17-22 (Cont)



$\sum F_y = R_A + R_D + 2(160.3) - 30(12) = 0$

Due to Symmetry, $R_A = R_D = 19.7$ kN



$V_{max} = 90$ kN

$|M_{max}| = 75.8$ kN·m

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{75.8 \text{ kN}\cdot\text{m}}{165\,000 \text{ kN/m}^2} = 0.459 \times 10^{-3} \text{ m}^3$$

From Appendix Table A-1(b):

W 410 x 0.38 ($S = 0.629 \times 10^{-3}$ m³)

W 360 x 0.44 ($S = 0.688 \times 10^{-3}$ m³)

W 300 x 0.44 ($S = 0.633 \times 10^{-3}$ m³)

Try W 410 x 0.38 ($d = 0.399$ m, $t_w = 0.00635$ m)

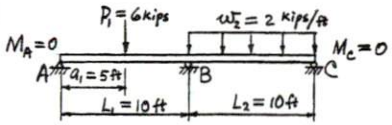
$$\tau_{avg} = \frac{V_{max}}{dt_w} = \frac{90 \text{ kN}}{(0.399 \text{ m})(0.00635 \text{ m})}$$

$= 35\,500$ kPa $= 35.5$ MPa $< \tau_{allow} = 100$ MPa

The section is satisfactory for shear.

Use W 410 x 0.38 ◄

17-23

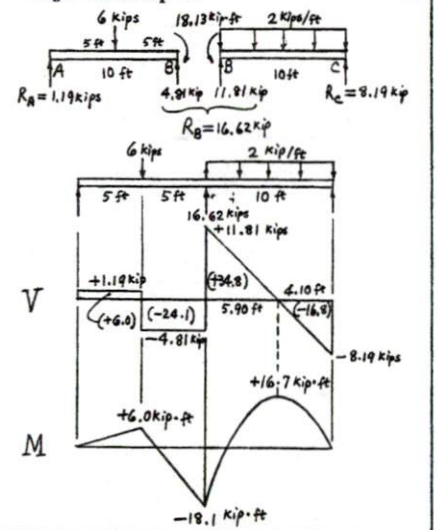


The three moment equation for the two spans is

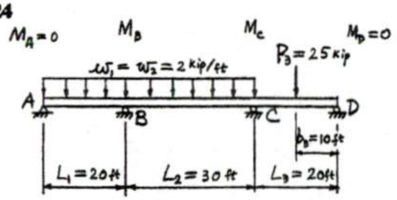
$$0 + 2M_B(10+10) + 0 = \frac{6(5)(10^2 - 5^2)}{10} - \frac{2(10)^3}{4}$$

$40 M_B = -725$

$M_B = -18.13$ kip·ft ◄



17-24



The three moment equation for spans 1 and 2 is

$$0 + 2M_B(20+30) + M_C(30) = \frac{(2)(20)^3}{4} - \frac{(2)(30)^3}{4}$$

$100 M_B + 30 M_C = -17\,500$ (a)

Cont'd)

17-24 (Cont)

The three moment equation for spans 2 and 3 is

$$M_B(30) + 2M_C(30+20) + 0 = -\frac{(2)(30)^3}{4} - \frac{(25)(10)(20^2 - 10^2)}{20}$$

$30 M_B + 100 M_C = -17\,250$ (b)

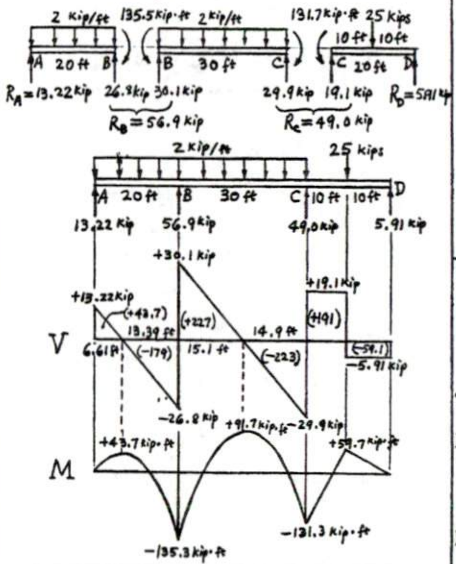
(a)/0.3 - (b): $303.3 M_B + 0 = -41\,083$

$M_B = -135.5$ kip·ft ◄

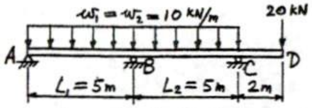
From (a):

$M_C = [-17\,500 - 100(-135.5)]/30$

$= -131.7$ kip·ft ◄



17-25

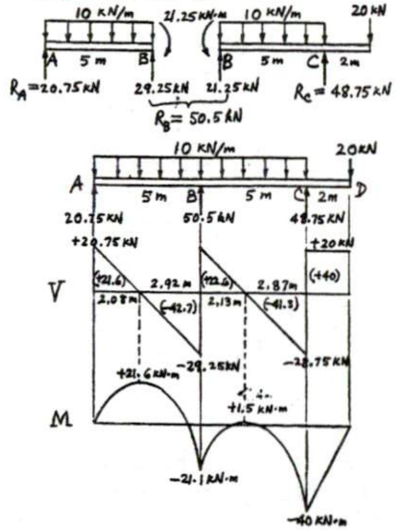


The three moment equation for the two spans is

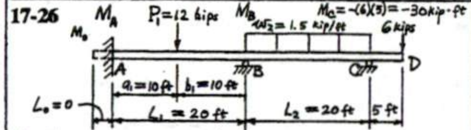
$$0 + 2M_B(5+5) + (-40)(5) = -\frac{10(5)^3}{4} - \frac{10(5)^3}{4}$$

$20 M_B - 200 = -625$

$M_B = -21.25$ kN·m ◄



17-26



The three moment equations are

$$M_O(0) + 2M_A(0+20) + M_B(20) = -\frac{(12)(10)(20^2 - 10^2)}{20}$$

$40 M_A + 20 M_B = -1800$

$2 M_A + M_B = -90$ (a)

$$M_A(20) + 2M_B(20+20) + (-30)(20) = -\frac{(12)(10)(20^2 - 10^2)}{20} - \frac{(1.5)(20)^3}{4}$$

$20 M_A + 80 M_B = 600 - 4800$

$M_A + 4 M_B = -210$ (b)

2(b) - (a): $7 M_B = 2(-210) - (-90) = -330$

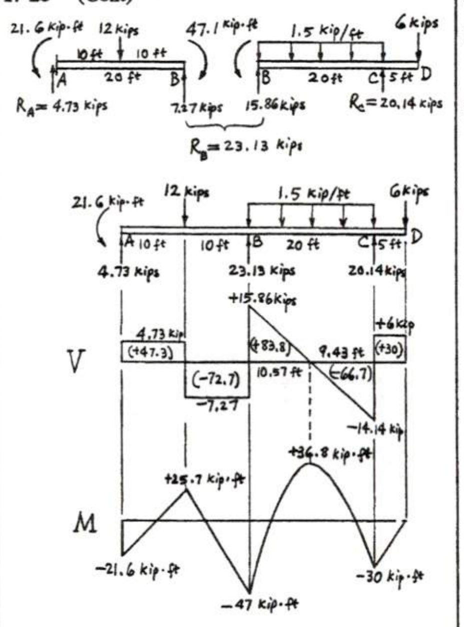
$M_B = -47.1$ kip·ft ◄

From (b):

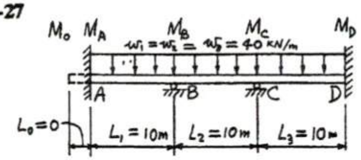
$M_A = -210 - 4(-47.1) = -21.6$ kip·ft ◄

(Cont'd)

17-26 (Cont)



17-27



Due to Symmetry, $M_A = M_D$, $M_B = M_C$

$$M_O(0) + 2M_A(0+10) + M_B(10) = -\frac{(40)(10)^3}{4}$$

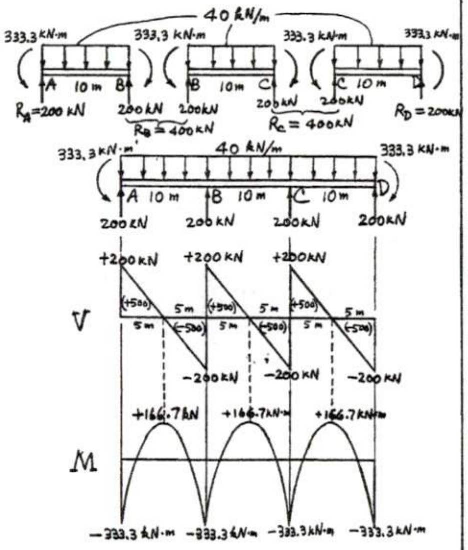
$$20M_A + 10M_B = -10\,000$$
$$2M_A + M_B = -1000 \qquad \text{(a)}$$

$$M_A(10) + 2M_B(10+10) + M_C(10) = -\frac{(40)(10)^3}{4} - \frac{(40)(10)^3}{4}$$

$$10M_A + 40M_B + 10M_C = -20\,000$$
$$10M_A + 50M_B = -20\,000$$
$$M_A + 5M_B = -2000 \qquad \text{(b)}$$

2(b) - (a): $9M_B = -3000$
$M_B = -333.3$ kN • m

From (b):
$M_A = -2000 - 5(-333.3) = -333.3$ kN • m



$V_{max} = 200$ kN
$|M_{max}| = 333.3$ kN • m

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{333.3 \text{ kN}\cdot\text{m}}{165\,000 \text{ kN/m}^2} = 2.02 \times 10^{-3} \text{ m}^3$$

From Appendix Table A-1(b):
W 530 x 0.90 ($S = 2.08 \times 10^{-3}$ m³)
W 410 x 1.30 ($S = 2.54 \times 10^{-3}$ m³)
W 360 x 1.20 ($S = 2.02 \times 10^{-3}$ m³)

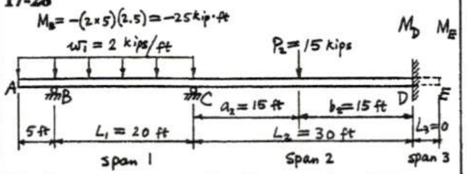
Try W 530 x 0.90 ($d = 0.533$ m, $t_w = 0.0102$ m)

$$\tau_{avg} = \frac{V_{max}}{dt_w} = \frac{200 \text{ kN}}{(0.533 \text{ m})(0.0102 \text{ m})}$$
$$= 36\,800 \text{ kPa} = 36.8 \text{ MPa} < \tau_{allow} = 100 \text{ MPa}$$

The section is satisfactory for shear.

Use W 530 x 0.90 ◀

17-28



The three moment equation for spans 1 and 2 is

$$(-25)(20) + 2M_C(20+30) + M_D(30) = -\frac{(2)(20)^3}{4} - \frac{(15)(15)(30^2 - 15^2)}{30}$$

$$100M_C + 30M_D = -8560 \qquad \text{(a)}$$

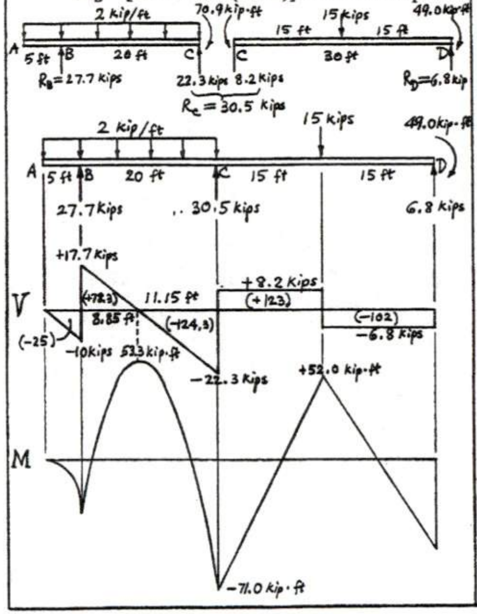
The three moment equation for spans 2 and 3 is

$$M_C(30) + 2M_D(30+0) + M_E(0) = -\frac{(15)(15)(30^2 - 15^2)}{30}$$

$$30M_C + 60M_D = -5060 \qquad \text{(b)}$$

2(a) - (b): $170M_C = -12\,060$
$M_C = -70.9$ kip • ft

From (a):
$M_D = [-8560 - 100(-70.9)]/30 = -49.0$ kip • ft



$|V_{max}| = 22.3$ kips
$|M_{max}| = 71.0$ kip • ft $= 852$ kip • in.

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{852 \text{ kip-in.}}{24 \text{ kip/in}^2} = 35.5 \text{ in}^3$$

From Appendix Table A-1(a):
W 16 x 26 ($S = 38.4$ in.³)
W 14 x 30 ($S = 42.0$ in.³)
W 12 x 30 ($S = 38.6$ in.³)

Try W 16 x 26 ($d = 15.69$ in., $t_w = 0.250$ in.)

$$\tau_{avg} = \frac{V_{max}}{dt_w} = \frac{22.3 \text{ kips}}{(15.69 \text{ in.})(0.250 \text{ in.})}$$
$$= 5.69 \text{ ksi} < \tau_{allow} = 14.5 \text{ ksi}$$

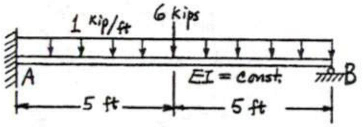
The section is satisfactory for shear.

Use W 16 x 26 ◀

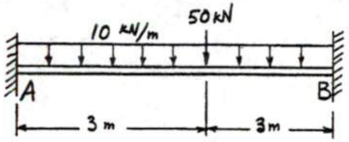
Test Problems for Chapter 17

The following problems may either be given to students for them to practice their problem solving skills or be used as test problems. The answers are provided on the next page.

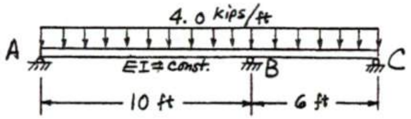
(1) The propped cantilever beam is subjected to the loads shown. Find the reactions at the supports and plot the shear force and bending moment diagrams for the beam.



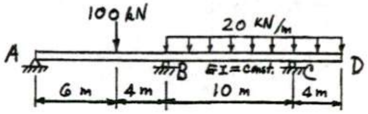
(2) The fixed beam is subjected to the loads shown. The uniform load includes the weight of the beam. Select the lightest W-shape for the beam. Assume that the allowable flexural stress is 165 MPa and the allowable shear stress is 100 MPa.



(3) The two-span continuous beam is subjected to a uniform load shown. Find the reactions at the supports.

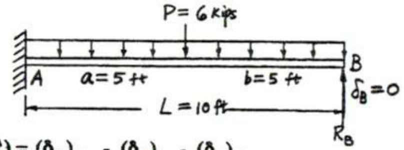


(4) The continuous beam is subjected to the loads shown. Use the three-moment theorem to determine the reactions at the supports and plot the shear force and bending moment diagrams for the beam.



Solutions to Test Problems for Chapter 17

(1)



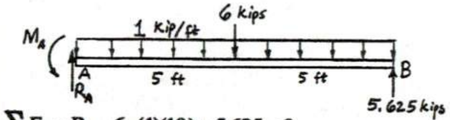
$$\delta_B(+\uparrow) = (\delta_B)_{R_B} - (\delta_B)_w - (\delta_B)_P$$

$$= \frac{R_B L^3}{3EI} - \frac{wL^4}{8EI} - \frac{Pa^2}{6EI}(3L - a)$$

$$= \frac{R_B(10)^3}{3EI} - \frac{(1)(10)^4}{8EI} - \frac{(6)(5)^2}{6EI}[3(10) - 5]$$

$$= \frac{1}{EI}(333.3R_B - 1250 - 625) = 0$$

$R_B = 5.625$ kips ↑ ◄

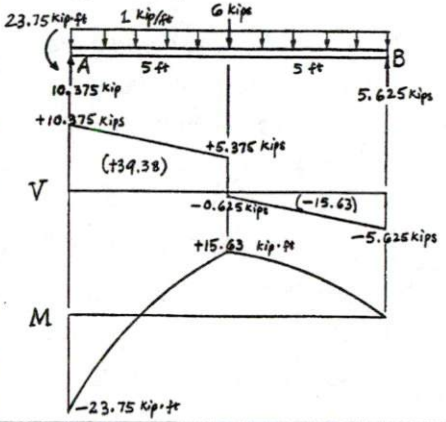


$\sum F_y = R_A - 6 - (1)(10) + 5.625 = 0$

$R_A = 10.375$ kips ↑ ◄

$\sum M_A = M_A - (1x10)(5) - (6)(5) + (5.625)(10) = 0$

$M_A = 23.75$ kip • ft ◄



(2)

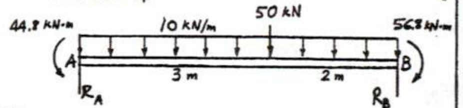
From Examples 17-3 and 17-5:

$$M_A = -\frac{wL^2}{12} - \frac{Pab^2}{L^2} = -\frac{(10)(5)^2}{12} - \frac{(50)(3)(2)^2}{5^2}$$

$= -44.8$ kN • m ◄

$$M_B = -\frac{wL^2}{12} - \frac{Pa^2b}{L^2} = -\frac{(10)(5)^2}{12} - \frac{(50)(3)^2(2)}{5^2}$$

$= -56.8$ kN • m ◄



$\sum M_B = -R_A(5) + 44.8 - 56.8 + (10x5)(2.5) + 50(2) = 0$

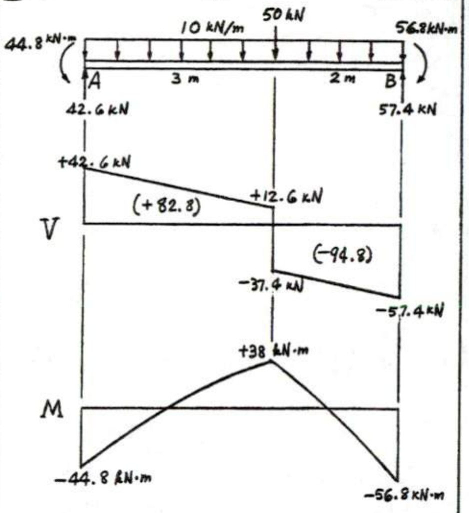
$R_A = 42.6$ kN ↑ ◄

$\sum M_A = -R_B(5) + 44.8 - 56.8 - (10x5)(2.5) - 50(3) = 0$

$R_B = 57.4$ kN ↑ ◄

Check:

$\sum F_y = 42.6 + 57.4 - 10(5) - 50 = 0$ (Checks)



$|V_{max}| = 57.4$ kN
$|M_{max}| = 56.8$ kN • m

(Cont'd)

Solutions to Test Problems for Chapter 17 (Cont'd)

(2) (Cont)

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{56.8 \text{ kN}\cdot\text{m}}{165\,000 \text{ kN/m}^2} = 0.344 \times 10^{-3} \text{ m}^3$$

From Appendix Table A-1(b):
W 300 x 0.32 ($S = 0.416 \times 10^{-3}$ m³)
W 250 x 0.32 ($S = 0.380 \times 10^{-3}$ m³)
W 200 x 0.41 ($S = 0.398 \times 10^{-3}$ m³)

Try W 300 x 0.32 ($d = 0.313$ m, $t_w = 0.00660$ m)

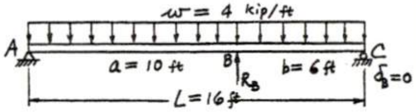
$$\tau_{avg} = \frac{V_{max}}{d t_w} = \frac{57.4 \text{ kN}}{(0.313 \text{ m})(0.00660 \text{ m})}$$

$= 27\,800 \text{ kPa} = 27.8 \text{ MPa} < \tau_{allow} = 100 \text{ MPa}$

The section is satisfactory for shear.

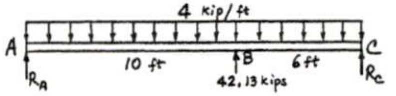
Use W 300 x 0.32 ◀

(3)



$$\delta_B(+\uparrow) = (\delta_B)_{R_B} - (\delta_B)_w$$
$$= \frac{R_B bx}{6EIL}(L^2 - x^2 - b^2) - \frac{wx}{24EI}(L^3 + x^3 - 2Lx^2)$$
$$= \frac{R_B(6)(10)}{6EI(16)}(16^2 - 10^2 - 6^2)$$
$$- \frac{(4)(10)}{24EI}(16^3 + 10^3 - 2 \times 16 \times 10^2)$$
$$= \frac{1}{EI}(75 R_B - 3160) = 0$$

$R_B = 42.13$ kips ↑ ◀



$\sum M_C = -R_A(16) - (42.13)(6) + (4 \times 16)(8) = 0$

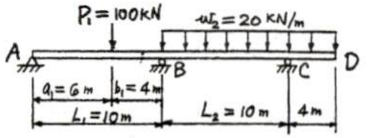
$R_A = 16.2$ kips ◀

$\sum M_A = R_C(16) + (42.13)(10) - (4 \times 16)(8) = 0$

$R_C = 5.67$ kips ◀

Check:
$\sum F_y = 16.2 + 42.13 + 5.67 - (4)(16) = 0$ (Checks)

(4)

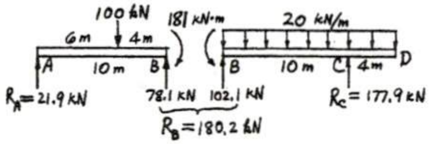


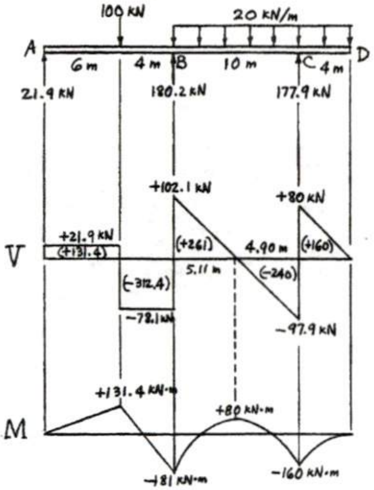
The moment at the external support C is
$M_C = -(20 \times 4)(2) = -160$ kN·m

The three moment equation for the two span is
$$0 + 2M_B(10 + 10) + (-160)(10) = -\frac{20(10)^3}{4} - \frac{(100)(6)(10^2 - 6^2)}{10}$$

$40 M_B - 1600 = -8840$

$M_B = -181$ kN·m ◀





18-1

From Appendix Table A-1(a), for W16 x 50:
$A = 14.7$ in.², $S_x = 81.0$ in.³

Loading:
Axial tensile force: $P = 10$ kips
Bending moment :

$$M = \frac{wL^2}{8} = \frac{(2 \text{ kip/ft})(20 \text{ ft})^2}{8}$$
$= 100$ kip·ft $= 1200$ kip·in.

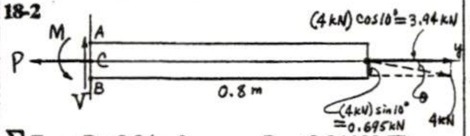
$$\sigma_{max}^{(T)} = +\frac{P}{A} + \frac{M}{S_x} = \frac{10 \text{ kips}}{14.7 \text{ in.}^2} + \frac{1200 \text{ kip}\cdot\text{in.}}{81.0 \text{ in.}^3}$$

$= 0.680$ ksi $+ 14.82$ ksi $= 15.5$ ksi ◀

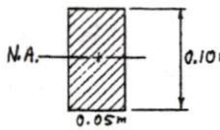
$$\sigma_{max}^{(C)} = +\frac{P}{A} - \frac{M}{S_x}$$

$= 0.680$ ksi $- 14.82$ ksi $= -14.14$ ksi ◀

18-2



$\sum F_x = -P + 3.94 = 0,$ $P = +3.94$ kN (T)
$\sum F_y = V - 0.695 = 0,$ $V = 0.695$ kN
$\sum M_C = M - 0.695(0.8) = 0,$ $M = 0.556$ kN·m



$A = (0.05)(0.10) = 0.005$ m²

$$S = \frac{(0.05)(0.10)^2}{6} = 8.33 \times 10^{-5} \text{ m}^3$$

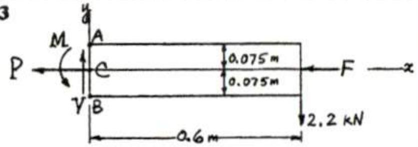
$$\sigma_A = +\frac{P}{A} + \frac{M}{S} = \frac{3.94 \text{ kN}}{0.005 \text{ m}^2} + \frac{0.556 \text{ kN}\cdot\text{m}}{8.33 \times 10^{-5} \text{ m}^3}$$
$= +788$ kPa $+ 6675$ kPa $= +7463$ kPa

$= 7.46$ MPa (T) ◀

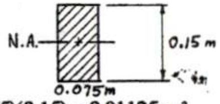
$$\sigma_B = +\frac{P}{A} - \frac{M}{S}$$
$= +788$ kPa $- 6675$ kPa $= -5887$ kPa

$= 5.89$ MPa (C) ◀

18-3



$\sum F_x = -P - F = 0,$ $P = -F$ (C)
$\sum F_y = V - 2.2 = 0,$ $V = 2.2$ kN
$\sum M_C = M - 2.2(0.6) = 0,$ $M = 1.32$ kN·m



$A = (0.075)(0.15) = 0.01125$ m²

$$S = \frac{(0.075)(0.15)^2}{6} = 2.81 \times 10^{-4} \text{ m}^3$$

(a)

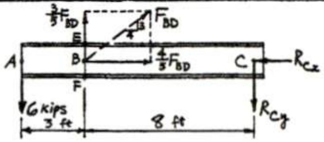
$$\sigma_A = +\frac{P}{A} + \frac{M}{S} = -\frac{F}{0.01125 \text{ m}^2} + \frac{1.32 \text{ kN}\cdot\text{m}}{2.81 \times 10^{-4} \text{ m}^3}$$
$= 88.9F + 47\,000 = 0$

$F = 52.8$ kN ◀

(b)

$$\sigma_B = -\frac{P}{A} - \frac{M}{S} = -\frac{52.8 \text{ kN}}{0.01125 \text{ m}^2} - \frac{1.32 \text{ kN}\cdot\text{m}}{2.81 \times 10^{-4} \text{ m}^3}$$
$= -9390$ kN/m² $= -9.39$ MPa (C) ◀

18-4



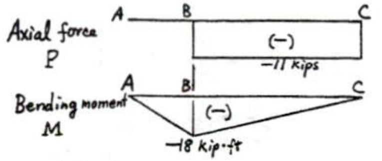
$$\sum M_C = -\frac{3}{5}F_{BD}(8) + (6)(11) = 0,$$
$F_{BD} = +13.75$ kips (T)

(Cont'd)

18-4 (Cont)

$\sum F_x = -R_{Cx} + \frac{4}{5}(13.75) = 0, \qquad R_{Cx} = 11 \text{ kips}$

$\sum F_y = -R_{Cy} + \frac{3}{5}(13.75) - 6 = 0, \qquad R_{Cy} = 2.25 \text{ kips}$



For two C9 x 15: $A = (2)(4.41) = 8.82 \text{ in.}^2$

$S = (2)(11.3) = 22.6 \text{ in.}^3$

The critical section is to the right of B:

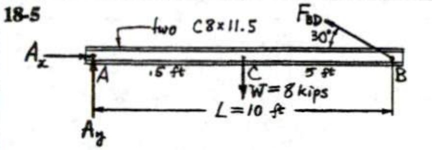
$$\sigma_{max}^{(T)} = \sigma_B = -\frac{P}{A} + \frac{M}{S} = -\frac{11}{8.82} + \frac{(18)(12)}{22.6}$$

$$= -1.25 \text{ ksi} + 9.56 \text{ ksi} = 8.31 \text{ ksi}$$ ◄

$$\sigma_{max}^{(C)} = \sigma_F = -\frac{P}{A} - \frac{M}{S} = -1.25 - 9.56$$

$$= 10.81 \text{ ksi}$$ ◄

18-5



$\sum M_A = F_{BD}\sin 30°(10 \text{ ft}) - (8 \text{ kips})(5 \text{ ft}) = 0$

$F_{BD} = 8 \text{ kips (T)}$

$\sum F_x = A_x - (8 \text{ kips})\cos 30° = 0$

$A_x = 6.93 \text{ kips}$

Axial force: $P = 6.93$ kips (C)

Max. bending moment at C:

$$M = \frac{wL}{4} = \frac{(8 \text{ kips})(10 \text{ ft})}{4}$$

$$= 20 \text{ kip}\cdot\text{ft} = 240 \text{ kip}\cdot\text{in.}$$

For a tentative selection of the channels, consider the bending moment only.

$$S_{req} = \frac{M/2}{\sigma_{allow}} = \frac{\frac{1}{2}(240 \text{ kip}\cdot\text{in.})}{15 \text{ kip/in.}^2} = 8 \text{ in.}^3$$

Try C8 x 11.5 ($A = 3.38 \text{ in.}^2$, $S = 8.14 \text{ in.}^3$)

$$\left|\sigma_{max}^{(C)}\right| = \left|\frac{\frac{1}{2}P}{A} - \frac{\frac{1}{2}M}{S}\right|$$

$$= \frac{\frac{1}{2}(6.93 \text{ kips})}{3.38 \text{ in.}^2} + \frac{\frac{1}{2}(240 \text{ kip}\cdot\text{in.})}{8.14 \text{ in.}^3}$$

$= 15.8 \text{ ksi} > \sigma_{allow}^{(C)} = 15 \text{ ksi}$ (N.G.)

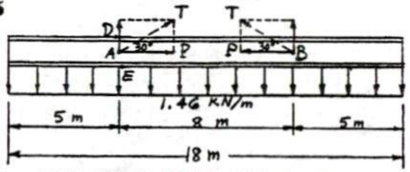
Try C9 x 13.4 ($A = 3.94 \text{ in.}^2$, $S = 10.6 \text{ in.}^3$)

$$\left|\sigma_{max}^{(C)}\right| = \frac{\frac{1}{2}(6.93 \text{ kips})}{3.94 \text{ in.}^2} + \frac{\frac{1}{2}(240 \text{ kip}\cdot\text{in.})}{10.6 \text{ in.}^3}$$

$= 12.2 \text{ ksi} < \sigma_{allow}^{(C)} = 15 \text{ ksi}$ (O.K.)

Use two C9 x 13.4 ◄

18-6



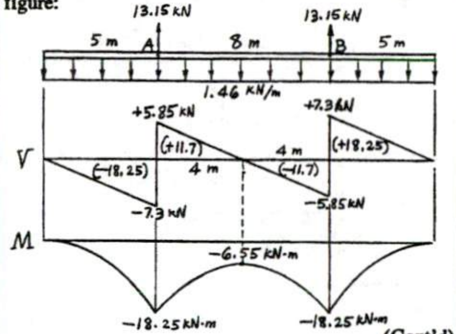
$\sum F_y = 2 T\sin 30° - (1.46 \text{ kN/m})(18 \text{ m}) = 0$

$T = 26.3 \text{ kN}$

Axial force between AB:

$P = T\cos 30° = 26.3\cos 30° = 22.8 \text{ kN (C)}$

Bending of the member is shown in the following figure:



(Cont'd)

18-6 (Cont)

From Appendix Table A-1(b), for W250 x 1.46:

$A = 19.0 \times 10^{-3} \text{ m}^2$

$S = 1.84 \times 10^{-3} \text{ m}^3$

$$\sigma_{max}^{(T)} = \sigma_D = \frac{M_{max}}{S} = \frac{18.25 \text{ kN}\cdot\text{m}}{1.84 \times 10^{-3} \text{ m}^3}$$

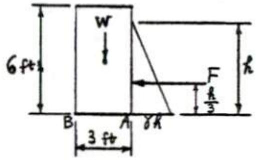
$= +9920 \text{ kPa} = +9.92 \text{ MPa}$ ◄

$$\sigma_{max}^{(C)} = \sigma_E = -\frac{P}{A} - \frac{M_{max}}{S}$$

$$= -\frac{22.8 \text{ kN}}{19.0 \times 10^{-3} \text{ m}^2} - \frac{18.25 \text{ kN}\cdot\text{m}}{1.84 \times 10^{-3} \text{ m}^3}$$

$= -11\,120 \text{ kPa} = -11.12 \text{ MPa}$ ◄

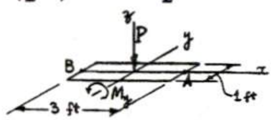
18-7



For 1 ft length of dam,

$W = (3 \times 6 \times 1)(150) = 2700 \text{ lb}$

$F = \left(\frac{1}{2}\gamma h\right) \times h \times 1 = \frac{1}{2}(62.4)h^2 = 31.2h^2 \text{ (lb)}$



$P = W = 2700 \text{ lb}$

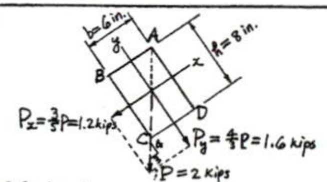
$M_y = F\frac{h}{3} = (31.2h^2)\frac{h}{3} = 10.4h^3$

$$\sigma_A = -\frac{P}{A} + \frac{M_y}{S_y} = -\frac{2700}{3 \times 1} + \frac{10.4h^3}{\frac{(1)(3)^2}{6}}$$

$= -900 + 6.93h^3 = 0$

$$h = \sqrt[3]{\frac{900}{6.93}} = 5.06 \text{ ft}$$ ◄

18-8



For 10 ft simple span, the maximum moments at the mid-span about x and y axes are, respectively

$$M_x = \frac{P_y L}{4} = \frac{(1.6 \text{ kips})(10 \text{ ft})}{4}$$

$= 4 \text{ kip}\cdot\text{ft} = 48 \text{ kip}\cdot\text{in.}$

$$M_y = \frac{P_x L}{4} = \frac{(1.2 \text{ kips})(10 \text{ ft})}{4}$$

$= 3 \text{ kip-ft} = 36 \text{ kip}\cdot\text{in.}$

$$S_x = \frac{bh^2}{6} = \frac{6(8)^2}{6} = 64 \text{ in.}^3$$

$$S_y = \frac{hb^2}{6} = \frac{8(6)^2}{6} = 48 \text{ in.}^3$$

$$\sigma_A = -\frac{M_x}{S_x} - \frac{M_y}{S_y} = -\frac{48 \text{ kip}\cdot\text{in.}}{64 \text{ in.}^3} - \frac{36 \text{ kip}\cdot\text{in.}}{48 \text{ in.}^3}$$

$= -0.75 \text{ ksi} - 0.75 \text{ ksi} = -1.50 \text{ ksi (C)}$ ◄

$$\sigma_B = -\frac{M_x}{S_x} + \frac{M_y}{S_y}$$

$= -0.75 \text{ ksi} + 0.75 \text{ ksi} = 0$ ◄

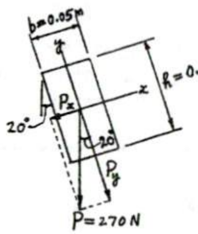
$$\sigma_C = +\frac{M_x}{S_x} + \frac{M_y}{S_y}$$

$= +0.75 \text{ ksi} + 0.75 \text{ ksi} = +1.50 \text{ ksi (T)}$ ◄

$$\sigma_D = +\frac{M_x}{S_x} - \frac{M_y}{S_y}$$

$= +0.75 \text{ ksi} - 0.75 \text{ ksi} = 0$ ◄

18-9



(Cont'd)

18-9 (Cont)

$P_x = (270\ \text{N}) \sin 20° = 92.3\ \text{N}$
$P_y = (270\ \text{N}) \cos 20° = 254\ \text{N}$

For a 2 m cantilever span, the maximum moments at the fixed end about x and y axes are, respectively,

$M_x = P_y L = (254\ \text{N})(2\ \text{m}) = 508\ \text{N}\cdot\text{m}$
$M_y = P_x L = (92.3\ \text{N})(2\ \text{m}) = 184.6\ \text{N}\cdot\text{m}$

$$S_x = \frac{bh^2}{6} = \frac{(0.05\ \text{m})(0.1\ \text{m})^2}{6} = 8.33 \times 10^{-5}\ \text{m}^3$$

$$S_y = \frac{hb^2}{6} = \frac{(0.1\ \text{m})(0.05\ \text{m})^2}{6} = 4.17 \times 10^{-5}\ \text{m}^3$$

$$\sigma^{(T)}_{max} = +\frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{508\ \text{N}\cdot\text{m}}{8.33 \times 10^{-5}\ \text{m}^3} + \frac{184.6\ \text{N}\cdot\text{m}}{4.17 \times 10^{-5}\ \text{m}^3}$$

$= 6.10 \times 10^6\ \text{N/m}^2 + 4.43 \times 10^6\ \text{N/m}^2$

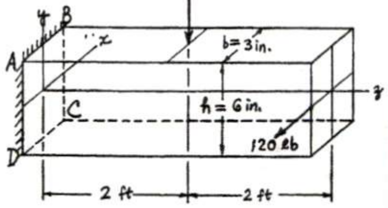
$= 10.53\ \text{MPa}$ ◀

$$\sigma^{(C)}_{max} = -\frac{M_x}{S_x} - \frac{M_y}{S_y}$$

$= -6.10 \times 10^6\ \text{N/m}^2 - 4.43 \times 10^6\ \text{N/m}^2$

$= -10.53\ \text{MPa}$ ◀

18-10



$M_x = (300\ \text{lb})(2\ \text{ft}) = 600\ \text{lb}\cdot\text{ft} = 7200\ \text{lb}\cdot\text{in.}$

$M_y = (120\ \text{lb})(4\ \text{ft}) = 480\ \text{lb}\cdot\text{ft} = 5760\ \text{lb}\cdot\text{in.}$

$$S_x = \frac{bh^2}{6} = \frac{(3\ \text{in.})(6\ \text{in.})^2}{6} = 18\ \text{in.}^3$$

$$S_y = \frac{hb^2}{6} = \frac{(6\ \text{in.})(3\ \text{in.})^2}{6} = 9\ \text{in.}^3$$

$$\sigma_A = +\frac{M_x}{S_x} - \frac{M_y}{S_y} = +\frac{7200\ \text{lb}\cdot\text{in.}}{18\ \text{in.}^3} - \frac{5760\ \text{lb}\cdot\text{in.}}{9\ \text{in.}^3}$$

$= +400\ \text{psi} - 640\ \text{psi} = -240\ \text{psi (C)}$ ◀

$$\sigma_B = +\frac{M_x}{S_x} + \frac{M_y}{S_y}$$

$= +400\ \text{psi} + 640\ \text{psi} = +1040\ \text{psi (T)}$ ◀

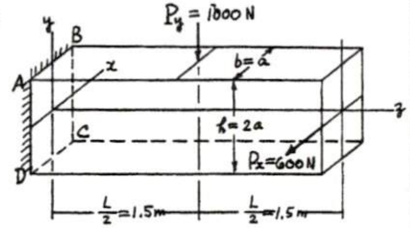
$$\sigma_C = -\frac{M_x}{S_x} + \frac{M_y}{S_y}$$

$= -400\ \text{psi} + 640\ \text{psi} = +240\ \text{psi (T)}$ ◀

$$\sigma_D = -\frac{M_x}{S_x} - \frac{M_y}{S_y}$$

$= -400\ \text{psi} - 640\ \text{psi} = -1040\ \text{psi (C)}$ ◀

18-11



$M_x = P_y(L/2) = (1000\ \text{N})(1.5\ \text{m}) = 1500\ \text{N}\cdot\text{m}$
$M_y = P_x L = (600\ \text{N})(3\ \text{m}) = 1800\ \text{N}\cdot\text{m}$

$$S_x = \frac{bh^2}{6} = \frac{(a)(2a)^2}{6} = \frac{2a^3}{3}$$

$$S_y = \frac{hb^2}{6} = \frac{(2a)(a)^2}{6} = \frac{a^3}{3}$$

$$\sigma^{(T)}_{max} = |\sigma^{(C)}_{max}| = \frac{M_x}{S_x} + \frac{M_y}{S_y}$$

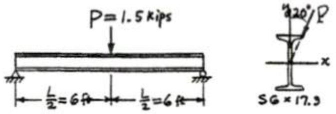
$$= \frac{1500\ \text{N}\cdot\text{m}}{2a^3/3} + \frac{1800\ \text{N}\cdot\text{m}}{a^3/3}$$

$$= \frac{7650\ \text{N}\cdot\text{m}}{a^3} = \sigma_{allow} = 10 \times 10^6\ \text{N/m}^2$$

$$a = \sqrt[3]{\frac{7650\ \text{N}\cdot\text{m}}{10 \times 10^6\ \text{N/m}^2}} = 0.0915\ \text{m}$$

$a_{req} = 91.5\ \text{mm}$ ◀

18-12



From Appendix Table A-2(a), for S 6 x 17.3:
$w = 17.3\ \text{lb/ft} = 0.0173\ \text{kip/ft}$
$S_x = 8.77\ \text{in.}^3,\ S_y = 1.30\ \text{in.}^3$

$P_x = P \sin 20° = (1.5\ \text{kip}) \sin 20° = 0.513\ \text{kip}$
$P_y = P \cos 20° = (1.5\ \text{kip}) \cos 20° = 1.41\ \text{kip}$

$$(M_x)_{max} = \frac{P_y L}{4} + \frac{wL^2}{8} = \frac{(1.41)(12)}{4} + \frac{(0.0173)(12)^2}{8}$$

$= 4.54\ \text{kip}\cdot\text{ft} = 54.5\ \text{kip}\cdot\text{in}$

$$(M_y)_{max} = \frac{P_x L}{4} = \frac{(0.513)(12)}{4}$$

$= 1.539\ \text{kip}\cdot\text{ft} = 18.5\ \text{kip}\cdot\text{in}$

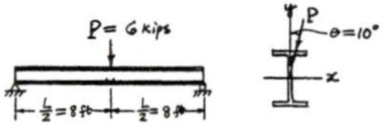
$$\sigma^{(T)}_{max} = |\sigma^{(C)}_{max}| = \frac{(M_x)_{max}}{S_x} + \frac{(M_y)_{max}}{S_y}$$

$$= \frac{54.5\ \text{kip}\cdot\text{in.}}{8.77\ \text{in.}^3} + \frac{18.5\ \text{kip}\cdot\text{in.}}{1.30\ \text{in.}^3}$$

$= 6.21\ \text{ksi} + 14.2\ \text{ksi} = 20.4\ \text{ksi}$

$= 20.4\ \text{ksi}$ ◀

18-13



For a preliminary selection, assume θ = 0°, i.e. P is vertical. Then

$$M_{max} = \frac{PL}{4} = \frac{(6\ \text{kips})(16\ \text{ft})}{4}$$

$= 24\ \text{kip}\cdot\text{ft} = 288\ \text{kip}\cdot\text{in.}$

$$S_{req} = \frac{M_{max}}{\sigma_{allow}} = \frac{288\ \text{kip}\cdot\text{in.}}{20\ \text{kip/in.}} = 14.4\ \text{in.}^3$$

Try W10 x 22 ($S_x = 23.2\ \text{in.}^3$, $S_y = 3.97\ \text{in.}^3$)
For θ = 10°, the components of P are:
$P_x = P \sin 10° = (6\ \text{kips}) \sin 10° = 1.04\ \text{kip}$
$P_y = P \cos 10° = (6\ \text{kips}) \cos 10° = 5.91\ \text{kip}$

$$(M_x)_{max} = \frac{P_y L}{4} + \frac{wL^2}{8} = \frac{(5.91)(16)}{4} + \frac{(0.022)(16)^2}{8}$$

$= 24.3\ \text{kip}\cdot\text{ft} = 292\ \text{kip}\cdot\text{in.}$

$$(M_y)_{max} = \frac{P_x L}{4} = \frac{(1.04)(16)}{4}$$

$= 4.16\ \text{kip}\cdot\text{ft} = 49.9\ \text{kip}\cdot\text{in.}$

$$\sigma_{max} = \frac{(M_x)_{max}}{S_x} + \frac{(M_y)_{max}}{S_y} = \frac{292}{23.2} + \frac{49.9}{3.97}$$

$= 12.6 + 12.6 = 25.2\ \text{ksi} > \sigma_{allow} = 20\ \text{ksi}$ (N.G.)

Try W12 x 30 ($S_x = 38.6\ \text{in.}^3$, $S_y = 6.24\ \text{in.}^3$)

$$(M_x)_{max} = \frac{P_y L}{4} + \frac{wL^2}{8} = \frac{(5.91)(16)}{4} + \frac{(0.030)(16)^2}{8}$$

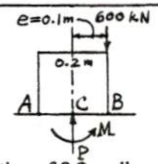
$= 24.6\ \text{kip}\cdot\text{ft} = 295\ \text{kip}\cdot\text{in.}$

$$\sigma_{max} = \frac{(M_x)_{max}}{S_x} + \frac{(M_y)_{max}}{S_y} = \frac{295}{38.6} + \frac{49.9}{6.24}$$

$= 7.64 + 8.00 = 15.6\ \text{ksi} < \sigma_{allow} = 20\ \text{ksi}$ (O.K.)

Use W 12 x 30 ◀

18-14



For circular section of 0.2 m diameter:

$A = \frac{\pi}{4}(0.2\ \text{m})^2 = 0.0314\ \text{m}^2$

$S = \frac{\pi}{32}(0.2\ \text{m})^3 = 7.85 \times 10^{-4}\ \text{m}^3$

$P = 600\ \text{kN (C)}$
$M = (600\ \text{kN})(0.1\ \text{m}) = 60\ \text{kN}\cdot\text{m}$

$$\sigma_A = -\frac{P}{A} + \frac{M}{S} = -\frac{600\ \text{kN}}{0.0314\ \text{m}^2} + \frac{60\ \text{kN}\cdot\text{m}}{7.85 \times 10^{-4}\ \text{m}^3}$$

$= -19\ 110\ \text{kN/m}^2 + 76\ 430\ \text{kN/m}^2$

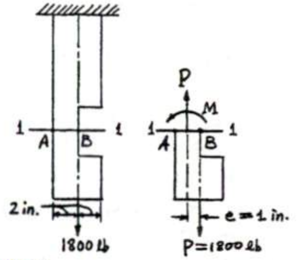
$= 57\ 300\ \text{kN/m}^2 = 57.3\ \text{MPa (T)}$ ◀

$$\sigma_B = -\frac{P}{A} - \frac{M}{S}$$

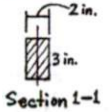
$= -19\ 110\ \text{kN/m}^2 - 76\ 430\ \text{kN/m}^2$

$= -95\ 500\ \text{kN/m}^2 = -95.5\ \text{MPa (C)}$ ◀

18-15



P = 1800 lb (T)
M = (P)(e) = (1800 lb)(1 in.) = 1800 lb • in.



$A = (2)(3) = 6 \text{ in.}^2$

$S = \dfrac{(3)(2)^2}{6} = 2 \text{ in.}^3$

$\sigma_A = +\dfrac{P}{A} - \dfrac{M}{S} = +\dfrac{1800 \text{ lb}}{6 \text{ in.}^2} - \dfrac{1800 \text{ lb} \cdot \text{in.}}{2 \text{ in.}^3}$

$= +300 \text{ psi} - 900 \text{ psi}$

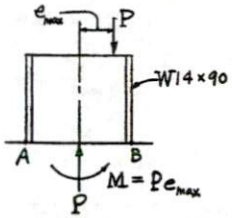
$= -600 \text{ psi (C)}$ ◄

$\sigma_B = +\dfrac{P}{A} + \dfrac{M}{S}$

$= +300 \text{ psi} + 900 \text{ psi}$

$= +1200 \text{ psi (T)}$ ◄

18-16



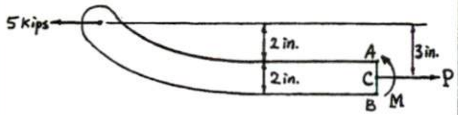
From Appendix Table A-1(a), for W14 x 90:
$A = 26.5 \text{ in.}^2$, $S = 143 \text{ in.}^3$

For the maximum eccentricity, the normal stress at A is zero. Thus,

$\sigma_A = -\dfrac{P}{A} + \dfrac{M}{S} = -\dfrac{P}{26.5} + \dfrac{P e_{max}}{143} = 0$

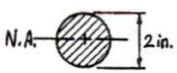
$e_{max} = 5.40 \text{ in.}$ ◄

18-17



$\sum F_x = P - 5 \text{ kips} = 0, \quad P = 5 \text{ kips(T)}$

$\sum M_c = M + (5 \text{ kips})(3 \text{ in.}) = 0, \quad M = -15 \text{ kip} \cdot \text{in.}$



$A = \dfrac{\pi(2)^2}{4} = 3.14 \text{ in.}^2$

$S = \dfrac{\pi(2)^3}{32} = 0.785 \text{ in.}^3$

$\sigma_A = +\dfrac{P}{A} + \dfrac{M}{S} = +\dfrac{5 \text{ kips}}{3.14 \text{ in.}^2} + \dfrac{15 \text{ kip} \cdot \text{in.}}{0.785 \text{ in.}^3}$

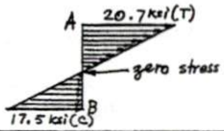
$= +1.59 \text{ ksi} + 19.11 \text{ ksi}$

$= +20.7 \text{ ksi (T)}$ ◄

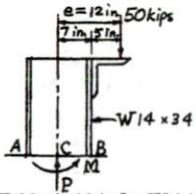
$\sigma_B = +\dfrac{P}{A} - \dfrac{M}{S}$

$= +1.59 - 19.11$

$= -17.5 \text{ ksi (C)}$ ◄



18-18



From Appendix Table A-1(a), for W 14 x 34,
d = 13.98 in.
$A = 10.0 \text{ in.}^2$
$S = 48.6 \text{ in.}^3$

P = 50 kips
M = (P)(e) = (50 kips)(12 in.) = 600 kip • in.

(Cont'd)

18.18 (Cont)

$\sigma_A = -\dfrac{P}{A} + \dfrac{M}{S} = -\dfrac{50}{10.0} + \dfrac{600}{48.6}$

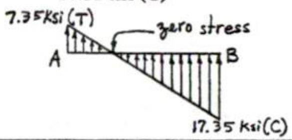
$= -5.0 + 12.35$

$= +7.35 \text{ ksi (T)}$ ◄

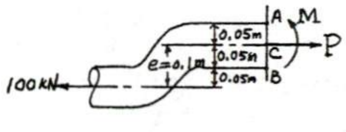
$\sigma_B = -\dfrac{P}{A} - \dfrac{M}{S}$

$= -5.0 - 12.35$

$= -17.35 \text{ ksi (C)}$ ◄



18-19



P = 100 kN (T)
M = (P)(e) = (100 kN)(0.1 m) = 10 kN • m

$A = \dfrac{\pi}{4}(0.1 \text{ m})^2 = 7.85 \times 10^{-3} \text{ m}^2$

$S = \dfrac{\pi}{32}(0.1 \text{ m})^3 = 9.82 \times 10^{-5} \text{ m}^3$

$\sigma_A = +\dfrac{P}{A} - \dfrac{M}{S} = +\dfrac{100}{7.85 \times 10^{-3}} - \dfrac{10}{9.82 \times 10^{-5}}$

$= +12\,740 \text{ kN/m}^2 - 101\,800 \text{ kN/m}^2$

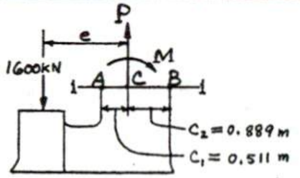
$= -89\,100 \text{ kPa} = 89.1 \text{ MPa (C)}$ ◄

$\sigma_B = +\dfrac{P}{A} + \dfrac{M}{S}$

$= +12\,740 \text{ kN/m}^2 + 101\,800 \text{ kN/m}^2$

$= +114\,500 \text{ kPa} = 114.5 \text{ MPa (T)}$ ◄

18-20



P = 1600 kN (T)
e = 0.9 + 0.511 = 1.41 m
M = Pe = (1600 kN)(1.41 m) = 2256 kN • m

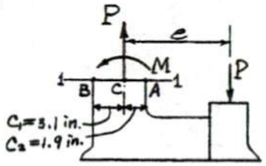
$\sigma_A = +\dfrac{P}{A} + \dfrac{Mc_1}{I} = +\dfrac{1600}{9.94 \times 10^{-2}} + \dfrac{(2256)(0.511)}{2.90 \times 10^{-2}}$

$= +55\,800 \text{ kN/m}^2 = 55.8 \text{ MPa (T)}$ ◄

$\sigma_B = +\dfrac{P}{A} - \dfrac{Mc_2}{I} = +\dfrac{1600}{9.94 \times 10^{-2}} + \dfrac{(2256)(0.889)}{2.90 \times 10^{-2}}$

$= -53\,100 \text{ kN/m}^2 = -53.1 \text{ MPa (C)}$ ◄

18-21



e = 8 in. + 1.9 in. = 9.9 in.
M = Pe

$\sigma_A = +\dfrac{P}{A} + \dfrac{Mc_2}{I} = +\dfrac{P \text{ (lb)}}{10 \text{ in.}^2} + \dfrac{P \text{ (lb)}(9.9 \text{ in.})(1.9 \text{ in.})}{27.2 \text{ in.}^4}$

$= 0.792 \, P \text{ (psi)} \le \sigma_{allow}^{(T)} = 4000 \text{ psi}$

$P \le 5050 \text{ lb}$

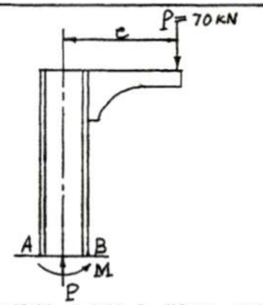
$\sigma_B = +\dfrac{P}{A} - \dfrac{Mc_1}{I} = +\dfrac{P \text{ (lb)}}{10 \text{ in.}^2} - \dfrac{P \text{ (lb)}(9.9 \text{ in.})(3.1 \text{ in.})}{27.2 \text{ in.}^4}$

$= -1.028 \, P \text{ (psi)} \le \sigma_{allow}^{(C)} = -12\,000 \text{ psi}$

$P \le 11\,670 \text{ lb}$

$P_{max} = 5050 \text{ lb}$ ◄

18-22



From Appendix Table A-1(b), for W410 x 0.53,

$d = 0.403$ m,
$A = 6.84 \times 10^{-3}$ m²,
$S = 0.926 \times 10^{-3}$ m³

$$e = 0.730 \text{ m} + \frac{0.403 \text{ m}}{2} = 0.932 \text{ m}$$

$$M = Pe = (70 \text{ kN})(0.932 \text{ m}) = 65.2 \text{ kN} \cdot \text{m}$$

$$\sigma_A = -\frac{P}{A} + \frac{M}{S} = -\frac{70 \text{ kN}}{6.84 \times 10^{-3} \text{ m}^2} + \frac{65.2 \text{ kN} \cdot \text{m}}{0.926 \times 10^{-3} \text{ m}^3}$$

$= -10\,230$ kPa $+ 70\,410$ kPa

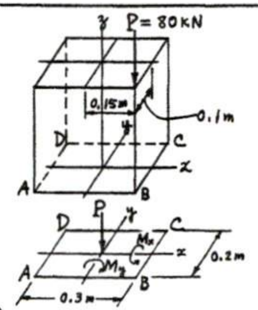
$= +60\,180$ kPa $= 60.18$ MPa (T) ◀

$$\sigma_B = -\frac{P}{A} - \frac{M}{S}$$

$= -10\,230$ kPa $- 70\,410$ kPa

$= -80\,640$ kPa $= -80.64$ MPa (C) ◀

18-23



$P = 80$ kN (C)
$M_x = (80 \text{ kN})(0.1 \text{ m}) = 8 \text{ kN} \cdot \text{m}$
$M_y = (80 \text{ kN})(0.15 \text{ m}) = 12 \text{ kN} \cdot \text{m}$

$A = (0.3 \text{ m})(0.2 \text{ m}) = 0.06 \text{ m}^2$

$$S_x = \frac{bh^2}{6} = \frac{(0.3 \text{ m})(0.2 \text{ m})^2}{6} = 0.002 \text{ m}^3$$

$$S_y = \frac{hb^2}{6} = \frac{(0.2 \text{ m})(0.3)^2}{6} = 0.003 \text{ m}^3$$

$$\sigma_A = -\frac{P}{A} - \frac{M_x}{S_x} + \frac{M_y}{S_y}$$

$$= -\frac{80 \text{ kN}}{0.06 \text{ m}^2} - \frac{8 \text{ kN} \cdot \text{m}}{0.002 \text{ m}^3} + \frac{12 \text{ kN} \cdot \text{m}}{0.003 \text{ m}^3}$$

$= -1330$ kPa $- 4000$ kPa $+ 4000$ kPa

$= -1330$ kPa $= -1.33$ MPa (C) ◀

$$\sigma_B = -\frac{P}{A} - \frac{M_x}{S_x} - \frac{M_y}{S_y}$$

$= -1330$ kPa $- 4000$ kPa $- 4000$ kPa

$= -9330$ kPa $= -9.33$ MPa (C) ◀

$$\sigma_C = -\frac{P}{A} + \frac{M_x}{S_x} - \frac{M_y}{S_y}$$

$= -1330$ kPa $+ 4000$ kPa $- 4000$ kPa

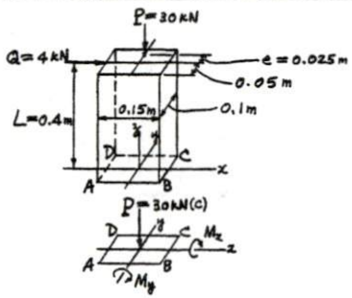
$= -1330$ kPa $= -1.33$ MPa (C) ◀

$$\sigma_D = -\frac{P}{A} + \frac{M_x}{S_x} + \frac{M_y}{S_y}$$

$= -1330$ kPa $+ 4000$ kPa $+ 4000$ kPa

$= +6670$ kPa $= +6.67$ MPa (T) ◀

18-24



$M_x = Pe = (30)(0.025) = 0.75 \text{ kN} \cdot \text{m}$
$M_y = QL = (4)(0.4) = 1.6 \text{ kN} \cdot \text{m}$

(Cont'd)

18-24 (Cont)

$A = (0.15 \text{ m})(0.1 \text{ m}) = 0.015 \text{ m}^2$

$$S_x = \frac{bh^2}{6} = \frac{(0.15 \text{ m})(0.1 \text{ m})^2}{6} = 2.5 \times 10^{-4} \text{ m}^3$$

$$S_y = \frac{hb^2}{6} = \frac{(0.1 \text{ m})(0.15)^2}{6} = 3.75 \times 10^{-4} \text{ m}^3$$

$$\sigma_A = -\frac{P}{A} + \frac{M_x}{S_x} + \frac{M_y}{S_y}$$

$$= -\frac{30 \text{ kN}}{0.015 \text{ m}^2} + \frac{0.75 \text{ kN} \cdot \text{m}}{2.5 \times 10^{-4} \text{ m}^3} + \frac{1.6 \text{ kN} \cdot \text{m}}{3.74 \times 10^{-4} \text{ m}^3}$$

$= -2000$ kPa $+ 3000$ kPa $+ 4270$ kPa

$= +5270$ kPa $= +5.27$ MPa (T) ◀

$$\sigma_B = -\frac{P}{A} + \frac{M_x}{S_x} - \frac{M_y}{S_y}$$

$= -2000$ kPa $+ 3000$ kPa $- 4270$ kPa

$= -3270$ kPa $= -3.27$ MPa (C) ◀

$$\sigma_C = -\frac{P}{A} - \frac{M_x}{S_x} - \frac{M_y}{S_y}$$

$= -2000$ kPa $- 3000$ kPa $- 4270$ kPa

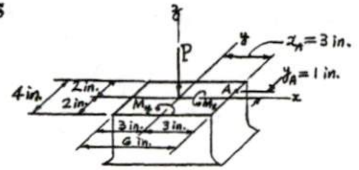
$= -9270$ kPa $= -9.27$ MPa (C) ◀

$$\sigma_D = -\frac{P}{A} - \frac{M_x}{S_x} + \frac{M_y}{S_y}$$

$= -2000$ kPa $- 3000$ kPa $+ 4270$ kPa

$= -730$ kPa $= -0.73$ MPa (C) ◀

18-25



$M_x = P(2 \text{ in.})$
$M_y = P(3 \text{ in.})$

$\sigma_A = E\varepsilon = (10 \times 10^6 \text{ psi})(7.2 \times 10^{-4}) = 7200$ psi

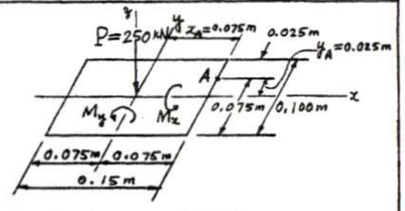
$$\sigma_A = -\frac{P}{A} + \frac{M_x y_A}{I_x} + \frac{M_y x_A}{I_y}$$

$$7200 = -\frac{P}{(6)(4)} + \frac{(2P)(1)}{\frac{1}{12}(6)(4)^3} + \frac{(3P)(3)}{\frac{1}{12}(4)(6)^3}$$

$= 0.1458$ P

$$P = \frac{7200}{0.1458} = 49\,400 \text{ lb} = 49.4 \text{ kips}$$ ◀

18-26



$M_x = (250 \text{ kN})(0.050 \text{ m}) = 12.5 \text{ kN} \cdot \text{m}$
$M_y = (250 \text{ kN})(0.075 \text{ m}) = 18.75 \text{ kN} \cdot \text{m}$

$A = (0.15 \text{ m})(0.100 \text{ m}) = 0.015 \text{ m}^2$

$$I_x = \frac{(0.15 \text{ m})(0.100 \text{ m})^3}{12} = 1.25 \times 10^{-5} \text{ m}^4$$

$$I_y = \frac{(0.100 \text{ m})(0.15 \text{ m})^3}{12} = 2.81 \times 10^{-5} \text{ m}^4$$

$$\sigma_A = -\frac{P}{A} + \frac{M_x y_A}{I_x} + \frac{M_y x_A}{I_y}$$

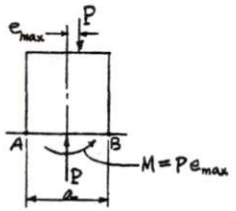
$$= -\frac{250 \text{ kN}}{0.015 \text{ m}^2} + \frac{(12.5 \text{ kN} \cdot \text{m})(0.025 \text{ m})}{1.25 \times 10^{-5} \text{ m}^4} + \frac{(18.75 \text{ kN} \cdot \text{m})(0.075 \text{ m})}{2.81 \times 10^{-5} \text{ m}^4}$$

$= -16\,700$ kPa $+ 25\,000$ kPa $+ 50\,000$ kPa

$= +58\,300$ kPa $= +58.3$ MPa

$$\varepsilon_A = \frac{\sigma_A}{E} = \frac{+58.3 \text{ MPa}}{70\,000 \text{ MPa}} = +8.33 \times 10^{-4}$$ ◀

18-27



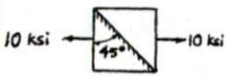
At the maximum eccentricity, the normal stress at A reduces to zero. Thus

$$\sigma_A = -\frac{P}{A} + \frac{M}{S} = -\frac{P}{\frac{\pi}{4}d^2} + \frac{Pe_{max}}{\frac{\pi}{32}d^3} = 0$$

From which, $e_{max} = \dfrac{d}{8}$

Diameter of kern $= 2e_{max} = \dfrac{d}{4}$ ◀

18-28



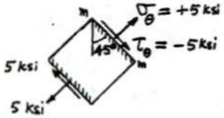
$\sigma_x = +10$ ksi
$\sigma_y = 0$
$\tau_x = 0$
$\theta = +45°$
$2\theta = +90°$

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_x \sin 2\theta$$

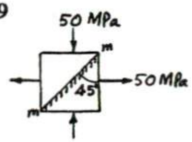
$$= \frac{10+0}{2} + \frac{10-0}{2}\cos 90° + 0 = +5\text{ ksi}$$ ◀

$$\tau_\theta = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_x \cos 2\theta$$

$$= -\frac{10-0}{2}\sin 90° + 0 = -5\text{ ksi}$$ ◀



18-29



$\sigma_x = +50$ MPa
$\sigma_y = -50$ MPa
$\tau_x = 0$
$\theta = -45°$
$2\theta = -90°$

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_x \sin 2\theta$$

$$= \frac{50+(-50)}{2} + \frac{50-(-50)}{2}\cos(-90°) + 0$$

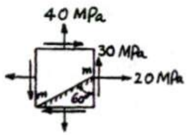
$$= 0$$ ◀

$$\tau_\theta = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_x \cos 2\theta$$

$$= -\frac{50-(-50)}{2}\sin(-90°) + 0$$

$$= +50\text{ MPa}$$ ◀

18-30



$\sigma_x = +20$ MPa
$\sigma_y = +40$ MPa
$\tau_x = +30$ MPa
$\theta = -60°$
$2\theta = -120°$

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_x \sin 2\theta$$

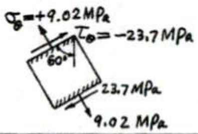
$$= \frac{20+40}{2} + \frac{20-40}{2}\cos(-120°) + 30\sin(-120°)$$

$$= +9.02\text{ MPa}$$ ◀

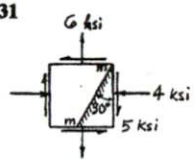
$$\tau_\theta = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_x \cos 2\theta$$

$$= -\frac{20-40}{2}\sin(-120°) + 30\cos(-120°)$$

$$= -23.7\text{ MPa}$$ ◀



18-31



$\sigma_x = -4$ ksi
$\sigma_y = +6$ ksi
$\tau_x = -5$ ksi
$\theta = -30°$
$2\theta = -60°$

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_x \sin 2\theta$$

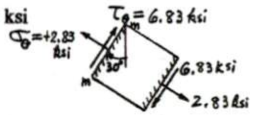
$$= \frac{-4+6}{2} + \frac{-4-6}{2}\cos(-60°) + (-5)\sin(-60°)$$

$$= +2.83\text{ ksi}$$ ◀

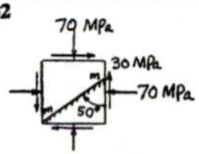
$$\tau_\theta = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_x \cos 2\theta$$

$$= -\frac{-4-6}{2}\sin(-60°) + (-5)\cos(-60°)$$

$$= -6.83\text{ ksi}$$ ◀



18-32



$\sigma_x = -70$ MPa
$\sigma_y = -70$ MPa
$\tau_x = +30$ MPa
$\theta = -50°$
$2\theta = -100°$

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_x \sin 2\theta$$

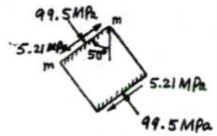
$$= \frac{-70+(-70)}{2} + \frac{-70-(-70)}{2}\cos(-100°) + 30\sin(-100°)$$

$$= -99.5\text{ MPa}$$ ◀

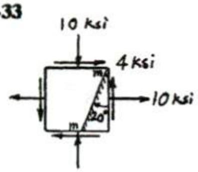
$$\tau_\theta = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_x \cos 2\theta$$

$$= -\frac{-70-(-70)}{2}\sin(-100°) + (30)\cos(-100°)$$

$$= -5.21\text{ MPa}$$ ◀



18-33



$\sigma_x = +10$ ksi
$\sigma_y = -10$ ksi
$\tau_x = +4$ ksi
$\theta = -20°$
$2\theta = -40°$

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_x \sin 2\theta$$

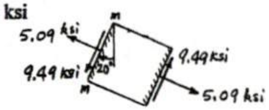
$$= \frac{10+(-10)}{2} + \frac{10-(-10)}{2}\cos(-40°) + 4\sin(-40°)$$

$$= +5.09\text{ ksi}$$ ◀

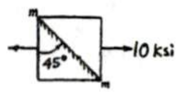
$$\tau_\theta = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_x \cos 2\theta$$

$$= -\frac{10-(-10)}{2}\sin(-40°) + 4\cos(-40°)$$

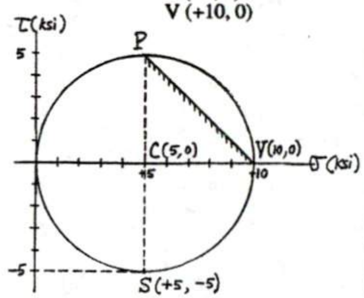
$$= +9.49\text{ ksi}$$ ◀

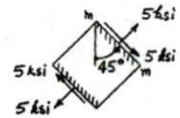


18-34

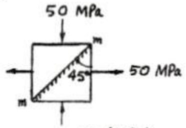


$\sigma_x = +10$ ksi
$\sigma_y = 0, \quad \tau_x = 0$
$\dfrac{\sigma_x + \sigma_y}{2} = +5$ ksi
C (+5, 0)
V (+10, 0)





18-35



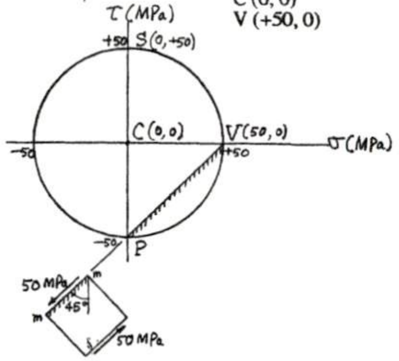
$\sigma_x = +50$ MPa

$\sigma_y = -50$ MPa, $\tau_x = 0$

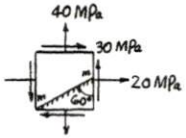
$\dfrac{\sigma_x + \sigma_y}{2} = 0$

C (0, 0)

V (+50, 0)



18-36



$\sigma_x = +20$ MPa

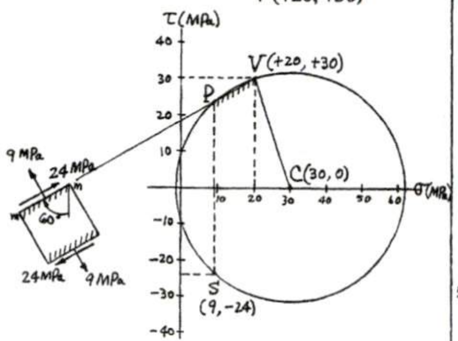
$\sigma_y = +40$ MPa

$\tau_x = +30$ MPa

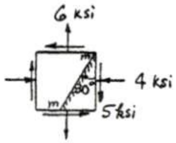
$\dfrac{\sigma_x + \sigma_y}{2} = +30$ MPa

C (+30, 0)

V (+20, +30)



18-37



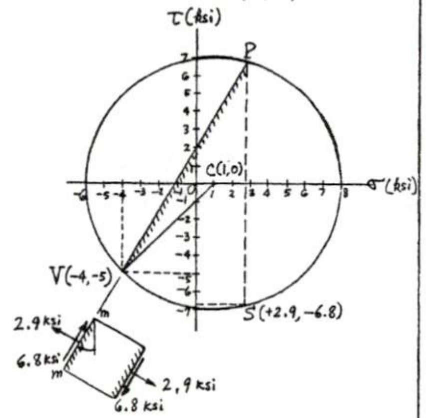
$\sigma_x = -4$ ksi

$\sigma_y = +6$ksi, $\tau_x = -5$ ksi

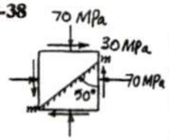
$\dfrac{\sigma_x + \sigma_y}{2} = +1$ ksi

C (+1, 0)

V (-4, -5)



18-38



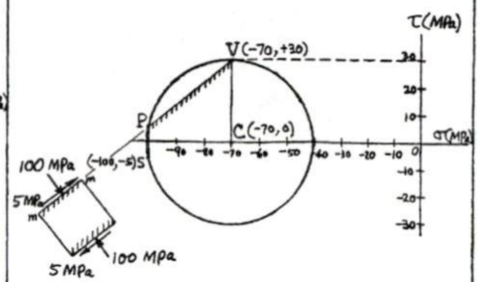
$\sigma_x = -70$ MPa

$\sigma_y = -70$ MPa, $\tau_x = +30$ MPa

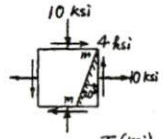
$\dfrac{\sigma_x + \sigma_y}{2} = -70$ MPa

C (-70, 0)

V (-70, +30)



18-39



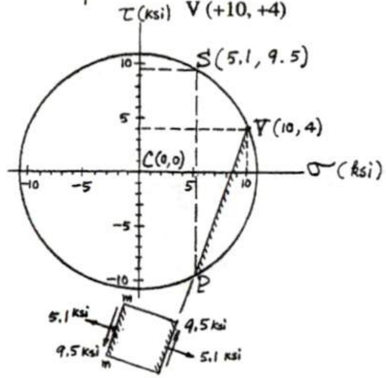
$\sigma_x = +10$ ksi

$\sigma_y = -10$ ksi, $\tau_x = +4$ ksi

$\dfrac{\sigma_x + \sigma_y}{2} = 0$

C (0, 0)

V (+10, +4)



18-40

(a) State of stress of the element:

$$\sigma_x = \sigma_l = \frac{pr}{2t} = \frac{(6\text{ MPa})(0.5\text{ m})}{2(0.02\text{ m})} = 75\text{ MPa} \blacktriangleleft$$

$$\sigma_y = \sigma_c = \frac{pr}{t} = 2\sigma_l = 150\text{ MPa} \blacktriangleleft$$

$$\tau_x = 0 \blacktriangleleft$$

(b) Stresses along the inclined plane m-m:

$\theta = -60°$, $2\theta = -120°$

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_x \sin 2\theta$$

$$= \frac{75 + 150}{2} + \frac{75 - 150}{2}\cos(-120°) + 0$$

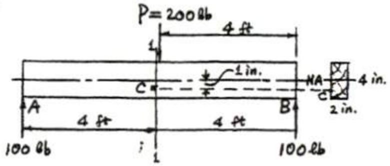
$$= +131.3\text{ MPa (T)} \blacktriangleleft$$

$$\tau_\theta = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_x \cos 2\theta$$

$$= -\frac{75 - 150}{2}\sin(-120°) + 0$$

$$= -32.5\text{ MPa} \blacktriangleleft$$

18-41



$V_{1\text{-}1} = +100$ lb

$M_{1\text{-}1} = +(100\text{ lb})(4\text{ ft}) = +400\text{ lb}\cdot\text{ft} = +4800\text{ lb}\cdot\text{in.}$

$I = \dfrac{1}{12}(2\text{ in.})(4\text{ in.})^3 = 10.67\text{ in}^4$

(a) State of stress at point C:

$$\sigma_x = \frac{My}{I} = +\frac{(4800\text{ lb}\cdot\text{in.})(1\text{ in.})}{10.67\text{ in.}^4} = 450\text{ psi} \blacktriangleleft$$

$$\sigma_y = 0 \blacktriangleleft$$

$$\tau_y = \frac{VQ}{It} = \frac{(100\text{ lb})(2\text{x}1\text{x}1.5\text{ in.}^3)}{(10.67\text{ in.}^4)(2\text{ in.})} = 14.1\text{ psi} \blacktriangleleft$$

(b) Stresses along the inclined plane m-m:

$\theta = +60°$, $2\theta = +120°$

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_x \sin 2\theta$$

$$= \frac{450 + 0}{2} + \frac{450 - 0}{2}\cos(120°) + (-14.1)\sin(120°)$$

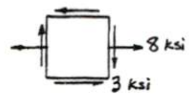
$$= +100.3\text{ psi} \blacktriangleleft$$

$$\tau_\theta = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_x \cos 2\theta$$

$$= -\frac{450 - 0}{2}\sin(120°) + (-14.1)\cos(120°)$$

$$= -187.8\text{ psi} \blacktriangleleft$$

18-42



$\sigma_x = +8$ ksi

$\sigma_y = 0$ ksi

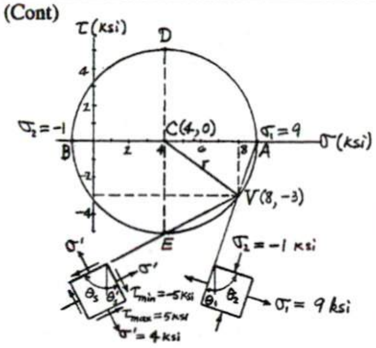
$\tau_x = -3$ ksi

C (4, 0)

V (8, -3)

(Cont'd)

18-42 (Cont)



$$b = \frac{\sigma_x - \sigma_y}{2} = \frac{8 - 0}{2} = 4 \text{ ksi}$$

$$r = \sqrt{b^2 + \tau_x^2} = \sqrt{4^2 + (-3)^2} = 5 \text{ ksi}$$

Principal Planes:

$$\theta_1 = \tan^{-1}\left[\frac{r - b}{\tau_x}\right] = \tan^{-1}\left[\frac{5 - 4}{-3}\right]$$

$$= \tan^{-1}\left[-\frac{1}{3}\right] = -18.4^\circ \blacktriangleleft$$

$$\theta_2 = \theta_1 + 90^\circ = -18.4^\circ + 90^\circ = 71.6^\circ \blacktriangleleft$$

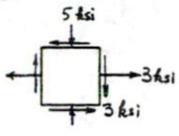
Planes for the maximum and minimum shear stresses:

$$\theta_s = -\tan^{-1}\left[\frac{b}{r + \tau_x}\right] = -\tan^{-1}\left[\frac{4}{5 + (-3)}\right]$$

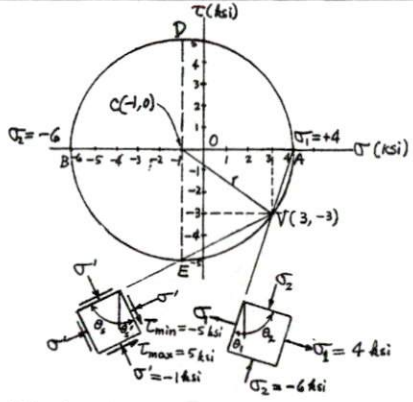
$$= -\tan^{-1}[2] = -63.4^\circ \blacktriangleleft$$

$$\theta_s' = \theta_s + 90^\circ = -63.4^\circ + 90^\circ = +26.6^\circ \blacktriangleleft$$

18-43



$\sigma_x = +3$ ksi
$\sigma_y = -5$ ksi
$\tau_x = -3$ ksi
C (-1, 0)
V (3, -3)



Principle planes:

$$b = \frac{\sigma_x - \sigma_y}{2} = \frac{3 - (-5)}{2} = 4 \text{ ksi}$$

$$r = \sqrt{b^2 + \tau_x^2} = \sqrt{4^2 + 3^2} = 5 \text{ ksi}$$

$$\theta_1 = \tan^{-1}\left[\frac{r - b}{\tau_x}\right] = \tan^{-1}\left[\frac{5 - 4}{-3}\right]$$

$$= \tan^{-1}\left[-\frac{1}{3}\right] = -18.4^\circ \blacktriangleleft$$

$$\theta_2 = \theta_1 + 90^\circ = -18.4^\circ + 90^\circ = +71.6^\circ \blacktriangleleft$$

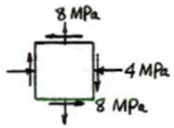
Planes for the maximum and minimum shear stresses:

$$\theta_s = -\tan^{-1}\left[\frac{b}{r + \tau_x}\right] = -\tan^{-1}\left[\frac{4}{5 + (-3)}\right]$$

$$= -\tan^{-1}[2] = -63.4^\circ \blacktriangleleft$$

$$\theta_s' = \theta_s + 90^\circ = -63.4^\circ + 90^\circ = +26.6^\circ \blacktriangleleft$$

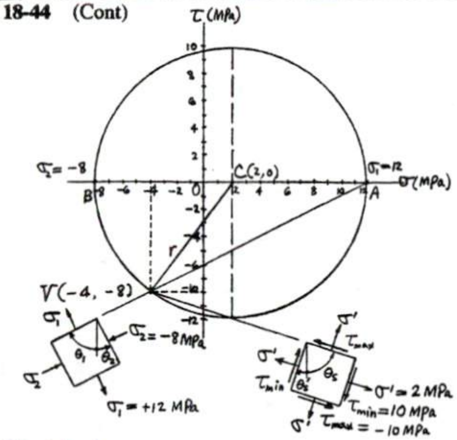
18-44



$\sigma_x = -4$ MPa
$\sigma_y = +8$ MPa
$\tau_x = -8$ MPa
C (2, 0)
V (-4, -8)

(Cont'd)

18-44 (Cont)



Principle planes:

$$b = \frac{\sigma_x - \sigma_y}{2} = \frac{-4 - 8}{2} = -6 \text{ MPa}$$

$$r = \sqrt{b^2 + \tau_x^2} = \sqrt{6^2 + 8^2} = 10 \text{ MPa}$$

$$\theta_1 = \tan^{-1}\left[\frac{r - b}{\tau_x}\right] = \tan^{-1}\left[\frac{10 - (-6)}{-8}\right]$$

$$= \tan^{-1}[-2] = -63.4^\circ \blacktriangleleft$$

$$\theta_2 = \theta_1 + 90^\circ = -63.4^\circ + 90^\circ = +26.6^\circ \blacktriangleleft$$

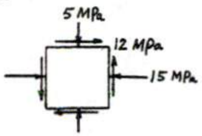
Planes for the maximum and minimum shear stresses:

$$\theta_s = -\tan^{-1}\left[\frac{b}{r + \tau_x}\right] = -\tan^{-1}\left[\frac{-6}{10 + (-8)}\right]$$

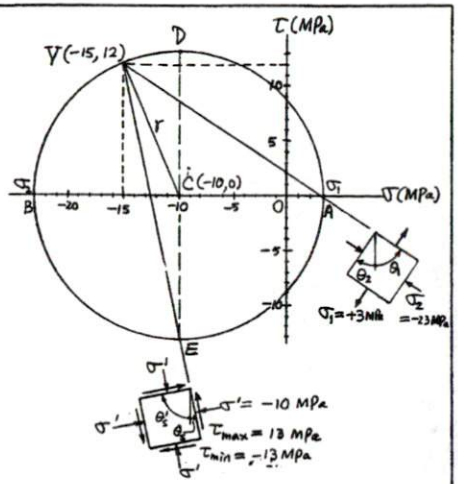
$$= -\tan^{-1}[-3] = +71.6^\circ \blacktriangleleft$$

$$\theta_s' = \theta_s - 90^\circ = +71.6^\circ - 90^\circ = -18.4^\circ \blacktriangleleft$$

18-45



$\sigma_x = -15$ MPa
$\sigma_y = -5$ MPa
$\tau_x = +12$ MPa
C (-10, 0)
V (-15, +12)



Principle planes:

$$b = \frac{\sigma_x - \sigma_y}{2} = \frac{-15 - (-5)}{2} = -5 \text{ MPa}$$

$$r = \sqrt{b^2 + \tau_x^2} = \sqrt{5^2 + 12^2} = 13 \text{ MPa}$$

$$\theta_1 = \tan^{-1}\left[\frac{r - b}{\tau_x}\right] = \tan^{-1}\left[\frac{13 - (-5)}{12}\right]$$

$$= \tan^{-1}\left[\frac{3}{2}\right] = +56.3^\circ \blacktriangleleft$$

$$\theta_2 = \theta_1 - 90^\circ = +56.3 - 90^\circ = -33.7^\circ \blacktriangleleft$$

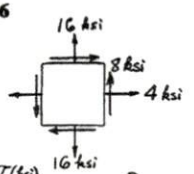
Planes for the maximum and minimum shear stresses:

$$\theta_s = -\tan^{-1}\left[\frac{b}{r + \tau_x}\right] = -\tan^{-1}\left[\frac{-5}{13 + 12}\right]$$

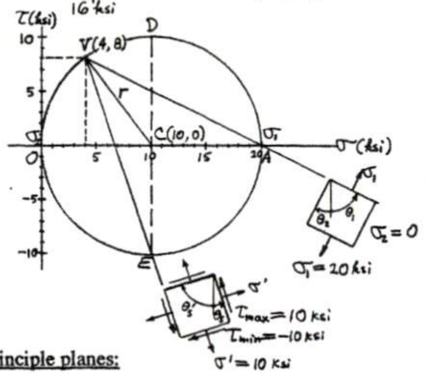
$$= -\tan^{-1}\left[-\frac{1}{5}\right] = +11.3^\circ \blacktriangleleft$$

$$\theta_s' = \theta_s - 90^\circ = +11.3^\circ - 90^\circ = -78.7^\circ \blacktriangleleft$$

18-46



$\sigma_x = +4$ ksi
$\sigma_y = +16$ ksi
$\tau_x = +8$ ksi
C (+10, 0)
V (+4, +8)



Principle planes:

$$b = \frac{\sigma_x - \sigma_y}{2} = \frac{4 - 16}{2} = -6 \text{ ksi}$$

$$r = \sqrt{b^2 + \tau_x^2} = \sqrt{6^2 + 8^2} = 10 \text{ ksi}$$

$$\theta_1 = \tan^{-1}\left[\frac{r - b}{\tau_x}\right] = \tan^{-1}\left[\frac{10 - (-6)}{8}\right]$$

$$= \tan^{-1}[2] = +63.4^\circ \quad ◀$$

$$\theta_2 = \theta_1 - 90^\circ = +63.4^\circ - 90^\circ = -26.6^\circ \quad ◀$$

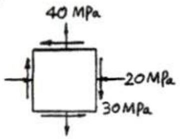
Planes for the maximum and minimum shear stresses:

$$\theta_s = -\tan^{-1}\left[\frac{b}{r + \tau_x}\right] = -\tan^{-1}\left[\frac{-6}{10 + 8}\right]$$

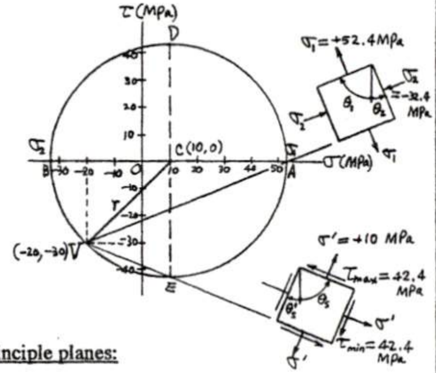
$$= -\tan^{-1}\left[-\frac{1}{3}\right] = +18.4^\circ \quad ◀$$

$$\theta_s' = \theta_s - 90^\circ = +18.4^\circ - 90^\circ = -71.6^\circ \quad ◀$$

18-47



$\sigma_x = -20$ MPa
$\sigma_y = +40$ MPa
$\tau_x = -30$ MPa
C (+10, 0)
V (-20, -30)



Principle planes:

$$b = \frac{\sigma_x - \sigma_y}{2} = \frac{-20 - 40}{2} = -30 \text{ MPa}$$

$$r = \sqrt{b^2 + \tau_x^2} = \sqrt{30^2 + 30^2} = 42.4 \text{ MPa}$$

$$\theta_1 = \tan^{-1}\left[\frac{r - b}{\tau_x}\right] = \tan^{-1}\left[\frac{42.4 - (-30)}{-30}\right]$$

$$= \tan^{-1}\left[-\frac{36.2}{15}\right] = -67.5^\circ \quad ◀$$

$$\theta_2 = \theta_1 + 90^\circ = -67.5^\circ + 90^\circ = +22.5^\circ \quad ◀$$

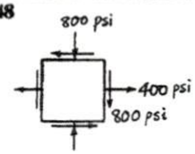
Planes for the maximum and minimum shear stresses:

$$\theta_s = -\tan^{-1}\left[\frac{b}{r + \tau_x}\right] = -\tan^{-1}\left[\frac{-30}{42.4 - 30}\right]$$

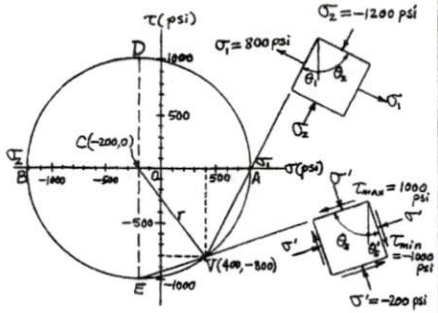
$$= -\tan^{-1}\left[-\frac{15}{12.4}\right] = +67.5^\circ \quad ◀$$

$$\theta_s' = \theta_s - 90^\circ = +67.5^\circ - 90^\circ = -22.5^\circ \quad ◀$$

18-48



$\sigma_x = +400$ psi
$\sigma_y = -800$ psi
$\tau_x = -800$ psi
C (-200, 0)
V (400, -800)



Principle planes:

$$b = \frac{\sigma_x - \sigma_y}{2} = \frac{400 - (-800)}{2} = 600 \text{ psi}$$

$$r = \sqrt{b^2 + \tau_x^2} = \sqrt{600^2 + 800^2} = 1000 \text{ psi}$$

$$\theta_1 = \tan^{-1}\left[\frac{r - b}{\tau_x}\right] = \tan^{-1}\left[\frac{1000 - 600}{-800}\right]$$

$$= \tan^{-1}\left[-\frac{1}{2}\right] = -26.6^\circ \quad ◀$$

$$\theta_2 = \theta_1 + 90^\circ = -26.6^\circ + 90^\circ = +63.4^\circ \quad ◀$$

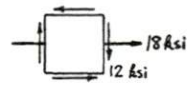
Planes for the maximum and minimum shear stresses:

$$\theta_s = -\tan^{-1}\left[\frac{b}{r + \tau_x}\right] = -\tan^{-1}\left[\frac{600}{1000 + (-800)}\right]$$

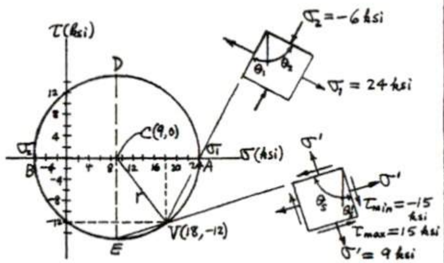
$$= -\tan^{-1}[3] = -71.6^\circ \quad ◀$$

$$\theta_s' = \theta_s + 90^\circ = -71.6^\circ + 90^\circ = +18.4^\circ \quad ◀$$

18-49



$\sigma_x = +18$ ksi
$\sigma_y = 0$
$\tau_x = -12$ ksi
C (9, 0)
V (18, -12)



Principle planes:

$$b = \frac{\sigma_x - \sigma_y}{2} = \frac{18 - 0}{2} = 9 \text{ ksi}$$

$$r = \sqrt{b^2 + \tau_x^2} = \sqrt{9^2 + 12^2} = 15 \text{ ksi}$$

$$\theta_1 = \tan^{-1}\left[\frac{r - b}{\tau_x}\right] = \tan^{-1}\left[\frac{15 - 9}{-12}\right]$$

$$= \tan^{-1}\left[-\frac{1}{2}\right] = -26.6^\circ \quad ◀$$

$$\theta_2 = \theta_1 + 90^\circ = -26.6^\circ + 90^\circ = +63.4^\circ \quad ◀$$

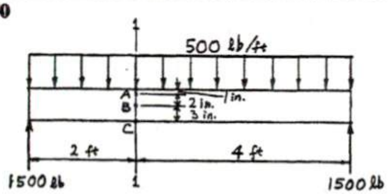
Planes for the maximum and minimum shear stresses:

$$\theta_s = -\tan^{-1}\left[\frac{b}{r + \tau_x}\right] = -\tan^{-1}\left[\frac{9}{15 - 12}\right]$$

$$= -\tan^{-1}[3] = -71.6^\circ \quad ◀$$

$$\theta_s' = \theta_s + 90^\circ = -71.6^\circ + 90^\circ = +18.4^\circ \quad ◀$$

18-50



$V_{1\text{-}1} = 1500 - 500(2) = +500 \text{ lb}$

$M_{1\text{-}1} = +(1500 \text{ lb})(2 \text{ ft}) - (500 \times 2 \text{ lb}) \dfrac{2 \text{ ft}}{2}$

$= +2000 \text{ lb} \cdot \text{ft} = 24\,000 \text{ lb} \cdot \text{in.}$

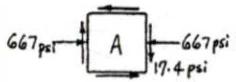
For full-size 4 in. x 6 in. section:

$I = \dfrac{1}{12}(4 \text{ in.})(6 \text{ in.})^3 = 72 \text{ in.}^4$

At point A

$|\sigma| = \dfrac{My}{I} = + \dfrac{(24\,000 \text{ lb} \cdot \text{in.})(2 \text{ in.})}{72 \text{ in.}^4} = 667 \text{ psi}$

$|\tau| = \dfrac{VQ}{It} = \dfrac{(500 \text{ lb})(4 \times 1 \text{ in.}^2)(2.5 \text{ in.})}{(72 \text{ in.}^4)(4 \text{ in.})} = 17.4 \text{ psi}$



$\sigma_x = -667 \text{ psi}$
$\sigma_y = 0$
$\tau_x = -17.4 \text{ psi}$

$a = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{-667 + 0}{2} = -333.5 \text{ psi}$

$b = \dfrac{\sigma_x - \sigma_y}{2} = \dfrac{-667 - 0}{2} = -333.5 \text{ psi}$

$r = \sqrt{b^2 + \tau_x^2} = \sqrt{333.5^2 + 17.4^2} = 334.0 \text{ ksi}$

$\sigma_1 = a + r = -333.5 + 334.0 = +0.5 \text{ psi}$ ◀

$\sigma_2 = a - r = -333.5 - 334.0 = -668 \text{ psi}$ ◀

$\theta_1 = \tan^{-1}\left[\dfrac{r - b}{\tau_x}\right] = \tan^{-1}\left[\dfrac{334 - (-333.5)}{-17.4}\right]$

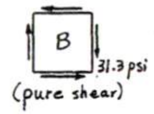
$= \tan^{-1}\left[-\dfrac{667.5}{17.4}\right] = -88.5^\circ$ ◀

$\theta_2 = \theta_1 + 90^\circ = -88.5^\circ + 90^\circ = +1.5^\circ$ ◀

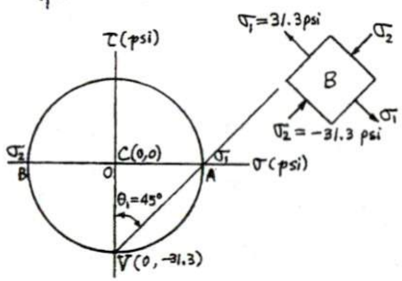
At point B:

$|\sigma| = 0$ (at the N.A.)

$|\tau| = |\tau_{max}| = 1.5 \dfrac{V}{A} = 1.5 \dfrac{500 \text{ lb}}{4 \times 6 \text{ in.}^2} = 31.3 \text{ psi}$



$\sigma_x = 0, \quad \sigma_y = 0$
$\tau_x = -31.3 \text{ psi}$
C (0, 0)
V (0, -31.3)



$\sigma_1 = +31.3 \text{ psi}$ ◀

$\sigma_2 = -31.3 \text{ psi}$ ◀

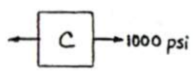
$\theta_1 = 45^\circ$ ◀

At point C:

$\sigma = \sigma_{max} = \dfrac{My}{I} = + \dfrac{(2000 \times 12 \text{ lb} \cdot \text{in.})(3 \text{ in.})}{72 \text{ in.}^4}$

$= 1000 \text{ psi}$

$\tau = 0$

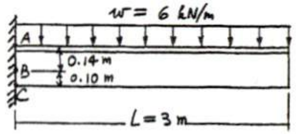


$\sigma_x = +1000 \text{ psi}$
$\sigma_y = 0$
$\tau_x = -31.3 \text{ psi}$

$\sigma_1 = +1000 \text{ psi}, \qquad \theta_1 = 0^\circ$ ◀

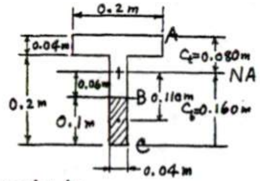
$\sigma_2 = 0$ ◀

18-51



$V_{max} = +wL = +(6 \text{ kN/m})(3 \text{ m}) = +18 \text{ kN}$

$|M_{max}| = \dfrac{wL^2}{2} = \dfrac{(6 \text{ kN/m})(3 \text{ m})^2}{2} = 27 \text{ kN} \cdot \text{m}$



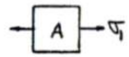
$I_{NA} = 8.53 \times 10^{-5} \text{ m}^4$

At point A:

$\sigma = \dfrac{Mc_t}{I} = + \dfrac{(27 \text{ kN} \cdot \text{m})(0.080 \text{ m})}{8.53 \times 10^{-5} \text{ m}^4}$

$= 25\,300 \text{ kN/m}^2 = 25.3 \text{ MPa (T)}$

$\tau = 0$



$\sigma_1 = +25.3 \text{ MPa}$ ◀

$\sigma_2 = 0$ ◀

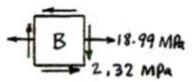
At point B:

$|\sigma| = \dfrac{My}{I} = + \dfrac{(27 \text{ kN} \cdot \text{m})(0.060 \text{ m})}{8.53 \times 10^{-5} \text{ m}^4}$

$= 18\,990 \text{ kN/m}^2 = 18.99 \text{ MPa (C)}$

$|\tau| = \dfrac{VQ}{It} = \dfrac{(18 \text{ kN})[(0.040 \times 0.100 \text{ m}^2)(0.110 \text{ m})]}{(8.53 \times 10^{-5} \text{ m}^4)(0.040 \text{ m})}$

$= 2320 \text{ kN/m}^2 = 2.32 \text{ MPa}$



$\sigma_x = -18.99 \text{ MPa}$
$\sigma_y = 0$
$\tau_x = -2.32 \text{ MPa}$

$a = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{-18.99 + 0}{2} = -9.50 \text{ MPa}$

$b = \dfrac{\sigma_x - \sigma_y}{2} = \dfrac{-18.99 - 0}{2} = -9.50 \text{ MPa}$

$r = \sqrt{b^2 + \tau_x^2} = \sqrt{9.50^2 + 2.32^2} = 9.78 \text{ MPa}$

$\sigma_1 = a + r = -9.50 + 9.78$

$= +0.28 \text{ MPa (T)}$ ◀

$\sigma_2 = a - r = -9.50 - 9.78$

$= -19.28 \text{ MPa (C)}$ ◀

At point C:

$|\sigma| = \dfrac{Mc_b}{I} = + \dfrac{(27 \text{ kN} \cdot \text{m})(0.160 \text{ m})}{8.53 \times 10^{-5} \text{ m}^4}$

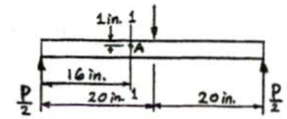
$= 50\,600 \text{ kN/m}^2 = 50.6 \text{ MPa (C)}$

$\tau = 0$

$\sigma_1 = 0$ ◀

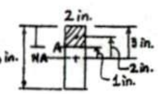
$\sigma_2 = 50.6 \text{ MPa}$ ◀

18-52



$V_{1\text{-}1} = \dfrac{P}{2} (\text{lb})$

$M_{1\text{-}1} = \dfrac{P}{2} \times 16 = 8P \text{ (lb} \cdot \text{in.)}$



$I = \dfrac{2(6)^3}{12} = 36 \text{ in.}^4$

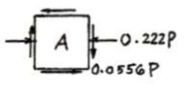
$Q = A'\bar{y}' = (2 \times 2)(2) = 8 \text{ in.}^3$

$|\sigma| = \dfrac{My}{I} = + \dfrac{(8P)(1)}{36} = 0.222P \text{ (psi)}$

$|\tau| = \dfrac{VQ}{It} = + \dfrac{\left(\frac{P}{2}\right)(8)}{(36)(2)} = 0.0556P \text{ (psi)}$

(Cont'd)

18-52 (Cont)



$\sigma_x = -0.222\,P$ (psi)
$\sigma_y = 0$
$\tau_x = -0.0556P$ (psi)

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_x^2}$$

$$= \sqrt{\left(\frac{-0.222P - 0}{2}\right)^2 + (-0.0556)^2}$$

$= 0.124P$

Equating the above expression to the given maximum shear stress of 100 psi, we find

$$P = \frac{100}{0.124} = 806\text{ lb}$$ ◀

18-53

The internal forces at the section through A are:
$P = P\cos 60° = 10\cos 60° = 5$ kN(T)
$V = P\sin 60° = 10\sin 60° = 8.66$ kN
$M = (10\sin 60°)(0.15) = 1.30$ kN • m

For the 20 mm x 100 mm section:

$A = (0.020\text{ m})(0.100\text{ m}) = 0.002\text{ m}^2$

$$I = \frac{(0.020\text{ m})(0.100\text{ m})^3}{12} = 1.67 \times 10^{-6}\text{ m}^4$$

The first moment of area A′ about the N.A. is
$Q = A'\bar{y}' = (0.020 \times 0.020)(0.040) = 1.6 \times 10^{-5}\text{ m}^3$

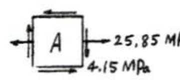
$$\sigma = \frac{P}{A} + \frac{My}{I}$$

$$= \frac{5\text{ kN}}{0.002\text{ m}^2} + \frac{(1.30\text{ kN}\cdot\text{m})(0.03\text{ m})}{1.67 \times 10^{-6}\text{ m}^4}$$

$= 2500\text{ kN/m}^2 + 23\,400\text{ kN/m}^2$
$= 25\,900\text{ kN/m}^2 = 25.9$ MPa

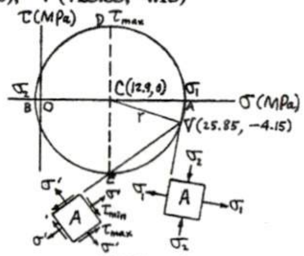
$$|\tau| = \frac{VQ}{It} = \frac{(8.66\text{ kN})(1.66 \times 10^{-5}\text{ m}^3)}{(1.67 \times 10^{-6}\text{ m}^4)(0.020\text{ m})}$$

$= 4150\text{ kN/m}^2 = 4.15$ MPa



$\sigma_x = +25.85$ MPa
$\sigma_y = 0$
$\tau_x = -4.15$ MPa

C (+12.9, 0), V (+25.85, -4.15)



$$a = b = \frac{\sigma_x \pm \sigma_y}{2} = \frac{25.85 \pm 0}{2} = 12.93\text{ MPa}$$

$$r = \sqrt{b^2 + \tau_x^2} = \sqrt{12.93^2 + 4.15^2} = 13.58\text{ MPa}$$

$\sigma_1 = a + r = 12.93 + 13.58 = +26.5$ MPa ◀

$\sigma_2 = a - r = 12.93 - 13.58 = -0.65$ MPa ◀

$\tau_{max} = +r = +13.58$ MPa ◀

$\tau_{min} = -r = -13.58$ MPa ◀

$\sigma' = a = 12.93$ MPa ◀

18-54

$$\sigma_x = \sigma_\ell = \frac{pr_i}{2t} = \frac{(6\text{ MPa})(0.250\text{m})}{2(0.015\text{m})} = 50\text{ MPa (T)}$$

$$\sigma_y = \sigma_c = \frac{pr_i}{t} = 2\sigma_\ell = 100\text{ MPa (T)}$$

$r_o = r_i + t = 0.250 + 0.015 = 0.265$ m

$$J = \frac{\pi(r_o^4 - r_i^4)}{2} = \frac{\pi(0.265^4 - 0.250^4)}{2} = 0.00161\text{ m}^4$$

$$|\tau_x| = \frac{Tc}{J} = \frac{Tr_o}{J} = \frac{(T)(0.265)}{0.00161}$$

$= 164.6\,T\text{ kN/m}^2 = 0.1646\,T$ MPa

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_x^2}$$

$$= \sqrt{\left(\frac{50 - 100}{2}\right)^2 + (0.1646T)^2}$$

$$= \sqrt{625 + 0.0271T^2} \le 100\text{ MPa}$$

$625 + 0.0271\,T^2 \le 10\,000$

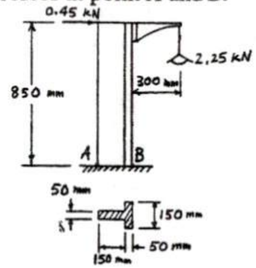
$$T \le \sqrt{\frac{10\,000 - 625}{0.0271}} = 588\text{ kN}\cdot\text{m}$$

$T_{max} = 588$ kN • m ◀

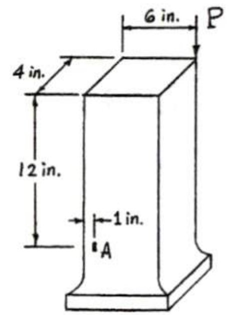
Test Problems for Chapter 18

The following problems may either be given to students for them to practice their problem solving skills or be used as test problems. The answers are provided on the next page.

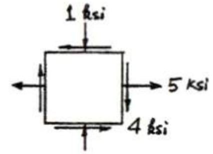
(1) For the light fixture subjected to the loads shown, determine the normal stresses at point *A* and *B*.



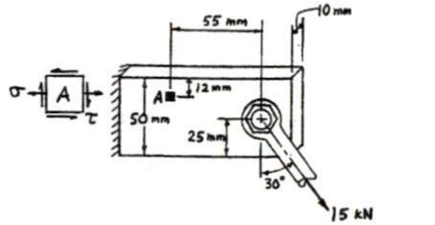
(2) An aluminum alloy block is subjected to an eccentric axial load *P* shown. The linear strain at *A* (measured by an electrical strain gage) is 500 x 10^{-6} in./in. due to the applied load *P*. If the modulus of elasticity of aluminum is $E = 10 \times 10^6$ psi, determine the magnitude of the force *P*.



(3) For the element subjected to the given state of plane stresses shown, determine (*a*) the principal stresses, the orientation of the principal planes, and (*b*) the maximum and the minimum shear stresses and the associated normal stresses and the planes where these stresses occurs.

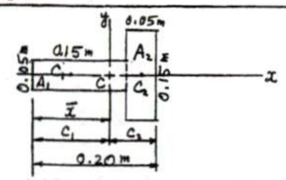


(4) A clevis transmits a load of 15 kN to the bracket shown. Determine (*a*) the state of stress of the element at point *A*, (*b*) the principal stresses and the orientation of the principal planes of the element.



Solutions to Test Problems for Chapter 18

(1)



$A = 0.150 \text{ m} \times 0.050 \text{ m} + 0.050 \text{ m} \times 0.150 \text{ m}$
$= 0.075 \text{ m}^2 + 0.075 \text{ m}^2 = 0.015 \text{ m}^2$

$$\bar{x} = \frac{(0.0075\text{m}^2)(0.075\text{m}) + (0.0075\text{m}^2)(0.175\text{m})}{0.0075 \text{ m}^2 + 0.0075 \text{ m}^2}$$

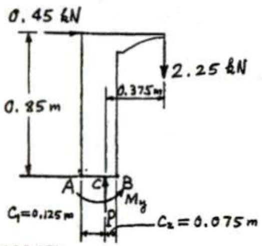
$= 0.125 \text{ m}$

$$I_y = \Sigma[I + A(\bar{y} - y)^2]$$

$$= \left[\frac{(0.150\text{m})(0.050\text{ m})^3}{12} + (0.0075\text{m}^2)(0.125\text{m} - 0.075)^2\right]$$

$$+ \left[\frac{(0.050\text{ m})(0.150\text{ m})^3}{12} + (0.0075\text{m}^2)(0.125\text{m} - 0.175)^2\right]$$

$= 5.31 \times 10^{-5} \text{ m}^4$



$P = 2.25 \text{ kN (C)}$
$M_y = (0.45 \text{ kN})(0.85 \text{ m}) + (2.25 \text{ kN})(0.375 \text{ m})$
$= 1.226 \text{ kN} \cdot \text{m}$

$$\sigma_A = -\frac{P}{A} + \frac{M_y c_1}{I_y} = -\frac{2.25 \text{ kN}}{0.015 \text{ m}^2} + \frac{(1.226 \text{ kN}\cdot\text{m})(0.125 \text{ m})}{5.31 \times 10^{-5} \text{ m}^4}$$

$= -150 \text{ kN/m}^2 + 2890 \text{ kN/m}^2$

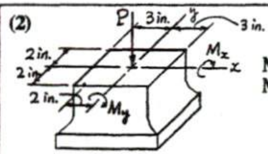
$= 2740 \text{ kN/m}^2 = 2.74 \text{ MPa (T)}$ ◀

$$\sigma_B = -\frac{P}{A} - \frac{M_y c_2}{I_y} = -\frac{2.25 \text{ kN}}{0.015 \text{ m}^2} + \frac{(1.226 \text{ kN}\cdot\text{m})(0.075 \text{ m})}{5.31 \times 10^{-5} \text{ m}^4}$$

$= -150 \text{ kN/m}^2 - 1730 \text{ kN/m}^2$

$= -1880 \text{ kN/m}^2 = -1.88 \text{ MPa (C)}$ ◀

(2)



$M_x = P(2 \text{ in.}) = 2P$
$M_y = P(3 \text{ in.}) = 3P$

$\sigma_A = E\varepsilon_A = (10 \times 10^6 \text{ psi})(500 \times 10^{-6}) = 5000 \text{ psi}$

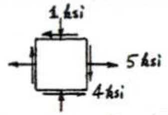
$$\sigma_A = -\frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y x}{I_y}$$

$$= -\frac{P}{24} + \frac{(2P)(2)}{\frac{(6)(4)^3}{12}} + \frac{(3P)(2)}{\frac{(4)(6)^3}{12}}$$

$= 0.1667 \text{ P} = 5000 \text{ psi}$

$$P = \frac{5000}{0.1667} = 30\,000 \text{ lb} = 30 \text{ kips}$$ ◀

(3)



$\sigma_x = 5 \text{ ksi}$
$\sigma_y = -1 \text{ ksi}$
$\tau_x = -4 \text{ ksi}$

$$a = \frac{\sigma_x + \sigma_y}{2} = \frac{5 + (-1)}{2} = 2 \text{ ksi}$$

$$b = \frac{\sigma_x - \sigma_y}{2} = \frac{5 - (-1)}{2} = 3 \text{ ksi}$$

$$r = \sqrt{b^2 + \tau_x^2} = \sqrt{3^2 + (-4)^2} = 5 \text{ ksi}$$

(a) Principal stresses and principal planes:

$\sigma_1 = a + r = 2 + 5 = 7 \text{ ksi}$ ◀

$\sigma_2 = a - r = 2 - 5 = -3 \text{ ksi}$ ◀

$$\theta_1 = \tan^{-1}\left[\frac{r - b}{\tau_x}\right] = \tan^{-1}\left[\frac{5 - 3}{-4}\right]$$

$$= \tan^{-1}\left[-\frac{1}{2}\right] = -26.6^\circ$$ ◀

$\theta_2 = \theta_1 + 90^\circ = -26.6^\circ + 90^\circ = +63.4^\circ$ ◀

(b) Maximum and minimum shear stresses and their planes:

$\tau_{max} = r = 5 \text{ ksi}$ ◀

$\tau_{max} = -r = -5 \text{ ksi}$ ◀

$\sigma' = a = 2 \text{ ksi}$ ◀

(Cont'd)

Solutions to Test Problems for Chapter 18 (Cont'd)

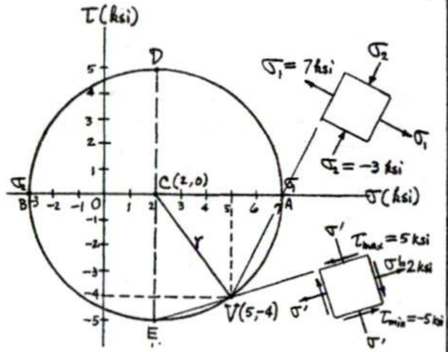
(3) (Cont)

$$\theta_s = -\tan^{-1}\left[\frac{b}{r + \tau_x}\right] = -\tan^{-1}\left[\frac{3}{5 + (-4)}\right]$$

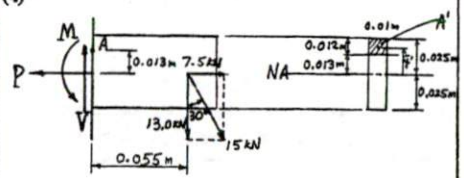
$= -\tan^{-1}[3] = -71.6^\circ$ ◀

$\theta_s' = \theta_s + 90^\circ = -71.6^\circ + 90^\circ = +18.4^\circ$ ◀

Using Mohr's circle to check the answers, we obtain:



(4)



The internal forces at the section through A are:

$P = 7.5 \text{ kN(T)}$
$V = 13.0 \text{ kN}$
$M = (13 \text{ kN})(0.055 \text{ m}) = 0.715 \text{ kN} \cdot \text{m}$

For the 10 mm x 50 mm section:

$A = (0.010 \text{ m})(0.050 \text{ m}) = 5.0 \times 10^{-4} \text{ m}^3$

$$I = \frac{(0.010 \text{ m})(0.050 \text{ m})^3}{12} = 1.042 \times 10^{-7} \text{ m}^4$$

The first moment of area A′ about the N.A. is

$Q = A'\bar{y}' = (0.010 \times 0.012)(0.019) = 2.28 \times 10^{-6} \text{ m}^3$

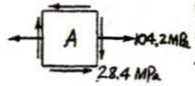
$$\sigma_x = \frac{P}{A} + \frac{My}{I}$$

$$= \frac{7.5 \text{ kN}}{5.0 \times 10^{-4} \text{ m}^2} + \frac{(0.715 \text{ kN}\cdot\text{m})(0.013 \text{ m})}{1.042 \times 10^{-7} \text{ m}^4}$$

$= 104\,200 \text{ kN/m}^2 = 104.2 \text{ MPa}$

$$|\tau_x| = \frac{VQ}{It} = \frac{(13.0 \text{ kN})(2.28 \times 10^{-6} \text{ m}^3)}{(1.042 \times 10^{-7} \text{ m}^4)(0.010 \text{ m})}$$

$= 28\,400 \text{ kN/m}^2 = 28.4 \text{ MPa}$



$\sigma_x = +104.2 \text{ MPa}$
$\sigma_y = 0$
$\tau_x = -28.4 \text{ MPa}$

$$a = \frac{\sigma_x + \sigma_y}{2} = \frac{104.2 + 0}{2} = 52.1 \text{ MPa}$$

$$b = \frac{\sigma_x - \sigma_y}{2} = \frac{104.2 - 0}{2} = 52.1 \text{ MPa}$$

$$r = \sqrt{b^2 + \tau_x^2} = \sqrt{52.1^2 + 28.4^2} = 59.3 \text{ MPa}$$

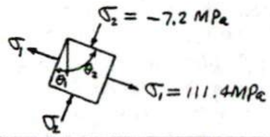
$\sigma_1 = a + r = 52.1 + 59.3 = +111.4 \text{ MPa}$ ◀

$\sigma_2 = a - r = 52.1 - 59.3 = -7.2 \text{ MPa}$ ◀

$$\theta_1 = \tan^{-1}\left[\frac{r - b}{\tau_x}\right] = \tan^{-1}\left[\frac{59.3 - 52.1}{-28.4}\right]$$

$= -14.2^\circ$ ◀

$\theta_2 = \theta_1 + 90^\circ = -14.2^\circ + 90^\circ = +75.8^\circ$ ◀



19-1

For the full-sized 2 in. x 4 in. section, the least radius of gyration is

$r_{min} = 0.289(2\text{ in.}) = 0.578\text{ in}$

For pinned ends, k = 1

$$\text{Slenderness ratio} = \frac{kL}{r_{min}} = \frac{1.0(8 \times 12\text{ in.})}{0.578\text{ in.}} = 166 \blacktriangleleft$$

19-2

From Appendix Table A-5(b), for 200 mm standard weight steel pipe:

r = 0.0747 m

For fixed-free ends, k = 2

$$\text{slenderness ratio} = \frac{kL}{r} = \frac{2(3\text{ m})}{0.0747\text{ m}} = 80.3 \blacktriangleleft$$

19-3

For square section: $a^2 = A, \quad a = \sqrt{A}$

$$r = 0.289a = 0.289\sqrt{A}$$

For circular section: $\frac{\pi}{4}d^2 = A, \quad d = 2\sqrt{\frac{A}{\pi}}$

$$r = \frac{d}{4} = \frac{1}{2}\sqrt{\frac{A}{\pi}} = 0.282\sqrt{A}$$

For the same cross-sectional area, a square column is better than a circular column, because the radius of gyration of a square section is greater than that of a circular section.

19-4

$\sigma_p = 33$ ksi

$$E = \frac{\sigma}{\varepsilon} = \frac{33\text{ ksi}}{0.00113} = 29.2 \times 10^3\text{ ksi}$$

$$\left(\frac{kL}{r}\right)_{min} = \sqrt{\frac{\pi^2 E}{\sigma_p}} = \sqrt{\frac{\pi^2(29.2 \times 10^3\text{ ksi})}{33\text{ ksi}}}$$

$$= 93.5 \blacktriangleleft$$

19-5

$$I = \frac{(0.1\text{ m})(0.1\text{ m})^3}{12} = 8.33 \times 10^{-6}\text{ m}^4$$

For pinned ends, k = 1

For square section, r = 0.289(0.1 m) = 0.0289 m

$$\frac{kL}{r} = \frac{1(2.5\text{ m})}{0.0289\text{ m}} = 86.5$$

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{kL}{r}\right)^2} = \frac{\pi^2(12 \times 10^9\text{ N/m}^2)}{(86.5)^2}$$

$$= 1.58 \times 10^7\text{ N/m}^2$$

$$P_{cr} = A\sigma_{cr} = (0.1 \times 0.1\text{ m}^2)(1.58 \times 10^7\text{ N/m}^2)$$

$$= 158 \times 10^3\text{ N} = 158\text{ kN} \blacktriangleleft$$

19-6

For circular section,

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.08\text{ m})^2 = 5.03 \times 10^{-3}\text{ m}^2$$

$$r = \frac{d}{4} = \frac{0.08\text{ m}}{4} = 0.02\text{ m}$$

For fixed ends, k = 0.5

$$\frac{kL}{r} = \frac{0.5(4\text{ m})}{0.02\text{ m}} = 100$$

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{kL}{r}\right)^2} = \frac{\pi^2(70 \times 10^6\text{ kN/m}^2)}{(100)^2}$$

$= 69.1 \times 10^3\text{ kN/m}^2 < \sigma_p = 230$ MPa (O.K.)

$$P_{cr} = A\sigma_{cr}$$

$$= (5.03 \times 10^{-3}\text{ m}^2)(69.1 \times 10^3\text{ kN/m}^2)$$

$$= 348\text{ kN} \blacktriangleleft$$

19-7

For L4 x 4 x $\frac{1}{2}$, A = 3.75 in.²

$r_{min} = r_z = 0.782$ in.

For fixed-pinned ends, k = 0.7

$$\frac{kL}{r} = \frac{0.7(18 \times 12\text{ in.})}{0.782\text{ in.}} = 193$$

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{kL}{r}\right)^2} = \frac{\pi^2(29 \times 10^3\text{ ksi})}{(193)^2}$$

$= 7.68\text{ ksi} < \sigma_p = 34$ ksi (O.K.)

$$P_{cr} = A\sigma_{cr} = (3.75\text{ in.}^2)(7.68\text{ kips/in.}^2)$$

$$= 28.8\text{ kips} \blacktriangleleft$$

19-8

For 2-in. standard steel pipe,

A = 1.07 in.², r = 0.787 in.

For fixed-free condition, k = 2

$$\frac{kL}{r} = \frac{2(4 \times 12\text{ in})}{0.0787\text{ in.}} = 122$$

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{kL}{r}\right)^2} = \frac{\pi^2(29 \times 10^3\text{ ksi})}{(122)^2}$$

$= 19.2\text{ ksi} < \sigma_p = 34$ ksi (O.K.)

$$P_{cr} = A\sigma_{cr} = (1.07\text{ in.}^2)(19.2\text{ kips/in.}^2)$$

$$= 20.5\text{ kips} \blacktriangleleft$$

19-9

For W12 x 87 section,

A = 25.6 in.², $r_{min} = r_y = 3.07$ in.

For pinned condition, k = 1

$$\frac{kL}{r} = \frac{1(25 \times 12\text{ in})}{3.07\text{ in.}} = 97.7$$

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{kL}{r}\right)^2} = \frac{\pi^2(29 \times 10^3\text{ ksi})}{(97.7)^2}$$

$= 30.0\text{ ksi} < \sigma_p = 34$ ksi (O.K.)

$$P_{cr} = A\sigma_{cr} = (25.6\text{ in.}^2)(30.0\text{ kip/in.}^2) = 768\text{ kips}$$

$$P_{allow} = \frac{P_{cr}}{F.S.} = \frac{768\text{ kips}}{2.0} = 384\text{ kips} \blacktriangleleft$$

19-10

For W300 x 0.58 section,

A = 7.61 x 10⁻³ m², $r_{min} = r_y = 0.0490$ m

For fixed ends, k = 0.5

$$\frac{kL}{r} = \frac{0.5(15\text{ m})}{0.0490\text{ m}} = 153$$

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{kL}{r}\right)^2} = \frac{\pi^2(200 \times 10^3\text{ MPa})}{(153)^2}$$

$= 84.3\text{ MPa} < \sigma_p = 200$ MPa (O.K.)

$$P_{cr} = A\sigma_{cr} = (7.61 \times 10^{-3}\text{ m}^2)(84.3\text{ MN/m}^2)$$

$$= 642 \times 10^{-3}\text{ MN} = 642\text{ kN}$$

$$P_{allow} = \frac{P_{cr}}{F.S.} = \frac{642\text{ kN}}{2.0} = 321\text{ kN} \blacktriangleleft$$

19-11

$P_{cr} = (F.S.)\,P_{allow} = 2(20\text{ kN}) = 40$ kN

For pinned ends, k = 1

From the Euler equation $P_{cr} = \frac{\pi^2 EI}{(kL)^2}$ we get

$$I = \frac{P_{cr}(kL)^2}{\pi^2 E} = \frac{(40\text{ kN})(1 \times 1.2\text{ m})^2}{\pi^2(200 \times 10^6\text{ kN/m}^2)}$$

$$= 2.918 \times 10^{-8}\text{ m}^4$$

For a square section of dimension b,

$$I = \frac{b(b)^3}{12} = \frac{b^4}{12}$$

$$b = \sqrt[4]{12\,I} = \sqrt[4]{12(2.918 \times 10^{-8}\text{ m}^4)}$$

$$= 0.0243\text{ m} = 24.3\text{ mm} \blacktriangleleft$$

19-12

$P_{cr} = (F.S.)\,P_{allow} = 2(5\text{ kips}) = 10\text{ kips}$

For fixed ends, $k = 0.5$

From the Euler equation $P_{cr} = \dfrac{\pi^2 EI}{L^2}$ we get

$$I = \frac{P_{cr}(kL)^2}{\pi^2 E} = \frac{(10\text{ kips})(0.5 \times 20 \times 12\text{ in.})^2}{\pi^2(29 \times 10^3\text{ kip/in.}^2)}$$

$$= 0.503\text{ in.}^4$$

For a circular section, $I = \dfrac{\pi d^4}{64}$

$$d = \sqrt[4]{\frac{64\,I}{\pi}} = \sqrt[4]{\frac{64(0.503\text{ in.}^4)}{\pi}} = 1.79\text{ in.}$$ ◀

19-13

For fixed-free ends, $k = 2$

For $\frac{3}{4}$ in. standard weight steel pipe,

$r = 0.334\text{ in.}$, $A = 0.333\text{ in.}^2$

$$\frac{kL}{r} = \frac{2(10 \times 12\text{ in.})}{0.334\text{ in.}} = 719$$

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{kL}{r}\right)^2} = \frac{\pi^2(29 \times 10^6\text{ lb/in.}^2)}{(719)^2} = 554\text{ lb/in.}^2$$

$$P_{cr} = A\sigma_{cr} = (0.333\text{ in.}^2)(554\text{ lb/in.}^2)$$

$$= 184\text{ lb}$$

Hence the 200 lb worker cannot get to the top of the pole before it buckles. ◀

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{200\text{ lb}}{0.333\text{ in}^2} = 600.6\text{ psi}$$

From $\sigma_{cr} = \dfrac{\pi^2 E}{\left(\frac{kL}{r}\right)^2}$ we get

$$\frac{kL}{r} = \sqrt{\frac{\pi^2 E}{\sigma_{cr}}} = \sqrt{\frac{\pi^2(29 \times 10^6\text{ psi})}{600.6\text{ psi}}} = 690$$

The height when buckling occurs is

$$L = \frac{690r}{k} = \frac{(690)(0.334\text{ in.})}{2}$$

$$= 115\text{ in.} = 9.61\text{ ft}$$ ◀

Note: Since a factor of safety is not used, it is not recommended to climb to this height. Once buckling occurs, the pole may suddenly deflect and snap and so it could be quite dangerous.

19-14

$r = 0.289(0.060\text{ m}) = 0.0173\text{ m}$

For pinned ends, $k = 1$

$$\frac{kL}{r} = \frac{1(3\text{ m})}{0.0173\text{ m}} = 173$$

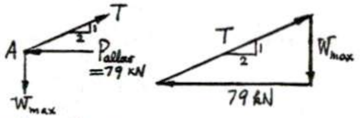
$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{kL}{r}\right)^2} = \frac{\pi^2(200 \times 10^6\text{ kN/m}^2)}{(173)^2}$$

$$= 65\,950\text{ kN/m}^2$$

$$P_{cr} = A\sigma_{cr} = (0.060\text{ m}^2)(65\,950\text{ kN/m}^2)$$

$$= 237\text{ kN}$$

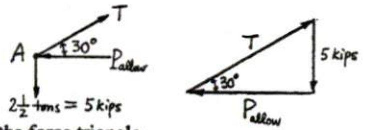
$$P_{allow} = \frac{P_{cr}}{F.S.} = \frac{237\text{ kN}}{3} = 79\text{ kN}$$



From the force diagram,

$$W_{max} = (79\text{ kN})/2 = 39.5\text{ kN}$$ ◀

19-15



From the force triangle,

$$P_{allow} = \frac{5\text{ kips}}{\tan 30^\circ} = 8.66\text{ kips}$$

$P_{cr} = (F.S.)\,P_{allow} = 3\,(8.66\text{ kips}) = 26.0\text{ kips}$

For pinned ends, $k = 1$

From $P_{cr} = \dfrac{\pi^2 EI}{(kL)^2}$ we get

$$I = \frac{P_{cr}(kL)^2}{\pi^2 E} = \frac{(26\text{ kips})(1 \times 120\text{ in.})^2}{\pi^2(29 \times 10^3\text{ kip/in.}^2)}$$

$$= 1.31\text{ in.}^4$$

Use $2\frac{1}{2}$-in. standard steel pipe ◀

(It provides a moment of inertia of 1.53 in.⁴ and is adequate for member AB.)

19-16

For the rectangular section

$r = 0.289\,(0.050\text{ m}) = 0.01445\text{ m}$

For pinned ends, $k = 1$

$$\frac{kL}{r} = \frac{1(2\text{ m})}{0.01445\text{ m}} = 138.4$$

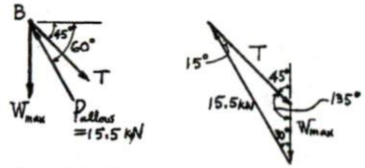
$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{kL}{r}\right)^2} = \frac{\pi^2(12 \times 10^6\text{ kN/m}^2)}{(138.4)^2}$$

$$= 6183\text{ kN/m}^2$$

$$P_{cr} = A\sigma_{cr} = (0.05 \times 0.10\text{ m}^2)(6183\text{ kN/m}^2)$$

$$= 30.9\text{ kN}$$

$$P_{allow} = \frac{P_{cr}}{F.S.} = \frac{30.9\text{ kN}}{2} = 15.5\text{ kN}$$



From the force triangle,

$$\frac{W_{max}}{\sin 15^\circ} = \frac{15.5\text{ kN}}{\sin 135^\circ}$$

$$W_{max} = \frac{(15.5\text{ kN})\sin 15^\circ}{\sin 135^\circ} = 5.67\text{ kN}$$ ◀

19-17

$W = 1\frac{1}{2}\text{ tons} = 3\text{ kips}$

$$\frac{F_{AC}}{\sin 80^\circ} = \frac{F_{AB}}{\sin 80^\circ} = \frac{3\text{ kips}}{\sin 20^\circ}$$

$F_{AC} = F_{AB} = 8.64\text{ kips} = P_{allow}$

$P_{cr} = (F.S.)\,P_{allow} = 3\,(8.64\text{ kips}) = 25.9\text{ kips}$

From $P_{cr} = \dfrac{\pi^2 EI}{(kL)^2}$ we get

$$I = \frac{P_{cr}(kL)^2}{\pi^2 E} = \frac{(25.9\text{ kips})(1 \times 3.5 \times 12\text{ in.})^2}{\pi^2(29 \times 10^3\text{ kip/in.}^2)}$$

$$= 160\text{ in.}^4$$

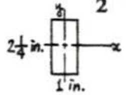
From $I = \dfrac{\pi}{64}d^4$ we get

$$d = \sqrt[4]{\frac{64I}{\pi}} = \sqrt[4]{\frac{64(0.160)}{\pi}} = 1.34\text{ in.}$$

$$d_{req} = 1.34\text{ in.}$$ ◀

19-18

For full size 1 in x $2\frac{1}{2}$ in. section:



$$\frac{kL_x}{r_x} = \frac{1(60)}{0.289 \times 2.25} = 92$$

$$\frac{kL_y}{r_y} = \frac{1(30)}{0.289 \times 1} = 104$$

$\frac{kL_x}{r_x}$ controls

$$\left(\frac{kL}{r}\right)_{min} = \sqrt{\frac{\pi^2 E}{\sigma_p}} = \sqrt{\frac{\pi^2(30\ 000\ \text{ksi})}{30\ \text{ksi}}} = 99$$

Since $\frac{kL_x}{r_x} = 92 < 99$, Euler eq. applies

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{kL}{r}\right)^2} = \frac{\pi^2(30\ 000\ \text{ksi})}{(104)^2} = 27.4\ \text{ksi}$$

$P_{cr} = A\sigma_{cr} = (1 \times 2.25)(27.4) = 61.6$ kips

$$P_{allow} = \frac{P_{cr}}{F.S.} = \frac{61.6\ \text{kips}}{2} = 30.8\ \text{kips}$$

The equilibrium of the beam requires:

$F(36+6) = P_{allow}(6)$

From which

$$F = \frac{6}{42}(30.8) = 4.4\ \text{kips}$$ ◀

19-19

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2(200 \times 10^3\ \text{MPa})}{250\ \text{MPa}}} = 126$$

$r = 0.289\ (0.10\ \text{m}) = 0.0289$ m

For pinned-ends, $k = 1$

$$\frac{kL}{r} = \frac{1(2.5\ \text{m})}{0.0289\ \text{m}} = 86.5 < C_c = 126$$

∴ J.B. Johnson formula applies.

$$\sigma_{cr} = \left[1 - \frac{(kL/r)^2}{2C_c^2}\right]\sigma_y = \left[1 - \frac{(86.5)^2}{2(126)^2}\right](250\ \text{MPa})$$

$= 191$ MPa

$P_{cr} = A\sigma_{cr} = (0.1 \times 0.2\ \text{m}^2)(191\ \text{MN/m}^2)$

$= 3.82$ MN ◀

19-20

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2(200 \times 10^3\ \text{MPa})}{250\ \text{MPa}}} = 126$$

$$r = \frac{d}{4} = \frac{0.1\ \text{m}}{4} = 0.025\ \text{m}$$

For fixed-pinned ends, $k = 0.7$

$$\frac{kL}{r} = \frac{0.7(3.5\ \text{m})}{0.025\ \text{m}} = 98 < C_c = 126$$

∴ J.B. Johnson formula applies.

$$\sigma_{cr} = \left[1 - \frac{(kL/r)^2}{2C_c^2}\right]\sigma_y = \left[1 - \frac{(98)^2}{2(126)^2}\right](250\ \text{MPa})$$

$= 174.4$ MPa

$$P_{cr} = A\sigma_{cr} = \frac{\pi}{4}(0.1\text{m}^2)(174.4\ \text{MN/m}^2)$$

$= 1.37$ MN ◀

19-21

$$Cc = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2(29 \times 10^3\ \text{ksi})}{50\ \text{ksi}}} = 107.0$$

For 5-in. standard weight steel pipe,
$A = 4.30$ in.², $r = 1.88$ in.

For fixed-free ends, $k = 2$

$$\frac{kL}{r} = \frac{2(6 \times 12\ \text{in.})}{1.88\ \text{in.}} = 76.6 < C_c^2 = 107.0$$

∴ J.B. Johnson formula applies.

$$\sigma_{cr} = \left[1 - \frac{(kL/r)^2}{2C_c^2}\right]\sigma_y = \left[1 - \frac{(76.6)^2}{2(107.0)^2}\right](50\ \text{ksi})$$

$= 37.2$ ksi

$P_{cr} = A\sigma_{cr} = (4.30\ \text{in.}^2)(37.2\ \text{kips/in.}^2)$

$= 160$ kips ◀

19-22

From Prob. 18-22, $C_c = 107.0$

For W14 x 74, $A = 21.8$ in.², $r_{min} = 2.48$ in.

For column with fixed ends, $k = 0.5$

$$\frac{kL}{r} = \frac{0.5(40 \times 12\ \text{in.})}{2.48\ \text{in.}} = 96.8 < C_c = 107.0$$

∴ J.B. Johnson formula applies.

$$\sigma_{cr} = \left[1 - \frac{(kL/r)^2}{2C_c^2}\right]\sigma_y = \left[1 - \frac{(96.8)^2}{2(126)^2}\right](50\ \text{ksi})$$

$= 29.5$ ksi

$P_{cr} = A\sigma_{cr} = (21.8\ \text{in.}^2)(29.5\ \text{kips/in.}^2)$

$= 644$ kips ◀

19-23

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2(200 \times 10^3\ \text{MPa})}{345\ \text{MPa}}} = 107.0$$

For W360 x 1.31,
$A = 17.1 \times 10^3$ in², $r_{min} = r_y = 0.0940$ m
For column with fixed-pinned ends, $k = 0.7$

$$\frac{kL}{r} = \frac{0.7(10\ \text{m})}{0.0940\ \text{m}} = 74.5 < C_c = 107.0$$

∴ J.B. Johnson formula applies.

$$\sigma_{cr} = \left[1 - \frac{(kL/r)^2}{2C_c^2}\right]\sigma_y = \left[1 - \frac{(74.5)^2}{2(107.0)^2}\right](345\ \text{MPa})$$

$= 261$ MPa

$P_{cr} = A\sigma_{cr} = (17.1 \times 10^{-3}\ \text{m}^2)(261 \times 10^3\ \text{kN/m}^2)$

$= 4470$ kN

$$P_{allow} = \frac{P_{cr}}{F.S.} = \frac{4470\ \text{kN}}{2.0} = 2235\ \text{kN}$$ ◀

19-24

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2(29 \times 10^3\ \text{ksi})}{50\ \text{ksi}}} = 107.0$$

For W12 x 65,
$A = 19.1$ in.², $r_{min} = r_y = 3.02$ in.

For column with fixed ends, $k = 0.5$

$$\frac{kL}{r} = \frac{0.5(25 \times 12\ \text{in.})}{3.02\ \text{in.}} = 49.7 < C_c = 107.0$$

∴ J.B. Johnson formula applies.

$$\sigma_{cr} = \left[1 - \frac{(kL/r)^2}{2C_c^2}\right]\sigma_y = \left[1 - \frac{(49.7)^2}{2(107.0)^2}\right](50\ \text{ksi})$$

$= 44.6$ ksi

$P_{cr} = A\sigma_{cr} = (19.1\ \text{in.}^2)(44.6\ \text{kip/in.}^2) = 852$ kips

$$P_{allow} = \frac{P_{cr}}{F.S.} = \frac{852\ \text{kips}}{2.0} = 426\ \text{kips}$$ ◀

19-25

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2(29 \times 10^3 \text{ ksi})}{50 \text{ ksi}}} = 107.0$$

For rectangular section,
$r = 0.289(1 \text{ in.}) = 0.289$ in.

For pinned ends, $k = 1$

$$\frac{kL}{r} = \frac{1(2 \times 12 \text{ in.})}{0.289 \text{ in.}} = 83.0 < C_c = 107.0$$

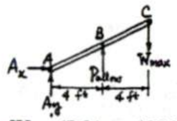
∴ J.B. Johnson formula applies.

$$\sigma_{cr} = \left[1 - \frac{(kL/r)^2}{2C_c^2}\right]\sigma_y = \left[1 - \frac{(83.0)^2}{2(126)^2}\right](50 \text{ ksi})$$

$$= 35.0 \text{ ksi}$$

$P_{cr} = A\sigma_{cr} = (1 \times 2 \text{ in.}^2)(35.0 \text{ kips/in.}^2)$
$= 70$ kips

$$P_{allow} = \frac{P_{cr}}{F.S.} = \frac{70 \text{ kips}}{2.5} = 28 \text{ kips}$$



$$\sum M_A = -W_{max}(8 \text{ ft}) + (28 \text{ kips})(4 \text{ ft}) = 0$$

$W_{max} = 14$ kips ◀

19-26

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2(200 \times 10^3 \text{ MPa})}{250 \text{ MPa}}} = 126$$

$$r = \frac{d}{4} = \frac{0.1 \text{ m}}{4} = 0.025 \text{ m}$$

The end condition of member AB is considered pinned on both sides, hence $k = 1$

$$\frac{kL}{r} = \frac{1(2 \text{ m})}{0.025 \text{ m}} = 80.0 < C_c = 126$$

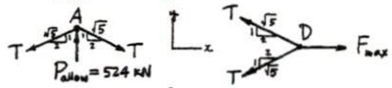
∴ J.B. Johnson formula applies.

$$\sigma_{cr} = \left[1 - \frac{(kL/r)^2}{2C_c^2}\right]\sigma_y = \left[1 - \frac{(80.0)^2}{2(126)^2}\right](250 \text{ MPa})$$

$$= 200 \text{ MPa}$$

$$P_{cr} = A\sigma_{cr} = \frac{\pi}{4}(0.1 \text{m})^2(200 \text{ MN/m}^2) = 1570 \text{ kN}$$

$$P_{allow} = \frac{P_{cr}}{F.S.} = \frac{1570 \text{ kN}}{3} = 524 \text{ kN}$$



$$\sum F_y = 524 - 2T \times \frac{1}{\sqrt{5}} = 0$$

$T = 586$ kN

$$\sum F_x = F_{max} - 2\left(586 \times \frac{2}{\sqrt{5}}\right) = 0$$

$F_{max} = 1048$ kN ◀

19-27

For 3 in. standard steel pipe:
$A = 2.23$ in.^{2} $r = 1.16$ in.

$$\frac{kL}{r} = \frac{(2.10)(5 \times 12 \text{ in.})}{1.16 \text{ in.}} = 109 < C_c = 126.1$$

∴ J.B. Johnson formula applies.

$$F.S. = \frac{5}{3} + \frac{3(kL/r)}{8C_c} - \frac{(kL/r)^3}{8C_c^3}$$

$$= \frac{5}{3} + \frac{3(109)}{8(126.1)} - \frac{(109)^3}{8(126.1)^3} = 1.91$$

$$\sigma_{allow} = \frac{\left[1 - \frac{(kL/r)^2}{2C_c^2}\right]\sigma_y}{F.S.} = \frac{\left[1 - \frac{(109)^2}{2(126.1)^2}\right](36 \text{ ksi})}{1.91}$$

$$= 11.81 \text{ ksi}$$

From Table 19-2, for $kL/r = 109$,
$\sigma_{allow} = 11.81$ ksi (checks)

$$P_{allow} = A\sigma_{allow} = (2.23 \text{ in.}^2)(11.81 \text{ kips/in.}^2)$$

$$= 26.3 \text{ kips}$$ ◀

19-28

$$\frac{kL}{r} = \frac{(2.10)(8 \times 12 \text{ in.})}{1.16 \text{ in.}} = 174 > C_c = 126.1$$

∴ Euler formula applies.

$$\sigma_{allow} = \frac{\pi^2 E/(kL/r)^2}{1.92} = \frac{\pi^2(29\ 000 \text{ ksi})/(174)^2}{1.92}$$

$$= 4.92 \text{ ksi}$$

From Table 19-2, for $kL/r = 174$
$\sigma_{allow} = 4.93$ ksi (checks)

$$P_{allow} = A\sigma_{allow} = (2.23 \text{ in.}^2)(4.92 \text{ kips/in.}^2)$$

$$= 11.0 \text{ kips}$$ ◀

19-29

For W8 x 40, $A = 11.7$ in.^{2}, $r_y = 2.04$ in.
For column with both ends fixed, $k = 0.65$

$$\frac{kL}{r} = \frac{(0.65)(20 \times 12 \text{ in.})}{2.04 \text{ in.}} = 76.5 < C_c = 107$$

∴ J.B. Johnson formula applies.

$$F.S. = \frac{5}{3} + \frac{3(kL/r)}{8C_c} - \frac{(kL/r)^3}{8C_c^3}$$

$$= \frac{5}{3} + \frac{3(76.5)}{8(107)} - \frac{(76.5)^3}{8(107)^3} = 1.89$$

$$\sigma_{allow} = \frac{\left[1 - \frac{(kL/r)^2}{2C_c^2}\right]\sigma_y}{F.S.} = \frac{\left[1 - \frac{(76.5)^2}{2(107)^2}\right](50 \text{ ksi})}{1.89}$$

$$= 19.7 \text{ kips}$$

From Table 19-3, By interpolation for $kL/r = 76.5$,
$\sigma_{allow} = 19.70$ ksi (checks)

$$P_{allow} = A\sigma_{allow} = (11.7 \text{ in.}^2)(19.7 \text{ kips/in.}^2)$$

$$= 230 \text{ kips}$$ ◀

19-30

$$\frac{kL}{r} = \frac{(0.65)(30 \times 12 \text{ in.})}{2.04 \text{ in.}} = 115 > C_c = 107$$

∴ Euler formula applies.

$$\sigma_{allow} = \frac{\pi^2 E/(kL/r)^2}{1.92} = \frac{\pi^2(29\ 000 \text{ ksi})/(115)^2}{1.92}$$

$= 11.27$ ksi

From Table 19-3, for $kL/r = 115$
$\sigma_{allow} = 11.29$ ksi (checks)

$$P_{allow} = A\sigma_{allow} = (11.7 \text{ in.}^2)(11.27 \text{ kips/in.}^2)$$

$$= 132 \text{ kips}$$ ◀

19-31

For W250 x 1.63,
$A = 21.2 \times 10^3$ m^2, $r_y = 0.0681$ m
For column with fixed-pinned ends, $k = 0.8$

$$\frac{kL}{r} = \frac{(0.8)(9 \text{ m})}{0.0681 \text{ m}} = 105.7 < C_c = 126.1$$

∴ J.B. Johnson formula applies.

$$\frac{kL/r}{C_c} = \frac{105.7}{126.1} = 0.838$$

$$F.S. = \frac{5}{3} + \frac{3(kL/r)}{8C_c} - \frac{(kL/r)^3}{8C_c^3}$$

$$= \frac{5}{3} + \frac{3(0.838)}{8} - \frac{(0.838)^3}{8} = 1.91$$

$$\sigma_{allow} = \frac{\left[1 - \frac{(kL/r)^2}{2C_c^2}\right]\sigma_y}{F.S.} = \frac{\left[1 - \frac{(0.838)^2}{2}\right](250 \text{ MPa})}{1.91}$$

$= 84.9$ MPa

From Table 19-3, By interpolation for
$kL/r = 105.7$ we get,
$\sigma_{allow} = 12.25 \text{ ksi} = 12.25 \times 6.895$
$= 84.5$ MPa (checks)

$P_{allow} = A\sigma_{allow}$

$$= (21.3 \times 10^3 \text{ m}^2)(84.9 \times 10^3 \text{ kN/m}^2)$$

$$= 1800 \text{ kN}$$ ◀

19-32

$$\frac{kL}{r} = \frac{(0.8)(12\text{ m})}{0.0681\text{ m}} = 141 > C_c = 126.1$$

$\therefore$ Euler formula applies.

$$\sigma_{allow} = \frac{\pi^2 E/(kL/r)^2}{1.92}$$
$$= \frac{\pi^2(200 \times 10^3\text{ MN/m}^2)/(141)^2}{1.92}$$
$$= 51.7\text{ MPa}$$

From Table 19-2, for kL/r = 141

$\sigma_{allow} = 7.51$ ksi $= 7.51$ ksi x 6.895
$= 51.8$ MPa (checks)

$P_{allow} = A\sigma_{allow}$
$= (21.2 \times 10^{-3}\text{ m}^2)(51.7 \times 10^3\text{ kN/m}^2)$
$= 1100$ kN ◄

19-33

For L127 x 127 x 12.7,
$A = 3.06 \times 10^{-3}\text{ m}^2$, $r_{min} = r_z = 0.025$ m
For fixed ends, $k = 0.65$

$$\frac{kL}{r} = \frac{(0.65)(3\text{ m})}{0.025\text{ m}} = 78 < C_c = 107.0$$

$\therefore$ J.B. Johnson formula applies.

$$\frac{kL/r}{C_c} = \frac{78}{107.0} = 0.729$$

$$F.S. = \frac{5}{3} + \frac{3(kL/r)}{8C_c} - \frac{(kL/r)^3}{8C_c^3}$$
$$= \frac{5}{3} + \frac{3(0.729)}{8} - \frac{(0.729)^3}{8} = 1.892$$

$$\sigma_{allow} = \frac{\left[1 - \frac{(kL/r)^2}{2C_c^2}\right]\sigma_y}{F.S.} = \frac{\left[1 - \frac{(0.729)^2}{2}\right](345\text{ MPa})}{1.892}$$
$$= 133.9\text{ MPa}$$

From Table 19-3, for kL/r = 78 we get,
$\sigma_{allow} = 19.41$ ksi $= 19.41 \times 6.895$
$= 133.8$ MPa (checks)

$P_{allow} = A\sigma_{allow}$
$= (3.06 \times 10^{-3}\text{ m}^2)(133.9 \times 10^3\text{ kN/m}^2)$
$= 410$ kN ◄

19-34

$$\frac{kL}{r} = \frac{(0.65)(4.5\text{ m})}{0.025\text{ m}} = 117 > C_c = 107.0$$

$\therefore$ Euler formula applies.

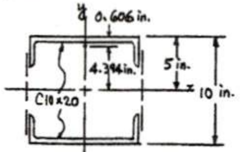
$$\sigma_{allow} = \frac{\pi^2 E/(kL/r)^2}{1.92}$$
$$= \frac{\pi^2(200 \times 10^3\text{ MPa})/(117)^2}{1.92}$$
$$= 75.1\text{ MPa}$$

From Table 19-3, for kL/r = 117 we get
$\sigma_{allow} = 10.91$ ksi $= 10.91 \times 6.895$
$= 75.2$ MPa (checks)

$P_{allow} = A\sigma_{allow}$
$= (3.06 \times 10^{-3}\text{ m}^2)(75.1 \times 10^3\text{ kN/m}^2)$
$= 230$ kN ◄

19-35

$\sigma_y = 36$ ksi, pinned ends, $k = 1$



$I_y = 2 \times 78.9 = 158\text{ in.}^4$

$I_x = 2\,[2.81 + (5.88)(4.394)^2] = 233\text{ in.}^4$

$$\text{least } r = r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{158}{2 \times 5.88}} = 3.67\text{ in.}$$

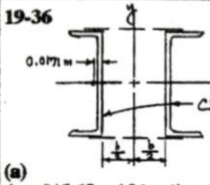
$$\frac{kL}{r} = \frac{(1.0)(25 \times 12\text{ in.})}{3.67\text{ in.}} = 82$$

(Cont'd)

19-35 (Cont)

From Table 19-2, for kL/r = 82, $\sigma_{allow} = 15.13$ ksi
$P_{allow} = A\sigma_{allow} = (2 \times 5.88\text{ in.}^2)(15.13\text{ kips/in.}^2)$
$= 178$ kips ◄

19-36



$\sigma_y = 345$ MPa
For pinned ends, $k = 1$

(a)
$A = 2(5.69 \times 10^{-3}\text{ m}^2) = 11.38 \times 10^{-3}\text{ m}^2$
$I_x = 2(67.4 \times 10^{-6}\text{ m}^4) = 134.8 \times 10^{-6}\text{ m}^4$

$$I_y = 2\left[2.14 \times 10^{-6}\text{ m}^4 + (5.69 \times 10^{-3}\text{ m}^2)\left(\frac{b}{2} + 0.0171\text{ m}\right)^2\right]$$
$$= I_x = 134.8 \times 10^{-6}\text{ m}^4$$

from which

$b = 0.180$ m $= 180$ mm ◄

(b)

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{134.8 \times 10^{-6}\text{ m}^4}{11.38 \times 10^{-3}\text{ m}^2}} = 0.1088\text{ m}$$

$$\frac{kL}{r} = \frac{(1.0)(12\text{ m})}{0.1088\text{ m}} = 110$$

From Table 19-3, for kL/r = 110
$\sigma_{allow} = 12.34$ ksi $= 12.34 \times 6.895$
$= 85.1$ MPa (checks)
$P_{allow} = A\sigma_{allow}$
$= (11.38 \times 10^{-3}\text{ m}^2)(85.1 \times 10^{-3}\text{ kN/m}^2)$
$= 968$ kN ◄

19-37

$\sigma_y = 36$ ksi
For pinned ends, $k = 1.0$
Assume r = 3 in.

$$\frac{kL}{r} = \frac{(1.0)(20 \times 12\text{ in.})}{3} = 80$$

From Table 19-2,
$\sigma_{allow} = 15.36$ ksi

$$A_{req} = \frac{P}{\sigma_{allow}} = \frac{400\text{ kips}}{15.36\text{ kips/in.}^2} = 26.0\text{ in.}^2$$

Section	A (in.²)	r_y (in.)
W14 x 90	26.5	3.70
W12 x 87	25.6	3.07
W10 x 100	29.4	2.65

Try W12 x 87

$$\frac{kL}{r} = \frac{(1.0)(20 \times 12\text{ in.})}{3.07\text{ in.}} = 78$$

From Table 19-2, $\sigma_{allow} = 15.58$ ksi
$P_{allow} = A\sigma_{allow} = (25.6\text{ in.}^2)(15.58\text{ kips/in.}^2)$
$= 399$ kips ≈ 400 kips (O.K.)

Use W12 x 87 ◄

19-38

$\sigma_y = 36$ ksi
For fixed-pinned ends, $k = 0.8$
Assume r = 3 in.

$$\frac{kL}{r} = \frac{(0.8)(35 \times 12\text{ in.})}{3} = 112$$

From Table 19-2, $\sigma_{allow} = 11.40$ ksi

$$A_{req} = \frac{P}{\sigma_{allow}} = \frac{150\text{ kips}}{11.40\text{ kips/in.}^2} = 13.2\text{ in.}^2$$

Section	A (in.²)	r_y (in.)
W14 x 53	15.6	1.92
W12 x 53	15.6	2.48
W10 x 60	17.6	2.57

Try W12 x 53

$$\frac{kL}{r} = \frac{(0.8)(35 \times 12\text{ in.})}{2.48\text{ in.}} = 135$$

From Table 19-2, $\sigma_{allow} = 8.19$ ksi
$P_{allow} = A\sigma_{allow} = (15.6\text{ in.}^2)(8.19\text{ kips/in.}^2)$
$= 127$ kips < 150 kips (N.G.)

Try W10 x 60

$$\frac{kL}{r} = \frac{(0.8)(35 \times 12\text{ in.})}{2.57\text{ in.}} = 131$$

From Table 19-2, $\sigma_{allow} = 8.70$ ksi
$P_{allow} = A\sigma_{allow} = (15.6\text{ in.}^2)(8.70\text{ kips/in.}^2)$
$= 153$ kips > 150 kips (O.K.)

Use W10 x 60 ◄

19-39

$\sigma_y = 250$ MPa
For pinned ends, $k = 1$
Assume $r = 0.070$ m

$$\frac{kL}{r} = \frac{(1)(10\ \text{m})}{0.070\ \text{m}} = 143$$

From Table 19-2,
$\sigma_{allow} = 7.30$ ksi
$\times 6.895 = 50.3$ MPa

$$A_{req} = \frac{P}{\sigma_{allow}} = \frac{600\ \text{kN}}{50.3 \times 10^3\ \text{kN/m}^2} = 11.9 \times 10^{-3}\ \text{m}^2$$

Section	A ($\times 10^{-3}$ m²)	r_y (m)
W360 x 0.99	12.9	0.0625
W300 x 0.95	12.3	0.0767
W250 x 1.12	14.6	0.0660

Since W300 x 0.95 has an area greater than $A_{req} = 11.9 \times 10^{-3}$ m² and a radius of gyration greater than $r = 0.070$ m, the section is satisfactory.

Use W300 x 0.95 ◀

19-40

$\sigma_y = 345$ MPa
For fixed ends, $k = 0.65$
Assume $r = 0.070$ m

$$\frac{kL}{r} = \frac{(0.65)(12\ \text{m})}{0.070\ \text{m}} = 111$$

From Table 19-3,
$\sigma_{allow} = 12.12$ ksi
$\times 6.895 = 83.6$ MPa

$$A_{req} = \frac{P}{\sigma_{allow}} = \frac{1500\ \text{kN}}{83.6 \times 10^3\ \text{kN/m}^2} = 17.9 \times 10^{-3}\ \text{m}^2$$

Section	A ($\times 10^{-3}$ m²)	r_y (m)
W360 x 1.31	17.1	0.0940
W300 x 1.27	16.5	0.0780
W250 x 1.46	19.0	0.0673

Try W3.27 x 1.27

$$\frac{kL}{r} = \frac{(0.65)(12\ \text{m})}{0.0780\ \text{m}} = 100$$

From Table 19-3,
$\sigma_{allow} = 14.71$ ksi
$\times 6.895 = 101.4$ MPa

$P_{allow} = A\sigma_{allow} = (16.5 \times 10^{-3}\ \text{m}^2)(101.4 \times 10^3\ \text{kN/m}^2)$
$\doteq 1670\ \text{kN} > 1500\ \text{kN}$ (O.K.)

Use W300 x 1.27 ◀

19-41

$\sigma_y = 50$ ksi
For fixed ends, $k = 0.65$
Assume $r = 2.5$ in.

$$\frac{kL}{r} = \frac{(0.65)(30 \times 12\ \text{in.})}{2.5\ \text{in.}} = 94$$

From Table 19-3, $\sigma_{allow} = 16.06$ ksi

$$A_{req} = \frac{P}{\sigma_{allow}} = \frac{200\ \text{kips}}{16.06\ \text{kips/in.}^2} = 12.5\ \text{in.}^2$$

Section	A (in.²)	r_y (in.)
W12 x 53	15.6	2.48
W10 x 49	14.4	2.54
W10 x 45	13.3	2.01

Try W10 x 45

$$\frac{kL}{r} = \frac{(0.65)(30 \times 12\ \text{in.})}{2.01\ \text{in.}} = 116$$

$\sigma_{allow} = 11.10$ ksi
$P_{allow} = A\sigma_{allow} = (13.3\ \text{in.}^2)(11.10\ \text{kips/in.}^2)$
$= 148\ \text{kips} < 200\ \text{kips}$ (N.G.)

Try W10 x 49

$$\frac{kL}{r} = \frac{(0.65)(30 \times 12\ \text{in.})}{2.54\ \text{in.}} = 92$$

From Table 19-3, $\sigma_{allow} = 16.50$ ksi
$P_{allow} = A\sigma_{allow} = (14.4\ \text{in.}^2)(16.50\ \text{kips/in.}^2)$
$= 238\ \text{kips} > 200\ \text{kips}$ (O.K.)

Use W10 x 49 ◀

19-42

$\sigma_y = 50$ ksi
For fixed ends, $k = 0.65$
Assume $r = 3.0$ in.

$$\frac{kL}{r} = \frac{(0.65)(40 \times 12\ \text{in.})}{3.0\ \text{in.}} = 104$$

From Table 19-3, $\sigma_{allow} = 13.77$ ksi

$$A_{req} = \frac{P}{\sigma_{allow}} = \frac{500\ \text{kips}}{13.77\ \text{kips/in.}^2} = 36.3\ \text{in.}^2$$

Section	A (in.²)	r_y (in.)
W24 x 131	38.5	2.97
W14 x 132	38.8	3.76
W14 x 109	32.0	3.73

Try W14 x 109

$$\frac{kL}{r} = \frac{(0.65)(40 \times 12\ \text{in.})}{3.73\ \text{in.}} = 84$$

From Table 19-3, $\sigma_{allow} = 18.41$ ksi
$P_{allow} = A\sigma_{allow} = (32.0\ \text{in.}^2)(18.41\ \text{kips/in.}^2)$
$= 589\ \text{kips} > 500\ \text{kips}$ (O.K.)

Use W14 x 109 ◀

19-43

$\sigma_y = 250$ MPa
For fixed-pinned ends, $k = 0.8$
Assume $r = 0.07$ m

$$\frac{kL}{r} = \frac{(0.8)(10\ \text{m})}{0.070\ \text{m}} = 114$$

From Table 19-2,
$\sigma_{allow} = 11.13$ ksi
$\times 6.895 = 76.7$ MPa

$$A_{req} = \frac{P}{\sigma_{allow}} = \frac{700\ \text{kN}}{76.7 \times 10^3\ \text{kN/m}^2} = 9.13 \times 10^{-3}\ \text{m}^2$$

Try 250 mm diameter standard steel pipe:
$A = 7.68 \times 10^{-3}$ m², $r = 0.0932$ m

$$\frac{kL}{r} = \frac{(0.8)(10\ \text{m})}{0.0932\ \text{m}} = 86$$

From Table 19-2,
$\sigma_{allow} = 14.67$ ksi
$\times 6.895 = 101.1$ MPa

$P_{allow} = A\sigma_{allow}$
$= (7.68 \times 10^{-3}\ \text{m}^2)(101.1.0 \times 10^3\ \text{kN/m}^2)$
$= 776\ \text{kN} > 700\ \text{kN}$ (O.K.)

Use 250 mm diameter standard steel pipe ◀

19-44

$\sigma_y = 50$ ksi
For pinned ends, $k = 1$
Assume $r = 2.0$ in.

$$\frac{kL}{r} = \frac{(1)(20 \times 12\ \text{in.})}{2.0\ \text{in.}} = 120$$

From Table 19-3, $\sigma_{allow} = 10.37$ ksi

$$A_{req} = \frac{P}{\sigma_{allow}} = \frac{70\ \text{kips}}{10.37\ \text{kips/in.}^2} = 6.75\ \text{in.}^2$$

Try 6 in. standard weight steel pipe.
$A = 5.58$ in.², $r = 2.25$ in.

$$\frac{kL}{r} = \frac{(1)(20 \times 12\ \text{in.})}{2.25\ \text{in.}} = 107$$

From Table 19-3, $\sigma_{allow} = 13.04$ ksi

$P_{allow} = A\sigma_{allow} = (5.58\ \text{in.}^2)(13.04\ \text{kips/in.}^2)$
$= 72.8\ \text{kips} > 70\ \text{kips}$ (O.K.)

Use 6 in. standard weight steel pipe ◀

Test Problems for Chapter 19

The following problems may either be given to students for them to practice their problem solving skills or be used as test problems. The answers are provided on the next page.

(1) A 10 m long steel column with a W300 x 1.27 section is made of A441 steel with E = 200 GPa and σ_y = 345 MPa. Determine the critical buckling load of the column if it has (*a*) pinned ends, and (*b*) fixed ends.

(2) A compression machine member with pinned ends has a length of 2 ft and a cross-section of 1 in. x 2 in. The member is made of A36 steel with E = 29 x 10³ ksi and σ_y = 36 ksi. Determine the maximum allowable axial compressive load tha can be supported by the member based on a factor of safety of 2.5.

(3) Select the lightest standard weight steel pipe section (in SI designation) to support an axial compressive load of 230 kN. The column has pinned ends and an unbraced length of 3.5 m. Use AISC specification and A441 steel with σ_y = 345 MPa.

(4) Select the lightest W-shape for a 30-ft column with fixed ends to support an axial compressive load of 350 kips. Use AISC specification and A242 steel with σ_y = 50 ksi.

Solutions to Test Problems for Chapter 19

(1)

For W300 x 1.27 section,

$A = 16.5 \times 10^{-3}\ m^2$

$r_{min} = r_y = 0.078\ m$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2(200\times10^3\ MPa)}{345\ MPa}} = 107.0$$

(a) For pinned ends, $k = 1.0$

$$\frac{kL}{r} = \frac{1.0(10\ m)}{0.075\ m} = 128.2 > C_c = 107.0$$

Euler formula applies.

$$\sigma_{cr} = \frac{\pi^2 E}{(kL/r)^2} = \frac{\pi^2(200\times10^3 MN/m^2)}{(128.2)^2}$$

$= 120.1\ MPa = 120.1 \times 10^3\ kN/m^2$

$P_{cr} = A\sigma_{cr}$

$= (16.5 \times 10^{-3}\ m^2)(120.1 \times 10^3\ kN/m^2)$

$= 1980\ kN$ ◄

(b) For fixed ends, $k = 0.5$

$$\frac{kL}{r} = \frac{0.5(10\ m)}{0.078\ m} = 64.1 < C_c = 107.0$$

J.B. Johnson formula applies.

$$\sigma_{cr} = \left[1 - \frac{(kL/r)^2}{2C_c^2}\right]\sigma_y = \left[1 - \frac{(64.1)^2}{2(107.0)^2}\right](345 MPa)$$

$= 283\ MPa = 283 \times 10^3\ kN/m^2$

$P_{cr} = A\sigma_{cr}$

$= (16.5 \times 10^{-3}\ m^2)(283 \times 10^3\ kN/m^2)$

$= 4670\ kN$ ◄

(2)

For 1 in x 2 in. section

$A = 2\ in.^2$

$r_{min} = 0.289(1\ in.) = 0.289\ in.$

For pinned ends, $k = 1.0$

For A36 steel, $C_c = 126.1$

$$\frac{kL}{r} = \frac{1.0(2 \times 12\ in.)}{0.289\ in.} = 83 < C_c = 126.1$$

J.B. Johnson formula applies.

$$\sigma_{cr} = \left[1 - \frac{(kL/r)^2}{2C_c^2}\right]\sigma_y = \left[1 - \frac{(83.0)^2}{2(126.1)^2}\right](36 ksi)$$

$= 28.2\ ksi$

$P_{cr} = A\sigma_{cr} = (2\ in.^2)(282\ kip/in.^2) = 56.4\ kips$

$$P_{allow} = \frac{P_{cr}}{F.S.} = \frac{56.4\ kip}{2.5} = 22.6\ kips$$ ◄

(3)

$\sigma_y = 345\ MPa$

For pinned ends, $k = 1.0$

Assume $r = 0.04\ m$

$$\frac{kL}{r} = \frac{1.0(3.5\ m)}{0.04\ m} = 88$$

From Table 19-3,

$\sigma_{allow} = 17.37\ ksi$

$\times 6.895 = 119.8\ MPa$

$$A_{req} = \frac{P}{\sigma_{allow}} = \frac{230\ kN}{119.8\times10^3\ kN/m^2} = 1.92 \times 10^{-3}\ m^2$$

Note that 90 mm norminal diameter obviously will not work. Hence we try 100 mm diameter standard steel pipe:

$A = 2.05 \times 10^{-3}\ m^2$, $r = 0.0384\ m$

(Cont'd)

Solutions to Test Problems for Chapter 19 (Cont'd)

(3) (Cont)

$$\frac{kL}{r} = \frac{(1.0)(3.5\text{ m})}{0.0384\text{ m}} = 91$$

From Table 19-3,

$\sigma_{allow} = 16.72$ ksi x 6.895 = 115.3 MPa

$$P_{allow} = A\sigma_{allow}$$
$$= (2.05 \times 10^{-3}\text{ m}^2)(115.3 \times 10^3\text{ kN/m}^2)$$
$$= 236\text{ kN} > 230\text{ kN} \quad \text{(O.K.)}$$

Use 100 mm standard steel pipe ◀

(4)

$\sigma_y = 50$ ksi

For fixed ends, $k = 0.65$

Assume $r = 3.0$ in.

$$\frac{kL}{r} = \frac{(0.65)(30 \times 12\text{ in.})}{3.0} = 78$$

From Table 19-3,

$\sigma_{allow} = 19.41$ ksi

$$A_{req} = \frac{P}{\sigma_{allow}} = \frac{350\text{ kips}}{19.41\text{ kips/in.}^2} = 18.03\text{ in.}^2$$

Section	A (in.²)	r_y (in.)
W14 x 90	26.5	3.70
W12 x 65	19.1	3.02
W10 x 77	22.6	2.60

Try W12 x 65

$$\frac{kL}{r} = \frac{(0.65)(30 \times 12\text{ in.})}{3.02\text{ in.}} = 78$$

From Table 19-3, $\sigma_{allow} = 19.41$ ksi

$$P_{allow} = A\sigma_{allow} = (19.1\text{ in.}^2)(19.41\text{ kips/in.}^2)$$
$$= 371\text{ kips} > 350\text{ kips} \quad \text{(O.K.)}$$

Use W12 x 65 ◀

20-1

Six $\frac{3}{4}$ in. diam. A 502-1 rivets

$$A_s = \frac{\pi}{4}\left(\frac{3}{4}\text{ in.}\right)^2 = 0.442\text{ in.}^2$$

$$P_s = nA_s\tau_{allow} = 6(0.442\text{ in.}^2)(15\text{ kips/in.}^2) = 39.8\text{ kips}$$

$$P_b = n(td)(\sigma_b)_{allow} = 6\left(\frac{1}{2} \times \frac{3}{4}\text{in.}^2\right)(1.35 \times 36\text{ kips/in.}^2)$$
$$= 109.4\text{ kips}$$

$$b_{net} = b - n(d + c) = 6 - 2\left(\frac{3}{4} + \frac{1}{8}\right) = 4.25\text{ in.}$$

$$P_t = b_{net}t(\sigma_t)_{allow} = (4.25\text{ in.})\left(\frac{1}{2}\text{in.}\right)(0.6 \times 36\text{ kips/in.}^2)$$
$$= 45.9\text{ kips}$$

Joint Strength: $P = P_s = 39.8$ kips ◀

$$\text{Joint Efficiency} = \frac{\text{Strength of Joint}}{\text{Strength of Solid Plate}} \times 100\%$$
$$= \frac{39.8\text{ kips}}{\left(6\text{ in.} \times \frac{1}{2}\text{ in.}\right)(0.6 \times 36\text{ kips/in.}^2)} \times 100\%$$
$$= 61.4\%$$ ◀

20-2

Six 20 mm diam. A 502-1 rivets

$$A_s = \frac{\pi}{4}(0.020\text{ m})^2 = 3.14 \times 10^{-4}\text{ m}^2$$

$$P_s = nA_s\tau_{allow} = 6(3.14 \times 10^{-4}\text{ m}^2)(103 \times 10^3\text{ kN/m}^2)$$
$$= 194\text{ kN}$$

$$P_b = n(td)(\sigma_b)_{allow}$$
$$= 6(0.013\text{ m} \times 0.020\text{ m})(1.35 \times 250 \times 10^3\text{ kN/m}^2)$$
$$= 527\text{ kN}$$

$$b_{net} = b - n(d + c) = 150\text{ mm} - 2(20\text{ mm} + 3\text{ mm})$$
$$= 104\text{ mm} = 0.104\text{ m}$$

$$P_t = b_{net}t(\sigma_t)_{allow}$$
$$= (0.104\text{ m})(0.013\text{ m})(0.6 \times 250 \times 10^3\text{ kN/m}^2)$$
$$= 203\text{ kN}$$

Joint Strength: $P = P_s = 194$ kN ◀

$$\text{Joint Efficiency} = \frac{\text{Strength of Joint}}{\text{Strength of Solid Plate}} \times 100\%$$
$$= \frac{194\text{ kN}}{(0.150\text{ m} \times 0.013\text{ m})(0.6 \times 250 \times 10^3\text{ kN/m}^2)} \times 100\%$$
$$= 66.3\%$$ ◀

20-3

Eight $\frac{7}{8}$ in. diam. A 502-2 rivets

$$A_s = \frac{\pi}{4}\left(\frac{7}{8}\text{ in.}\right)^2 = 0.601\text{ in.}^2$$

$$P_s = n(2A_s)\tau_{allow} = 8(2 \times 0.601\text{ in.}^2)(20\text{ kips/in.}^2)$$
$$= 192\text{ kips}$$

$$P_b = n(td)(\sigma_b)_{allow} = 8\left(\frac{5}{8} \times \frac{7}{8}\text{ in.}^2\right)(1.35 \times 36\text{ kips/in.}^2)$$
$$= 213\text{ kips}$$

$$b_{net} = b - n(d + c) = 12 - 4\left(\frac{7}{8} + \frac{1}{8}\right) = 8\text{ in.}$$

$$P_t = b_{net}t(\sigma_t)_{allow} = (8\text{ in.})\left(\frac{5}{8}\text{in.}\right)(0.6 \times 36\text{ kips/in.}^2)$$
$$= 108\text{ kips}$$

Joint Strength: $P = P_t = 108$ kips ◀

$$\text{Joint Efficiency} = \frac{P}{P_{solid\ plate}} \times 100\%$$
$$= \frac{108\text{ kips}}{\left(12\text{ in.} \times \frac{5}{8}\text{in.}\right)(0.6 \times 36\text{ kips/in.}^2)} \times 100\%$$
$$= 66.7\%$$ ◀

20-4

Eight 22 mm diam. A 502-2 rivets

$$A_s = \frac{\pi}{4}(0.022\text{ m})^2 = 3.80 \times 10^{-4}\text{ m}^2$$

$$P_s = nA_s\tau_{allow} = 8(2 \times 3.80 \times 10^{-4}\text{ m}^2)(138 \times 10^3\text{ kN/m}^2)$$
$$= 839\text{ kN}$$

$$P_b = n(td)(\sigma_b)_{allow}$$
$$= 8(0.016\text{ m} \times 0.022\text{ m})(1.35 \times 250 \times 10^3\text{ kN/m}^2)$$
$$= 950\text{ kN}$$

$$b_{net} = b - n(d + c) = 300\text{ mm} - 4(22\text{ mm} + 3\text{ mm})$$
$$= 200\text{ mm} = 0.200\text{ m}$$

$$P_t = b_{net}t(\sigma_t)_{allow}$$
$$= (0.200\text{ m})(0.016\text{ m})(0.6 \times 250 \times 10^3\text{ kN/m}^2)$$
$$= 480\text{ kN}$$

Joint Strength: $P = P_t = 480$ kN ◀

(Cont'd)

20-4 (Cont)

$$\text{Joint Efficiency} = \frac{P}{P_{\text{solid plate}}} \times 100\%$$

$$= \frac{480 \text{ kN}}{(0.016\text{m} \times 0.300\text{ m})(0.6 \times 250 \times 10^3 \text{kN/m}^2)} \times 100\%$$

$$= 66.7\%$$ ◀

20-5

$\frac{7}{8}$ in. diam. A 502-2 rivets.

The angle is L $5 \times 3 \times \frac{3}{8}$.

$$A_{net} = A_{angle} - t(d + c) = 2.86 - \frac{3}{8}\left(\frac{7}{8} + \frac{1}{8}\right) = 2.49 \text{ in.}^2$$

Allowable Tensile Strength $= P_t = A_{net}(\sigma_t)_{allow}$

$$= (2.49 \text{ in}^2)(0.6 \times 36 \text{ kips/in}^2) = 53.8 \text{ kips}$$ ◀

20-6

$$A_s = \frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 0.601 \text{ in.}^2$$

P_s per rivet $= A_s\,\tau_{allow} = (0.601 \text{ in.}^2)(20 \text{ kips/in.}^2)$
$= 12.02$ kips

P_b per rivet $= (td)(\sigma_b)_{allow}$

$$= \left(\frac{1}{2} \times \frac{7}{8} \text{ in.}^2\right)(1.35 \times 36 \text{ kips/in.}^2) = 21.3 \text{ kips}$$

$$n = \frac{P_t}{P_s \text{ per rivet}} = \frac{53.8 \text{ kips}}{12.02 \text{ kips}} = 4.47$$

Use 5 rivets ◀

20-7

Four 1 in. diam. A 502-1 rivets

$$A_s = \frac{\pi}{4}(1 \text{ in.})^2 = 0.785 \text{ in.}^2$$

2L $5 \times 3 \times \frac{3}{8}$: $A = 2.86$ in.^{2}

$$A_{net} = 2[A_{angle} - t(d + c)]$$

$$= 2\left[2.86 - \frac{3}{8}\left(1 + \frac{1}{8}\right)\right] = 4.88 \text{ in.}^2$$

$$P_t = A_{net}(\sigma_t)_{allow} = (4.88 \text{ in.}^2)(0.6 \times 36 \text{ kips/in.}^2) = 105 \text{ kips}$$

$$P_s = 4(2\,A_s)\tau_{allow} = 4(2 \times 0.785 \text{ in.}^2)(15 \text{ kips/in.}^2) = 94.2 \text{ kips}$$

Since twice of the thickness of the angle is greater than the thickness of the gusset plate, bearing on gusset plate is more critical. Thus

$$P_b = 4(t_g d)(\sigma_b)_{allow} = 4\left(\frac{5}{8} \times 1 \text{ in.}^2\right)(135 \times 36 \text{ kips/in.}^2) = 122 \text{ kips}$$

Joint Strength: $P = P_s = 94.2$ kips ◀

$$\text{Joint Efficiency} = \frac{P}{2A_{angle}(\sigma_t)_{allow}} \times 100\%$$

$$= \frac{94.2 \text{ kips}}{2(2.86 \text{ in.}^2)(0.6 \times 36 \text{ kips/in.}^2)} \times 100\%$$

$$= 76.2\%$$ ◀

20-8

Four 25 mm diam. A 502-1 rivets:

$$A_s = \frac{\pi}{4}(0.025 \text{ m})^2 = 4.91 \times 10^{-4} \text{ m}^2$$

2L127 x 76 x 9.5: $A_{angle} = 1.85 \times 10^{-3}$ m^2

$$A_{net} = 2[A_{angle} - t(d + c)]$$
$$= 2[1.85\times10^{-3} \text{ m}^2 - (0.0095 \text{ m})(0.025 \text{ m} + 0.003 \text{ m})]$$
$$= 3.168 \times 10^{-3} \text{ m}^2$$

$$P_t = A_{net}(\sigma_t)_{allow}$$
$$= (3.168 \times 10^{-3} \text{ m}^2)(0.6 \times 250 \times 10^3 \text{ kN/m}^2)$$
$$= 475 \text{ kN}$$

$$P_s = 4(2\,A_s)\tau_{allow}$$
$$= 8(4.91 \times 10^{-4} \text{ m}^2)(103 \times 10^3 \text{ kN/m}^2)$$
$$= 405 \text{ kN}$$

Bearing on the gusset plate:

$$P_b = 4(t_g d)(\sigma_b)_{allow}$$
$$= 4(0.016 \text{ m} \times 0.025 \text{ m})(1.35 \times 250 \times 10^3 \text{ kN/m}^2)$$
$$= 540 \text{ kN}$$

Joint Strength: $P = P_s = 405$ kN ◀

$$\text{Joint Efficiency} = \frac{P}{2A_{angle}(\sigma_t)_{allow}} \times 100\%$$

$$= \frac{405 \text{ kN}}{2(1.85 \times 10^{-3} \text{ m}^2)(0.6 \times 250 \times 10^3 \text{kN/m}^2)} \times 100\%$$

$$= 73.0\%$$ ◀

20-9

$\frac{7}{8}$ in. diam. A 502-1 rivets:

$$A_s = \frac{\pi}{4}\left(\frac{7}{8} \text{ in.}\right)^2 = 0.601 \text{ in.}^2$$

$$P_s = 4(2\,A_s)\tau_{allow}$$
$$= 4(2 \times 0.601 \text{ in.}^2)(15 \text{ kips/in.}^2)$$
$$= 72.1 \text{ kips}$$

Bearing of 4 rivets on the web of W18 x 60 beam ($t_w = 0.415$ in.):

$$P_b = 4(t_w d)(\sigma_b)_{allow}$$
$$= 4\left(0.415 \text{ in.} \times \frac{7}{8} \text{ in.}\right)(1.35 \times 36 \text{ kips/in.}^2)$$
$$= 70.6 \text{ kips}$$

Since twice the thickness of the angle and the thickness of the flange of W12 x 87 column (t_f = 0.810 in.) are both greater than the thickness of the web of the beam, the bearings on the angles and on the column are not critical.

Joint Strength: $P = P_b = 70.6$ kips ◀

20-10

For a 22 mm diam. A502-1 rivets:

$$A_s = \frac{\pi}{4}(0.022 \text{ m})^2 = 3.80 \times 10^{-4} \text{ m}^2$$

$$P_s = 4(2\,A_s)\tau_{allow}$$
$$= 4(2 \times 3.80 \times 10^{-4} \text{ m}^2)(103 \times 10^3 \text{ kN/m}^2)$$
$$= 313 \text{ kN}$$

Bearing of 4 rivets on the web of W460 x 0.88 beam ($t_w = 0.0105$ m):

$$P_b = 4(t_w d)(\sigma_b)_{allow}$$
$$= 4(0.0105 \text{ m} \times 0.022 \text{ m})(1.35 \times 250 \times 10^3 \text{ kN/m}^2)$$
$$= 312 \text{ kN}$$

Since twice the thickness of the clipped angle L102 x 102 x 19.1(2 x 19.1 = 38.2 mm) and thickness of the flange of W300 x 1.27 column (t_f = 20.6 mm) are both greater than the thickness of the web of the beam, the bearings on the angles and on the column are not critical.

Joint Strength: $P = P_b = 312$ kN ◀

20-11

$\frac{3}{4}$ in. diam. A502-1 rivets:

$$A_s = \frac{\pi}{4}\left(\frac{3}{4} \text{ in.}\right)^2 = 0.442 \text{ in.}^2$$

$$P_s = 4(2\,A_s)\tau_{allow}$$
$$= 4(2 \times 0.442 \text{ in.}^2)(15 \text{ kips/in.}^2)$$
$$= 53.0 \text{ kips}$$

Bearing of 4 rivets on the web of W18 x 60 beam ($t_w = 0.355$ in.):

$$P_b = 4(t_w d)(\sigma_b)_{allow}$$
$$= 4\left(0.335 \times \frac{3}{4} \text{ in.}^2\right)(1.35 \times 36 \text{ kips/in.}^2)$$
$$= 51.8 \text{ kips}$$

Since twice the thickness: $2\left(\frac{5}{16} \text{ in.}\right)$ of the clipped angle and flange thickness of W12 x 65 column (t_f = 0.605 in.) are both greater than the thickness of the web of the beam (t_w = 0.355 in.), the bearings on the clipped angles and on the column are not critical.

Joint Strength: $P = P_b = 51.8$ kips

From $R = \frac{wL}{2} = P$ we find

$$w = \frac{2P}{L} = \frac{2(51.8 \text{ kips})}{12 \text{ ft}} = 8.63 \text{ kip/ft}$$

For a simple beam: $M_{max} = \frac{wL^2}{8}$

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{wL^2}{8S} = \sigma_{allow}$$

For W18 x 50 beam, $S = 88.9$ in.^{3}

For A36 steel, $\sigma_{allow} = 0.66(36 \text{ ksi})$

$$w = \frac{8S\sigma_{allow}}{L^2} = \frac{8(88.9 \text{ in.}^3)(23.8 \text{ ksi})}{(12 \times 12 \text{ in.})^2}$$

$$= 0.816 \text{ kip/in.} = 9.79 \text{ kip/ft}$$

$$w_{allow} = 8.63 \text{ kip/ft}$$ ◀

20-12

20 mm diam. A502-1 rivets:

$A_s = \frac{\pi}{4}(0.020\text{ m})^2 = 3.14 \times 10^{-4}\text{ m}^2$

$P_s = 4(2A_s)\tau_{allow}$
$= 4(2 \times 3.14 \times 10^{-4}\text{ m}^2)(103 \times 10^3\text{ kN/m}^2)$
$= 259\text{ kN}$

Bearing of 4 rivets on the web of W460 x 0.73 beam ($t_w = 0.00902$ m):

$P_b = 4(t_w d)(\sigma_b)_{allow}$
$= 4(0.00902\text{ m} \times 0.020\text{ m})(1.35 \times 250 \times 10^3\text{ kN/m}^2)$
$= 244\text{ kN}$

Since twice the thickness of the clipped angle L102 x 89 x 7.9 (2t = 2 x 7.9 = 15.8 mm) and flange thickness of W300 x 0.95 column (t_f = 15.4 mm) are both greater than the web thickness of the beam, the bearings on the clipped angles and on the column are not critical.

Joint Strength: $P = P_b = 244\text{ kN}$

Solving w from $R = \frac{wL}{2} = P$ we get:

$$w = \frac{2P}{L} = \frac{2(244\text{ kN})}{3.65\text{ m}} = 134\text{ kN/m}$$

For a simple beam: $M_{max} = \frac{wL^2}{8}$

$$\sigma = \frac{M_{max}}{S} = \frac{wL^2}{8S} = \sigma_{allow}$$

For W460 x 0.73 beam, $S = 1.46 \times 10^{-3}\text{ m}^3$

For A36 steel, $\sigma_{allow} = 0.66(250 \times 10^3\text{ kN/m}^2)$
$= 165 \times 10^3\text{ kN/m}^2$

$$w = \frac{8S\sigma_{allow}}{L^2} = \frac{8(1.46 \times 10^{-3}\text{ m}^3)(165 \times 10^3\text{ kN/m}^2)}{(3.65\text{ m})^2}$$

$= 145\text{ kN/m}$

$w_{allow} = 134\text{ kN/m}$ ◀

20-13

Six $\frac{3}{4}$ in. diam. A235 bearing-type bolts.

$$A_s = \frac{\pi}{4}\left(\frac{3}{4}\text{ in.}\right)^2 = 0.442\text{ in.}^2$$

$P_s = nA_s\tau_{allow}$
$= 6(0.442\text{ in.}^2)(22\text{ kips/in.}^2)$
$= 58.3\text{ kips}$

From the solution to Prob. 20-1,
$P_b = 109.4$ kips
$P_t = 45.9$ kips

Joint Strength: $P = P_t = 45.9$ kips ◀

$$\text{Joint Efficiency} = \frac{P}{P_{solid\ plate}} \times 100\%$$

$$= \frac{45.9\text{ kips}}{\left(6\text{ in.} \times \frac{1}{2}\text{ in.}\right)(0.6 \times 36\text{ kips/in.}^2)} \times 100\%$$

$= 70.8\ \%$ ◀

20-14

Six 20 mm A325 bearing-type bolts:

$A_s = \frac{\pi}{4}(0.020\text{ m})^2 = 3.14 \times 10^{-4}\text{ m}^2$

$P_s = nA_s\tau_{allow}$
$= 6(3.14 \times 10^{-4}\text{ m}^2)(152 \times 10^3\text{ kN/m}^2)$
$= 286\text{ kN}$

From the solution to Prob. 20-2,
$P_b = 527$ kN
$P_t = 203$ kN

Joint Strength: $P = P_t = 203$ kN ◀

$$\text{Joint Efficiency} = \frac{P}{P_{solid\ plate}}$$

$$= \frac{203\text{ kN}}{(0.150\text{ m})(0.013\text{ m})(0.6 \times 250 \times 10^3\text{ kN/m}^2)}$$

$= 0.694 = 69.4\ \%$ ◀

20-15

Eight $\frac{7}{8}$ in. diam. A490 bearing-type bolts:

$$A_s = \frac{\pi}{4}\left(\frac{7}{8}\text{ in.}\right)^2 = 0.601\text{ in.}^2$$

$P_s = n(2A_s)\tau_{allow}$
$= 8(2 \times 0.601\text{ in.}^2)(32\text{ kips/in.}^2)$
$= 308\text{ kips}$

From the solution to Prob. 20-3
$P_b = 213$kips
$P_t = 108$ kips

Joint Strength: $P = P_t = 108$ kips ◀

$$\text{Joint Efficiency} = \frac{P}{P_{solidplate}} \times 100\%$$

$$= \frac{108\text{ kips}}{\left(12\text{ in} \times \frac{5}{8}\text{in.}\right)(0.6 \times 36\text{ kips/in.}^2)} \times 100\%$$

$= 66.7\ \%$ ◀

20-16

Eight 22 mm diam. A490 bearing-type bolts:

$A_s = \frac{\pi}{4}(0.022\text{ m})^2 = 3.80 \times 10^{-4}\text{ m}^2$

$P_s = n(2A_s)\tau_{allow}$
$= 8(2 \times 3.80 \times 10^{-4}\text{ m}^2)(221 \times 10^3\text{ kN/m}^2)$
$= 1340\text{ kN}$

From the solution to Prob. 20-4,
$P_b = 950$ kN
$P_t = 480$ kN

Joint Strength = $P = P_t = 480$ kN ◀

$$\text{Joint Efficiency} = \frac{P}{P_{solid\ plate}}$$

$$= \frac{480\text{ kN}}{(0.300\text{ m} \times 0.016\text{ m})(0.6 \times 250 \times 10^3\text{kN/m}^2)}$$

$= 0.667 = 66.7\ \%$ ◀

20-17

$\frac{7}{8}$ in. diam. A490 bearing-type bolts.

From Prob.20-5,
$P_t = 53.8$ kips

P_s per rivet $= A_s\tau_{allow} = (0.601\text{ in.}^2)(32\text{ kips/in.}^2)$
$= 19.23$ kips

P_b per rivet $= (td)(\sigma_b)_{allow}$

$$= \left(\frac{1}{2} \times \frac{7}{8}\text{ in.}^2\right)(1.35 \times 36\text{ kips/in.}^2) = 21.3\text{ kips}$$

$$n = \frac{P_t}{P_s\text{ per rivet}} = \frac{53.8}{19.23} = 2.80$$

Use 3 bolts ◀

20-18

Four 1 in. diam. A490 friction-type bolts.

$$A_s = \frac{\pi}{4}(1\text{ in.})^2 = 0.785\text{ in.}^2$$

$P_s = n(2A_s)\tau_{allow}$
$= 4(2 \times 0.785\text{ in.}^2)(20\text{ kips/in.}^2)$
$= 126\text{ kips}$

From the solution to Prob. 20-7,
$P_t = 105$ kips

Since slip is not supposed to occur in the friction-type bolt connection, bearing strength needs not be considered.

Joint Strength: $P = P_t = 105$ kips ◀

$$\text{Joint Efficiency} = \frac{P}{2A_{angle}(\sigma_t)_{allow}} \times 100\%$$

$$= \frac{105\text{ kips}}{2(2.86\text{ in.}^2)(0.6 \times 36\text{ kips/in.}^2)} \times 100\%$$

$= 85.0\ \%$ ◀

20-19

Four 25 mm A490 friction-type bolts:

$A_s = \frac{\pi}{4}(0.025\ m)^2 = 4.91 \times 10^{-4}\ m^2$

$P_s = n(2A_s)\tau_{allow}$
$= 4(2 \times 4.91 \times 10^{-4}\ m^2)(138 \times 10^3\ kN/m^2)$
$= 542\ kN$

From the solution to Prob. 20-8,
$A_{angle} = 1.85 \times 10^{-3}\ m^2$
$P_t = 475\ kN$

For friction-type bolt connection, bearing strength need not be considered.

Joint Strength: $P = P_t = 475\ kN$ ◄

$$\text{Joint Efficiency} = \frac{P}{2A_{angle}(\sigma_t)_{allow}} \times 100\%$$

$$= \frac{475\ kN}{2(1.85 \times 10^{-3}\ m^2)(0.6 \times 250 \times 10^3\ kN/m^2)} \times 100\%$$

$= 85.6\ \%$ ◄

20-20

$\frac{7}{8}$ in. diam. A325 bearing-type bolts.

$A_s = \frac{\pi}{4}\left(\frac{7}{8}\ in.\right)^2 = 0.601\ in.^2$

$P_s = n(2A_s)\tau_{allow}$
$= 4(2 \times 0.601\ in.^2)(20\ kips/in.^2)$
$= 106\ kips$

From the solution to Prob. 20-9,
$P_b = 70.6\ kips$

Joint Strength: $P = P_b = 70.6\ kips$ ◄

20-21

Four 22 mm A490 friction-type bolts.

$A_s = \frac{\pi}{4}(0.022\ m)^2 = 3.80 \times 10^{-4}\ m^2$

$P_s = n(2A_s)\tau_{allow}$
$= 4(2 \times 3.80 \times 10^{-4}\ m^2)(138 \times 10^3\ kN/m^2)$
$= 420\ kN$

For friction-type bolts, bearing strength need not be considered.

Joint Strength: $P = P_t = 420\ kN$ ◄

20-22

$\frac{3}{4}$ in. diam. A490 bearing-type bolts:

$A_s = \frac{\pi}{4}\left(\frac{3}{4}\ in.\right)^2 = 0.442\ in.^2$

$P_s = n(2A_s)\tau_{allow}$
$= 4(2 \times 0.442\ in.^2)(20\ kip/in.^2)$
$= 70.7\ kips$

For friction-type bolts, bearing strength need not be considered. Thus the strength of the joint is
$P = P_s = 70.7\ kips$

$R = \frac{wL}{2} = P$

$w = \frac{2P}{L} = \frac{2(70.7\ kips)}{12\ ft} = 11.8\ kip/ft$

From the solution to Prob 20-11, to keep the flexural stress within the allowable limit, the load should be within:
$w = 9.79\ kips/ft$
Hence

$w_{allow} = 9.79\ kips/ft$ ◄

20-23

Four 20 mm A490 friction-type bolts:

$A_s = \frac{\pi}{4}(0.020\ m)^2 = 3.14 \times 10^{-4}\ m^2$

$P_s = n(2A_s)\tau_{allow}$
$= 4(2 \times 3.14 \times 10^{-4}\ m^2)(138 \times 10^3\ kN/m^2)$
$= 347\ kN$

For friction-type bolts, bearing strength need not be considered. Thus the strength of the joint is

$P = P_s = 347\ kN$ ◄

$R = \frac{wL}{2} = P$

(Cont'd)

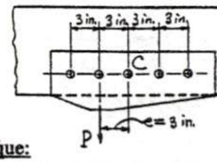
22-23 (Cont)

$w = \frac{2P}{L} = \frac{2(347\ kN)}{3.65\ m} = 190\ kN/m$

From the solution to Prob 20-12, to keep the flexural stress within the allowable limit, the load should be within:
$w = 145\ kN/m$
Hence

$w_{allow} = 145\ kN/m$ ◄

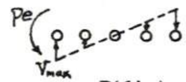
20-24



<u>Due to Torque:</u>

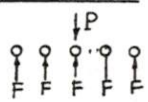
$\sum x^2 = 2(3\ in.)^2 + 2(6\ in.)^2 = 90\ in.^2$

$K = \frac{Pe}{\sum x^2} = \frac{P(3\ in.)}{90\ in.^2} = \frac{P}{30\ in.}$



$V_{max} = Kx_{max} = \frac{P(6\ in.)}{30\ in.} = \frac{P}{5}$

<u>Due to direct shear force:</u>



$F = \frac{P}{5}$

<u>The maximum shear load on the left rivet :</u>

$F_{max} = \frac{P}{5} + \frac{P}{5} = 0.4P$

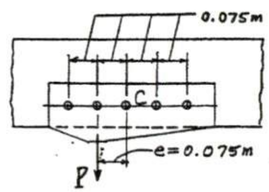
For $\frac{7}{8}$ in. rivet: $A = \frac{\pi}{4}\left(\frac{7}{8}\ in.\right)^2 = 0.601\ in.^2$

The allowable shear load per rivet is
$A\tau_{allow}$ - $(0.601\ in.^2)(15\ kip/in.^2) = 9.02\ kips$

Equating the maximum shear load to the allowable shear load we get

$P = \frac{9.02}{0.4} = 22.6\ kips$ ◄

20-25



$\sum x^2 = 2(0.075\ m)^2 + 2(0.150\ m)^2 = 0.05625\ m^2$

$K = \frac{Pe}{\sum x^2} = \frac{P(0.075\ m)}{0.05625\ m^2} = 1.33\ P/m$

$V_{max} = Kx_{max} = (1.33\ P/m)(0.150\ m) = 0.2\ P$

The direct shear force on each rivet is $\frac{P}{5} = 0.2P.$

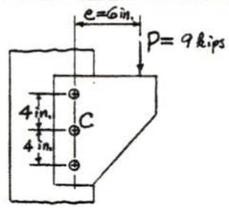
The allowable shear load per rivet is

$A\tau_{allow} = \frac{\pi}{4}(0.02\ m)^2(103 \times 10^3\ kN/m^2) = 32.4\ kN$

Equating this load to the total shear load of 0.2P + 0.2P on the left rivet, we get

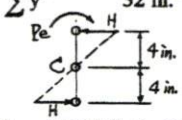
$P = \frac{32.4\ kN}{0.4} = 80.9\ kN$ ◄

20-26



$\sum y^2 = 2(4\ in.)^2 = 32\ in.^2$

$K = \frac{Pe}{\sum y^2} = \frac{(9\ kips)(6\ in.)}{32\ in.^2} = 1.689\ kip/in.$



$H = Ky = (1.689\ kip/in.)(4\ in.) = 6.75\ kips$

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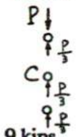
20-26 (Cont)



$\frac{P}{3} = \frac{9\text{ kips}}{3} = 3\text{ kips}$

The maximum load is

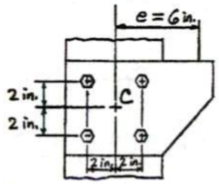
$F = \sqrt{(3\text{ kips})^2 + (6.75\text{ kips})^2} = 7.39\text{ kips}$

$A_{req} = \frac{F}{\tau_{allow}} = \frac{7.39\text{ kips}}{20\text{ kip/in.}^2} = 0.370\text{ in.}^2 = 0.7854\,d^2$

$d = \sqrt{\frac{0.370\text{ in.}^2}{0.7854}} = 0.686\text{ in.}$

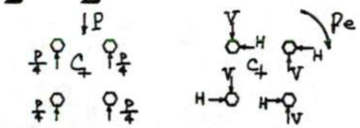
Use $\frac{3}{4}$ in. bolts ◄

20-27



$\sum x^2 + \sum y^2 = 4\,(2\text{ in.})^2 + 4\,(2\text{ in.})^2 = 32\text{ in.}^2$

$K = \frac{Pe}{\sum x^2 + \sum y^2} = \frac{P(6\text{ in.})}{32\text{ in}^2} = \frac{P}{5.33\text{ in.}}$



Due to direct shear force: The shear force in each bolt is $\frac{P}{4} = 0.25P$

Due to torque:

$H = Ky = \frac{P}{5.33\text{ in.}}(2\text{ in.}) = 0.375\,P$

$V = Kx = \frac{P}{5.33\text{ in.}}(2\text{ in.}) = 0.375\,P$

The maximum shear force:

$F_{max} = \sqrt{H^2 + \left(V + \frac{P}{4}\right)^2}$

$= \sqrt{(0.375P)^2 + (0.375\,P + 0.25\,P)^2} = 0.729\,P$

For $\frac{3}{4}$ in. bolts:

$A = \frac{\pi}{4}\left(\frac{3}{4}\text{ in.}\right)^2 = 0.442\text{ in.}^2$

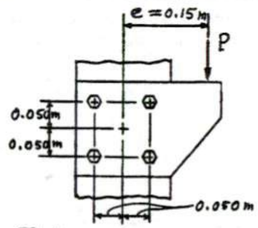
The allowable load per bolt is

$F_{allow} = A\tau_{allow} = (0.442\text{ in.}^2)(20\text{ kip/in.}^2)$
$= 8.84\text{ kips} = 0.729\,P$

Equating the maximum load to the allowable load, we get

$P = \frac{8.84\text{ kips}}{0.729} = 12.1\text{ kips}$ ◄

20-28



$\sum x^2 + \sum y^2 = 4\,(0.050\text{ m})^2 + 4\,(0.050\text{ m})^2$
$= 0.02\text{ m}^2$

$K = \frac{Pe}{\sum x^2 + \sum y^2} = \frac{P(0.150\text{ m})}{0.02\text{ m}^2} = 7.5\,P/\text{m}$

$H = Ky = (7.5\,P/\text{m})(0.050\text{ m}) = 0.375\,P$
$V = Kx = (7.5\,P/\text{m})(0.050\text{ m}) = 0.375\,P$

Due to direct shear force, shear force per bolt is:
$\frac{P}{4} = 0.25\,P$

The maximum load is

$F_{max} = \sqrt{H^2 + (V + 0.25\,P)^2}$

$= \sqrt{(0.375P)^2 + (0.375\,P + 0.25\,P)^2} = 0.729\,P$

(Cont'd)

20-28 (Cont)

For 20 mm bolts,

$A = \frac{\pi}{4}(0.020\text{ m})^2 = 3.14 \times 10^{-4}\text{ m}^2$

For A490 friction type bolt, $\tau_{allow} = 138$ MPa

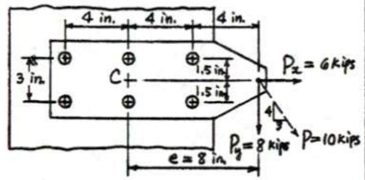
The allowable load per bolt is

$F_{allow} = A\tau_{allow} = (3.14 \times 10^{-4}\text{ m}^2)(138 \times 10^3\text{ m}^2)$
$= 43.3\text{ kN} = 0.729\,P$

Equating the allowable load to the maximum load, we get

$P = \frac{43.3\text{ kN}}{0.729} = 59.4\text{ kN}$ ◄

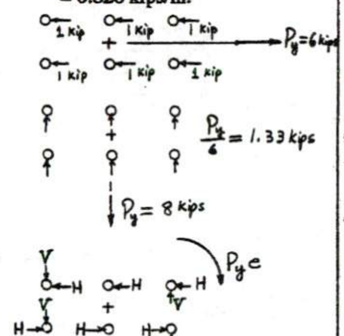
20-29



$\sum x^2 + \sum y^2 = 4\,(4\text{ in.})^2 + 6\,(2\text{ in.})^2 = 77.5\text{ in.}^2$

$K = \frac{P_x(0) + P_y e}{\sum x^2 + \sum y^2} = \frac{(8\text{ kips})(8\text{ in.})}{77.5\text{ in.}^2}$

$= 0.826\text{ kips/in.}$



$H = Ky = (0.826\text{ kips/in.})(1.5\text{ in.}) = 1.24\text{ kips}$
$V = Kx = (0.826\text{ kips/in.})(4\text{ in.}) = 3.30\text{ kips}$

The maximum load on the rivet is

$F_{max} = \sqrt{(H + 1)^2 + (V + 1.33)^2}$

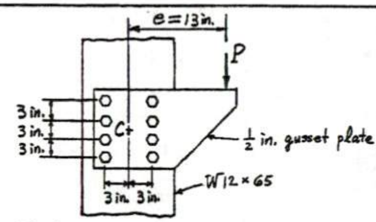
$= \sqrt{(1.24 + 1)^2 + (3.30 + 1.33)^2} = 5.14\text{ kips}$

For $\frac{3}{4}$ in. rivets:

$A = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.442\text{ in.}^2$

$\tau_{max} = \frac{F_{max}}{A} = \frac{5.14\text{ kips}}{0.442\text{ in.}^2} = 11.6\text{ ksi}$ ◄

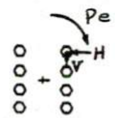
20-30



$\sum x^2 + \sum y^2 = 8\,(3\text{ in.})^2 + 4\,(1.5\text{ in.})^2 + 4\,(4.5\text{ in.})^2$

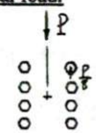
$= 162\text{ in.}^2$

$K = \frac{Pe}{\sum x^2 + \sum y^2} = \frac{P(13\text{ in.})}{162\text{ in}^2} = 0.0802\,P/\text{in.}$



Due to torque:
$H = Ky = (0.0802\,P/\text{in.})(4.5\text{ in.}) = 0.3609\,P$
$V = Kx = (0.0802\,P/\text{in.})(3\text{ in.}) = 0.241\,P$

Due to direct shear load:



$\frac{P}{8} = 0.125\,P$

(Cont'd)

20-30 (Cont)

The maximum load:

$$F_{max} = \sqrt{H^2 + \left(V + \frac{P}{8}\right)^2}$$

$$= \sqrt{(0.3609\ P)^2 + (0.241\ P + 0.125\ P)^2}$$

$$= 0.514 \text{ kips}$$

For $\frac{7}{8}$ in. bolts: $A = \frac{\pi}{4}\left(\frac{7}{8} \text{ in.}\right)^2 = 0.601 \text{ in.}^2$

For A 325 bearing-type bolts: $\tau_{allow} = 22$ ksi

$(F_s)_{allow} = A\tau_{allow} = (0.601 \text{ in.}^2)(22 \text{ kip/in.}^2)$
$= 13.22$ kips

Equating the maximum load to the allowable shear force, we get

$0.514\,P = 13.22$ kips

$P = 25.7$ kips

Allowable bearing stress (for bearing-type bolts) on A36 steel:

$(\sigma_b)_{allow} = 1.35 \times 36 \text{ ksi} = 48.5 \text{ ksi}$

For W12 x 65 column, $t_f = 0.605$ in.

Thus the thickness of the gusset plate $t = \frac{1}{2}$ in. is more critical for bearing stress.

$(F_b)_{allow} = td(\sigma_b)_{allow}$

$= \left(\frac{1}{2} \text{ in.}\right)\left(\frac{7}{8} \text{ in.}\right)(48.6 \text{ ksi}) = 21.3$ kips

Equating the maximum load to the allowable bearing force, we get

$0.514\,P = 21.3$ kips

$P = 41.4$ kips

$P_{allow} = 25.7$ kips ◄

20-31

Allowable load of an E = 70, $\frac{1}{2}$ in. fillet weld:

$q = 0.212\ \sigma_u$ (size)

$= 0.212\ (70 \text{ kip/in.}^2)\left(\frac{1}{2} \text{ in.}\right) = 7.42$ kip/in.

Length of weld: $L = 2(8 \text{ in.}) = 16$ in.

Strength of the weld:

$P = qL = (7.42 \text{ kips/in.})(16 \text{ in.})$

$= 119$ kips ◄

$$\text{Joint Efficiency} = \frac{P}{P_{plate}} \times 100\%$$

$$= \frac{119 \text{ kips}}{\left(8 \text{ in.} \times \frac{3}{4} \text{ in.}\right)(0.6 \times 36 \text{ kips/in.}^2)} \times 100\%$$

$= 91.8\ \%$ ◄

20-32

Strength of plate:

$P_{plate} = \left(8 \text{ in.} \times \frac{3}{4} \text{ in.}\right)(0.6 \times 36 \text{ kips/in.}^2) = 130$ kips

To develop full strength of the plate, the strength of the weld must be no less than the strength of the plate. Thus

$P = qL = P_{plate}$

$0.212\ \sigma_u$ (size)$L = P_{plate}$

$$\text{size} = \frac{P_{plate}}{0.212\sigma_u L} = \frac{130 \text{ kips}}{0.212\ (70 \text{ kip/in.}^2)(16 \text{ in.})}$$

$= 0.548$ in.

Use $\frac{9}{16}$ in. (= 0.5625 in.) fillet weld ◄

20-33

Allowable load of an E70, $\frac{1}{2}$ in. fillet weld:

$q = 0.212\ \sigma_u$ (size)

$= 0.212\ (70 \text{ kip/in}^2)\left(\frac{1}{2} \text{ in.}\right) = 7.42$ kips/in.

(Cont'd)

Length of weld:

$L = 2(10 \text{ in.}) = 20$ in.

Strength of the weld:

$P = qL = (7.42 \text{ kips/in.})(20 \text{ in.})$

$= 148$ kips ◄

$$\text{Joint Efficiency} = \frac{P}{P_{plate}} \times 100\%$$

$$= \frac{148 \text{ kips}}{\left(10 \text{ in.} \times \frac{3}{4} \text{ in.}\right)(0.6 \times 36 \text{ kips/in.}^2)} \times 100\%$$

$= 91.4\ \%$ ◄

20-34

Strength of Plate:

$P_{plate} = \left(10 \text{ in.} \times \frac{3}{4} \text{in.}\right)(0.6 \times 36 \text{ kips/in.}^2) = 162$ kips

To develop full strength of the plate, the strength of the weld must be no less than the strength of the plate. Thus

$P = qL = P_{plate}$

$0.212\ \sigma_u$ (size)$L = P_{plate}$

$$\text{size} = \frac{P_{plate}}{0.212\sigma_u L} = \frac{162 \text{ kips}}{0.212\ (70 \text{ kip/in.}^2)(20 \text{ in.})}$$

$= 0.546$ in.

Use $\frac{9}{16}$ in. (0.5625 in.) fillet weld ◄

20-35

Strength of plate:

$P_{plate} = \left(8 \text{ in.} \times \frac{1}{2} \text{ in.}\right)(0.6 \times 36 \text{ kips/in.}^2) = 86.4$ kips

Allowable load of an E70, $\frac{5}{16}$ in. fillet weld:

$q = 0.212\ \sigma_u$ (size)

$= 0.212\ (70 \text{ kip/in.}^2)\left(\frac{5}{16} \text{ in.}\right) = 4.64$ kip/in.

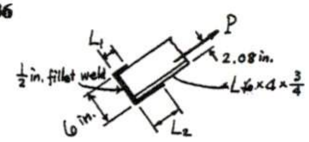
To develop the full strength of the plate, we must have

$$L = \frac{P}{q} = \frac{86.4 \text{ kips}}{4.64 \text{ kips/in.}} = 18.6 \text{ in.}$$

Use $L = 19$ in. ◄

$L_1 = \frac{1}{2}(19 \text{ in.} - 8 \text{ in.}) = 5\frac{1}{2}$ in. ◄

20-36



Strength of $L6 \times 4 \times \frac{3}{4}$ angle:

$P = A(0.6\ \sigma_y)$
$= (6.94 \text{ in.}^2)(0.6 \times 36 \text{ kip/in.}^2) = 149.9$ kips

$q = 0.212\ \sigma_u$ (size)

$= 0.212\ (90 \text{ kip/in.}^2)\left(\frac{1}{2} \text{ in.}\right) = 9.54$ kip/in.

Total length required:

$$L = \frac{P}{q} = \frac{149.9 \text{ kips}}{9.54 \text{ kip/in.}} = 15.7 \text{ in.}$$

Use $L = 16$ in. ◄

Consider moment of weld about L_2, we have

$L_1(6) + 6(3) + L_2(0) = L(2.08) = 16(2.08)$

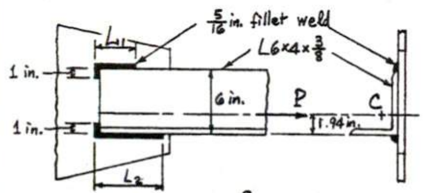
$$L_1 = \frac{16(2.08) - 6(3)}{6} = 2.55 \text{ in.}$$

Use $L_1 = 2\frac{1}{2}$ in. ◄

$L_2 = 16 - 6 - 2.5 = 7.5$ in.

Use $L_2 = 7\frac{1}{2}$ in. ◄

20-37



Tensile strength of L6 x 4 x $\frac{3}{8}$ angle:

$P = A(0.6\ \sigma_y)$
$= (3.61\ in.^2)\ (0.6 \times 36\ kip/in.^2) = 78.0\ kips$

Allowable load of an E80, $\frac{5}{16}$ in. fillet weld:

$q = 0.212\ \sigma_u$ (size)

$= 0.212\ (80\ kip/in.^2)\left(\frac{5}{16}\ in.\right) = 5.3\ kip/in.$

The total required length of weld is:

$$L = \frac{P}{q} = \frac{78.0\ kips}{5.3\ kips/in.} = 14.7\ in.$$

Use L = 15 in. ◀

The centroid C of the L6 x 4 x $\frac{3}{8}$ angle is 1.94 in. from the short leg. To have the centroid of the fillet weld to be located at the same distance, total moment of weld segments about L_2 must be equal to the moment of total length L (through C) about the same line. Thus

$L_1(6) + 1(5.5) + 1(0.5) = 15(1.94)$

from which we get

$L_1 = 3.85$ in.

Use $L_1 = 3\frac{7}{8}$ in. ◀

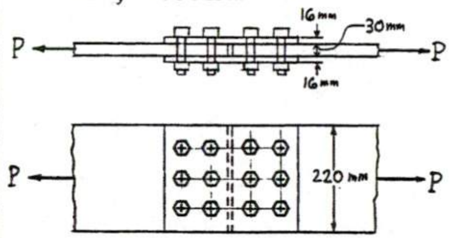
$L_2 = 15 - 3\frac{7}{8} - 2 = 9\frac{1}{8}$ in.

Use $L_2 = 9\frac{1}{8}$ in. ◀

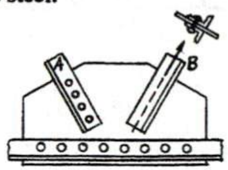
Test Problems for Chapter 20

The following problems may either be given to students for them to practice their problem solving skills or be used as test problems. The answers are provided on the next page.

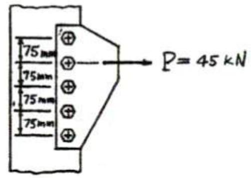
(1) Determine the strength and efficiency of the butt joint shown. The bearing type, 22-mm-diameter bolts are made of A325 steel. The plates are made of A36 steel with $\sigma_y = 250$ MPa.



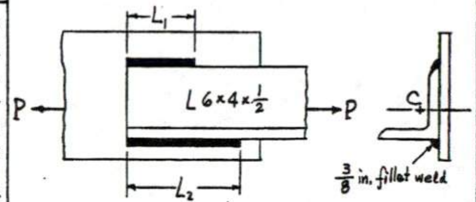
(2) The tension member *B* shown consists of two L4 X 3 X 3/8 angle riveted back to back to a 5/8-in. gusset plate. Determine the required number of 7/8-in. diameter rivets of A502 grade 2 steel so that the tensile strength of the net section of the angle can be develoed. The angles and the gusset plate are of A36 steel.



(3) A steel plate is attached to a machine with five 19-mm-diameter bolts as shown. Determine the maximum shear stress in the bolts.

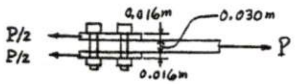


(4) The structural joint shown is fillet-welded with E-90 electrode and has a strength equal to that of steel angle with $\sigma_y = 36$ ksi. Determine the proper length L_1 and L_2 if the load is applied through the centroid *C* of the angle.



Solutions to Test Problems for Chapter 20

(1)



Six 22 mm diam. A 490 bolts.

$\tau_{allow} = 152$ MPa

$A_s = \dfrac{\pi}{4}(0.022\text{ m})^2 = 3.80 \times 10^{-4}\text{ m}^2$

$P_s = nA_s\tau_{allow} = 6(2 \times 3.80\text{x}10^{-4}\text{ m}^2)(152\text{x}10^3\text{ kN/m}^2)$
$= 693$ kN

$P_b = n(td)(\sigma_b)_{allow}$
$= 6(0.030\text{ m} \times 0.022\text{ m})(1.35 \times 250 \times 10^3\text{ kN/m}^2)$
$= 1337$ kN

$b_{net} = b - n(d + c) = 220\text{ mm} - 3(22\text{ mm} + 3\text{ mm})$
$= 145\text{ mm} = 0.145\text{ m}$

$P_t = b_{net}t(\sigma_t)_{allow}$
$= (0.145\text{ m})(0.030\text{ m})(0.6 \times 250 \times 10^3\text{ kN/m}^2)$
$= 623$ kN

Joint Strength: $P = P_t = 623$ kN ◀

Joint Efficiency $= \dfrac{P}{P_{solid\ plate}}$

$= \dfrac{623\text{ kN}}{(0.220\text{m} \times 0.030\text{m})(0.6\text{x}250\text{x}10^3\text{kN/m}^2)}$

$= 0.63 = 63\%$ ◀

(2)

The member is made of two L 4 x 3 x $\frac{3}{8}$ angles.

$A_{net} = 2A_{angle} - 2t(d + c)$
$= 2(2.48) - 2\left(\frac{3}{8}\right)\left(\frac{7}{8} + \frac{1}{8}\right) = 4.21\text{ in.}^2$

$P_t = A_{net}(\sigma_t)_{allow}$
$= (4.21\text{ in.}^2)(0.6 \times 36\text{ kips/in.}^2) = 90.9$ kips

$\frac{7}{8}$ in. diam. A 502-2 revets.

$A_s = \dfrac{\pi}{4}\left(\dfrac{7}{8}\text{in.}\right)^2 = 0.601\text{ in.}^2$

P_s per rivet $= 2A_s\tau_{allow} = 2(0.601\text{ in.}^2)(20\text{ kips/in.}^2)$
$= 24.04$ kips

P_b per rivet $= (t_s d)(\sigma_b)_{allow}$

$= \left(\dfrac{5}{8}\text{ in.} \times \dfrac{7}{8}\text{ in.}\right)(1.35 \times 36\text{ kips/in.}^2)$

$= 26.6$ kips

The number of rivets required is

$$n = \frac{P_t}{P_s\text{ per rivet}} = \frac{90.9\text{ kips}}{24.04\text{ kips}} = 3.78$$

Use 4 rivets ◀

(3)

The load is equivalent to:
$P = 45$ kN
$M = (45\text{ kN})(0.075\text{ m}) = 3.375\text{ kN}\cdot\text{m}$

Due to P:

$F = \dfrac{P}{5} = \dfrac{45\text{ kN}}{5} = 9$ kN

Due to M:

$\sum y^2 = 2(0.15\text{ m})^2 + 2(0.075\text{ m})^2 = 0.05635\text{ m}^2$

$K = \dfrac{M}{\sum y^2} = \dfrac{3.375\text{ kN}\cdot\text{m}}{0.05625\text{ m}^2} = 60$ kN/m

$H = Ky = (60\text{ kN/m})(0.15\text{ m}) = 9$ kN

The largest shear load on the upper bolt is

$F + H = 9\text{ kN} + 9\text{ kN} = 18$ kN

The maximum shear stress is

$$\tau_{max} = \frac{18\text{ kN}}{\frac{\pi}{4}(0.019\text{ m})^2}$$

$= 63\ 500\text{ kN/m}^2 = 63.5$ MPa ◀

Solutions to Test Problems for Chapter 20 (Cont'd)

(4)

The tensile strength of L6 x 4 x $\frac{1}{2}$ angle:

$P_t = A_{angle}(0.6\ \sigma_y)$
$= (4.75\text{ in.}^2)(0.6 \times 36\text{ kip/in.}^2) = 102.6$ kips

Allowable load of an E90, $\frac{3}{8}$ in. fillet weld:

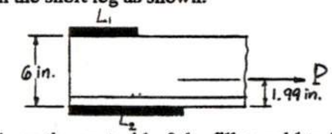
$q = 0.212\ \sigma_u$ (size)
$= 0.212(90\text{ kip/in.}^2)\left(\frac{3}{8}\text{ in.}\right) = 7.16$ kip/in.

The total required length of weld is:

$$L = \frac{P}{q} = \frac{102.6\text{ kips}}{7.16\text{ kips/in.}} = 14.3\text{ in.}$$

Use L = 15 in. ◀

The centroid C of the L6 x 4 x $\frac{3}{8}$ angle is 1.99 in. from the short leg as shown.



To have the centroid of the fillet weld to be located at the same position, the total moment of weld segments about L_2 must be equal to the moment of the total length L (through C) about the same line. Thus

$L_1(6) + L_2(0) = 15(1.99)$

from which we get

$L_1 = 4.98$ in.

Use $L_1 = 5$ in. ◀

$L_2 = 15 - 5 = 10$ in.

Use $L_2 = 10$ in. ◀

Solution to Computer Program Assignment C1-1

```
'
' * C01-1 *   Solution to two linear equations with two unknowns
'
' Clear screen
        CLS
'
' Display the introductory remarks
        PRINT "Want to see the introductory remarks?"
        INPUT "-- Y / N "; RMK$
        IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 190
100     PRINT "REMARKS: (1) This program can be used to solve two equations"
        PRINT "             with two unknowns."
        PRINT "         (2) The input data are the coefficients of x and y"
        PRINT "             and the right hand constant of each equation."
        PRINT "         (3) The input data are printed on the line printer"
        PRINT "             immediately after data are entered."
        PRINT "             Make sure your printer is on."
'
' Input problem I.D. and print problem I.D.
190     PRINT "Enter problem designation:"
        INPUT "-- Prob. I.D. "; ID$
        LPRINT "TWO LINEAR EQUATIONS WITH TWO UNKNOWNS":    LPRINT
        LPRINT "Solution to "; ID$
        LPRINT :    LPRINT "The given equations are:":    LPRINT
'
' Enter coefficients of x and y and right hand constants, print equation
        PRINT "Enter coefficients of x and y and right hand constant--"
       FOR I = 1 TO 2
        PRINT "    For Equation #"; I; ": "; :    INPUT A(I), B(I), K(I)
        LPRINT TAB(6); A(I); "x";
        IF B(I) >= 0 THEN LPRINT " + ";
        IF B(I) < 0 THEN LPRINT " - ";
        LPRINT ABS(B(I)); "y = "; K(I)
       NEXT I
'
' Compute the second order determinate and the solution
        D = A(1) * B(2) - A(2) * B(1)
        DX = K(1) * B(2) - K(2) * B(1)
        DY = A(1) * K(2) - A(2) * K(1)
        IF D = 0 GOTO 410
        X = DX / D:    Y = DY / D
'
' Print results
410     LPRINT :    LPRINT "The determinants are:":    LPRINT
        LPRINT TAB(7); "D  = "; :    LPRINT USING "######.###"; D
        LPRINT TAB(7); "Dx = "; :    LPRINT USING "######.###"; DX
        LPRINT TAB(7); "Dy = "; :    LPRINT USING "######.###"; DY
        IF D = 0 GOTO 490
        LPRINT :    LPRINT "The solution to the equations is:":    LPRINT
        LPRINT TAB(7); "x = "; :    LPRINT USING "#######.###"; X
        LPRINT TAB(7); "y = "; :    LPRINT USING "#######.###"; Y
        GOTO 510
490     LPRINT
        IF DX = 0 AND DY = 0 GOTO 500
        LPRINT "The equations are inconsistent, there is no solution"
        GOTO 510
500     LPRINT "The equations are dependent, there is no unique solution"
510 END
```


C1-1 (a)

```
TWO LINEAR EQUATIONS WITH TWO UNKNOWNS

Solution to EXAMPLE 1-16

The given equations are:

     .9397 x -  .766 y =  0
     .342 x +  .6428 y =  100

The determinants are:

      D  =      0.866
      Dx =     76.600
      Dy =     93.970

The solution to the equations is:

      x =      88.452
      y =     108.509
```

C1-1 (b)

```
TWO LINEAR EQUATIONS WITH TWO UNKNOWNS

Solution to Problem 1-51

The given equations are:

     3.45 x -  2.65 y =  2.77
     1.86 x +  3.76 y =  9.85

The determinants are:

      D  =     17.901
      Dx =     36.518
      Dy =     28.830

The solution to the equations is:

      x =       2.040
      y =       1.611
```

C1-1 (c)

```
TWO LINEAR EQUATIONS WITH TWO UNKNOWNS

Solution to Problem 1-52

The given equations are:

     .1736 x -  .6428 y =  0
     .9848 x -  .766 y =  200

The determinants are:

      D  =      0.500
      Dx =    128.560
      Dy =     34.720

The solution to the equations is:

      x =     257.093
      y =      69.433
```

Solution to Computer Program Assignment C1-2

```
' * C01-2 *   Solution to three linear equations with three unknowns
'
' Clear screen
     CLS
'
' Display the introductory remarks
     PRINT "Want to see the introductory remarks?"
     INPUT "-- Y / N "; RMK$
     IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 190
100  PRINT "REMARKS: (1) This program can be used to solve three equations"
     PRINT "             with three unknowns."
     PRINT "         (2) The input data are the coefficients of x and y"
     PRINT "             and the right hand constant of each equation."
     PRINT "         (3) The input data are printed on the line printer"
     PRINT "             immediately after data are entered."
     PRINT "             Make sure your printer is on."
'
' Input problem I.D. and print problem I.D.
190  PRINT "Enter problem designation:"
     INPUT "-- Prob. I.D. "; ID$
     LPRINT "THREE LINEAR EQUATIONS WITH THREE UNKNOWNS":   LPRINT
     LPRINT "Solution to "; ID$
     LPRINT :   LPRINT "The given equations are:":   LPRINT
'
' Enter coefficients of x and y and right hand constants, print equation
     PRINT "Enter coefficients of x and y and right hand constant:"
    FOR I = 1 TO 3
     PRINT "--For Equation #"; I; ": "; :   INPUT A(I), B(I), C(I), K(I)
     LPRINT TAB(6); A(I); "x";
     IF B(I) >= 0 THEN LPRINT " + ";
     IF B(I) < 0 THEN LPRINT " - ";
     LPRINT ABS(B(I)); "y";
     IF C(I) >= 0 THEN LPRINT " + ";
     IF C(I) < 0 THEN LPRINT " - ";
     LPRINT ABS(C(I)); "z = "; K(I)
    NEXT I
'
' Compute the third order determinate and the solution
     D = A(1) * B(2) * C(3) + A(3) * B(1) * C(2) + A(2) * B(3) * C(1)
     D = D - A(3) * B(2) * C(1) - A(1) * B(3) * C(2) - A(2) * B(1) * C(3)
     DX = K(1) * B(2) * C(3) + K(3) * B(1) * C(2) + K(2) * B(3) * C(1)
     DX = DX - K(3) * B(2) * C(1) - K(1) * B(3) * C(2) - K(2) * B(1) * C(3)
     DY = A(1) * K(2) * C(3) + A(3) * K(1) * C(2) + A(2) * K(3) * C(1)
     DY = DY - A(3) * K(2) * C(1) - A(1) * K(3) * C(2) - A(2) * K(1) * C(3)
     DZ = A(1) * B(2) * K(3) + A(3) * B(1) * K(2) + A(2) * B(3) * K(1)
     DZ = DZ - A(3) * B(2) * K(1) - A(1) * B(3) * K(2) - A(2) * B(1) * K(3)
     IF D = 0 GOTO 510
     X = DX / D:   Y = DY / D:   Z = DZ / D
'
' Print results
510  LPRINT :   LPRINT "The determinants are:":   LPRINT
     LPRINT TAB(7); "D  = "; :   LPRINT USING "######.###"; D
     LPRINT TAB(7); "Dx = "; :   LPRINT USING "######.###"; DX
     LPRINT TAB(7); "Dy = "; :   LPRINT USING "######.###"; DY
     LPRINT TAB(7); "Dz = "; :   LPRINT USING "######.###"; DZ
     IF D = 0 GOTO 610
     LPRINT :   LPRINT "The solution to the equations is:":   LPRINT
     LPRINT TAB(7); "x = "; :   LPRINT USING "#######.###"; X
     LPRINT TAB(7); "y = "; :   LPRINT USING "#######.###"; Y
     LPRINT TAB(7); "z = "; :   LPRINT USING "#######.###"; Z
     GOTO 660
610  LPRINT
     IF DX = 0 AND DY = 0 AND DZ = 0 GOTO 650
     LPRINT "The equations are inconsistent, there is no solution"
     GOTO 660
650  LPRINT "The equations are dependent, there is no unique solution"
660 END
```


C1-2 (a)

```
THREE LINEAR EQUATIONS WITH THREE UNKNOWNS

Solution to EXAMPLE 1-20

The given equations are:

     3 x +  0 y +  1 z =  0
     2 x -  1 y +  4 z =  8
     4 x -  3 y +  1 z = -7

The determinants are:

      D  =     31.000
      Dx =    -31.000
      Dy =     62.000
      Dz =     93.000

The solution to the equations is:

      x =      -1.000
      y =       2.000
      z =       3.000
```

C1-2 (b)

```
THREE LINEAR EQUATIONS WITH THREE UNKNOWNS

Solution to Problem 1-53

The given equations are:

     -.429 x +  .231 y +  0 z =  1920
     -.857 x -  .923 y -  .923 z =  2880
      .286 x -  .308 y -  .385 z =  2160

The determinants are:

      D  =      -0.168
      Dx =     -67.951
      Dy =   -1519.946
      Dz =    2106.260

The solution to the equations is:

      x =      405.228
      y =     9064.255
      z =   -12560.766
```

C1-2 (c)

```
THREE LINEAR EQUATIONS WITH THREE UNKNOWNS

Solution to Problem 1-54

The given equations are:

     -.444 x -  .857 y +  .667 z =  0
      .444 x +  .429 y +  .667 z =  17
      .778 x -  .286 y -  .333 z =  0

The determinants are:

      D  =      -0.900
      Dx =      -8.094
      Dy =      -6.308
      Dz =     -13.493

The solution to the equations is:

      x =        8.994
      y =        7.009
      z =       14.992
```

Solution to Computer Program Assignment C2-1

```
'  * C02-1 *  Resultant of a concurrent coplanar force system
'
' Clear screen, compute pi and the conversion factors
      CLS :   PI = 4 * ATN(1):    DR = PI / 180:  RD = 180 / PI
'
' Display the introductory remarks
      PRINT "Want to see the introductory remarks?"
      INPUT "-- Y / N "; RMK$
      IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100   PRINT "REMARKS: (1) This program can be used to compute the resultant"
      PRINT "             of a concurrent coplanar force system."
      PRINT "         (2) You need to input the magnitude and the direction"
      PRINT "             angle of each force. The direction angles must be"
      PRINT "             in the standard position."
      PRINT "         (3) The input data and the computed results are"
      PRINT "             printed on the printer immediately after"
      PRINT "             data are entered. Make sure your printer is"
      PRINT "             on before entering data."
'
' Input problem I.D.
200   PRINT "Enter problem designation:"
      INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
      PRINT "Select the system of units:"
      PRINT "        1 -- SI Units"
      PRINT "        2 -- US customary units"
      INPUT "-- 1 / 2"; U
      IF U = 1 THEN UF$ = "N"
      IF U = 2 THEN UF$ = "lb"
'
' Print problem I.D. and heading
      LPRINT "RESULTANT OF A CONCURRENT COPLANAR FORCE SYSTEM"
      LPRINT :   LPRINT "Solution to "; ID$:    LPRINT
      LPRINT "____________________________________________________________"
      LPRINT
      LPRINT "Force  Magnitude  Direction Angle     x-compt       y-compt"
      LPRINT TAB(12); UF$; TAB(23); "Degrees"; TAB(42); UF$; TAB(55); UF$
      LPRINT "____________________________________________________________"
      LPRINT
'
' Enter the number of given forces
      PRINT "Enter the number of given forces:"
      INPUT "-- No. of Force"; NF
'
' Initialize sum
      RX = 0:    RY = 0
'
' Enter the data of the given forces and compute their components
      PRINT "Input the magnitude and the direction angle ";
      PRINT "(in standard postion) of each force:"
'
   FOR I = 1 TO NF
      PRINT "-- Mag.("; UF$; ") & Angle° of Force No."; :   PRINT I;
      INPUT F, AD
'
' Compute the force components
      A = AD * DR:    FX = F * COS(A):    FY = F * SIN(A)
'
' Print the given forces
      LPRINT "F"; USING "#"; I;
```


Solution to Computer Program Assignment C2-1 Continued

```
      LPRINT TAB(4); USING "########.##"; F;
      LPRINT TAB(21); USING "####.##"; AD;
      LPRINT TAB(34); USING "########.##"; FX;
      LPRINT TAB(47); USING "########.##"; FY
'
' Compute the sum of force components
      RX = RX + FX:     RY = RY + FY
'
   NEXT I
'
' Find the magnitude of the resultant
      R = SQR(RX * RX + RY * RY)
'
' Find the acute angle alpha
      IF ABS(RX / R) < .00001 THEN RX = 0
      IF ABS(RY / R) < .00001 THEN RY = 0
      IF ABS(RX) > 0 THEN A = ATN(RY / RX)
      ALPHA = A * RD
      IF RX = 0 AND RY > 0 THEN ALPHA = 90
      IF RX = 0 AND RY < 0 THEN ALPHA = -90
'
' Find the direction angle of the resultant in the standard position
      IF RX >= 0 THEN THETA = ALPHA
      IF RX < 0 THEN THETA = 180 + ALPHA
      IF THETA > 180 THEN THETA = THETA - 360
'
      LPRINT "____________________________________________________________"
      LPRINT
      LPRINT "R"; TAB(4); USING "########.##"; R;
      LPRINT TAB(21); USING "####.##"; THETA;
      LPRINT TAB(34); USING "########.##"; RX;
      LPRINT TAB(47); USING "########.##"; RY
      LPRINT "____________________________________________________________"
END
```

C2-1 (a) RESULTANT OF A CONCURRENT COPLANAR FORCE SYSTEM

Solution to EXAMPLE 2-9

Force	Magnitude N	Direction Angle Degrees	x-compt N	y-compt N
F1	4000.00	0.00	4000.00	0.00
F2	9000.00	60.00	4500.00	7794.23
F3	8000.00	105.00	-2070.55	7727.41
F4	25000.00	150.00	-21650.63	12500.00
F5	3000.00	200.00	-2819.08	-1026.06
R	32468.64	123.75	-18040.27	26995.58

C2-1 (b) RESULTANT OF A CONCURRENT COPLANAR FORCE SYSTEM

Solution to Problem 2-25

Force	Magnitude lb	Direction Angle Degrees	x-compt lb	y-compt lb
F1	16.00	255.00	-4.14	-15.45
F2	20.00	-30.00	17.32	-10.00
F3	15.00	36.87	12.00	9.00
F4	10.00	90.00	-0.00	10.00
R	25.99	-14.38	25.18	-6.45

C2-1 (c) RESULTANT OF A CONCURRENT COPLANAR FORCE SYSTEM

Solution to Problem 2-26

Force	Magnitude N	Direction Angle Degrees	x-compt N	y-compt N
F1	10000.00	-25.00	9063.08	-4226.18
F2	8000.00	35.00	6553.22	4588.61
F3	7000.00	130.00	-4499.51	5362.31
F4	12000.00	180.00	-12000.00	-0.00
R	5792.47	98.77	-883.22	5724.74

Solution to Computer Program Assignment C2-2

```
' * C02-2 *   Resultant of a parallel coplanar force system
'
' Clear screen
    CLS
'
' Display the introductory remarks
    PRINT "Want to see the introductory remarks?"
    INPUT "-- Y / N "; RMK$
    IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100 PRINT "REMARKS: (1) This program can be used to compute the resultant"
    PRINT "             of a parallel coplanar force system."
    PRINT "         (2) The forces are assumed to act in the vertical"
    PRINT "             position. Downward forces are positive and"
    PRINT "             upward as negative."
    PRINT "         (3) You must input the magnitude and location of each"
    PRINT "             force."
    PRINT "         (4) The input data and the computed results are"
    PRINT "             printed on the printer immediately after"
    PRINT "             data are entered. Make sure your printer is on"
    PRINT "             before entering data."
'
' Input problem I.D.
200 PRINT "Enter problem designation:"
    INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
    PRINT "Select the system of units:"
    PRINT "        1 -- SI Units"
    PRINT "        2 -- US customary units"
    INPUT "-- 1 / 2"; U
    IF U = 2 GOTO 300
    UF$ = "N": UL$ = "m": UM$ = "N-M"
    GOTO 400
300 UF$ = "lb": UL$ = "ft": UM$ = "lb-ft"
'
' Print problem I.D. and heading
400 LPRINT "RESULTANT OF A PARALLEL COPLANAR FORCE SYSTEM"
    LPRINT :   LPRINT "Solution to "; ID$:    LPRINT
    LPRINT "_____________________________________________________"
    LPRINT :   LPRINT "Force   Magnitude   Dist. from A   Moment about A"
    LPRINT TAB(12); UF$; TAB(25); UL$; TAB(40); UM$
    LPRINT "_____________________________________________________"
    LPRINT
'
' Enter the number of given forces
    PRINT "Enter the number of given forces:"
    INPUT "-- No. of Forces"; N
'
' Initialize sum
    R = 0:    M = 0
'
' Enter the data of the given forces and compute their components
    PRINT "Input the magnitude and position of each force"
'
  FOR I = 1 TO N
    PRINT "-- Mag.("; UF$; ", downward as +) & Dist. from A ("; UL$; ")";
    INPUT F, D
'
' Compute the moments about A
    MF = F * D
```

Solution to Computer Program Assignment C2-2 Continued

```
' Print the given forces and its moment about A
     LPRINT "F"; USING "#"; I;
     LPRINT TAB(5); USING "########.##"; F;
     LPRINT TAB(21); USING "####.###"; D;
     LPRINT TAB(35); USING "########.##"; MF
'
' Compute the sum of forces and moments
     R = R + F:     M = M + MF
'
  NEXT I
'
' Compute the location of the resultant
     X = M / R
'
     LPRINT "_____________________________________________________"
     LPRINT
     LPRINT "R"; TAB(5); USING "########.##"; R;
     LPRINT TAB(21); USING "####.###"; X;
     LPRINT TAB(35); USING "########.##"; M
     LPRINT "_____________________________________________________"
END
```

C2-2 (a)

RESULTANT OF A PARALLEL COPLANAR FORCE SYSTEM

Solution to Problem 2-60

Force	Magnitude N	Dist. from A m	Moment about A N-M
F1	400.00	0.000	0.00
F2	500.00	3.000	1500.00
F3	-300.00	5.000	-1500.00
F4	200.00	7.000	1400.00
R	800.00	1.750	1400.00

C2-2 (b)

RESULTANT OF A PARALLEL COPLANAR FORCE SYSTEM

Solution to Problem 2-61

Force	Magnitude lb	Dist. from A ft	Moment about A lb-ft
F1	-90.00	3.000	-270.00
F2	130.00	6.000	780.00
F3	60.00	9.000	540.00
R	100.00	10.500	1050.00

Solution to Computer Program Assignment C3-1

```
' * C03-1 *    Equilibrium of a concurrent coplanar force system
'
' Clear screen, compute pi and the conversion factors
     CLS :    PI = 4 * ATN(1):     DR = PI / 180:  RD = 180 / PI
'
' Display the introductory remarks
     PRINT "Want to see the introductory remarks?"
     INPUT "-- Y / N "; RMK$
     IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100  PRINT "REMARKS: (1) This program deals with the equilibrium of"
     PRINT "             concurrent coplanar force system. It can be used"
     PRINT "             to analyze a truss joint."
     PRINT "          (2) You need to input the direction angles of the two"
     PRINT "             unknown forces and the magnitude and the"
     PRINT "             direction angles of each known forces."
     PRINT "          (3) The input data and the computed results are"
     PRINT "             printed on the printer immediately after"
     PRINT "             data are entered. Make sure your printer is on"
     PRINT "             before entering data."
'
' Input problem I.D.
200  PRINT "Enter problem designation:"
     INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
     PRINT "Select the system of units:"
     PRINT "        1 -- SI Units"
     PRINT "        2 -- US customary units"
     INPUT "-- 1 / 2"; U
     IF U = 1 THEN UF$ = "N"
     IF U = 2 THEN UF$ = "lb"
'
' Print problem I.D. and heading
     LPRINT "EQUILIBRIUM OF A CONCURRENT COPLANAR FORCE SYSTEM"
     LPRINT :   LPRINT "Solution to "; ID$:     LPRINT
     LPRINT "______________________________________________________________"
     LPRINT :   LPRINT "Force  Magnitude  Direction Angle      ";
     LPRINT "x-compt       y-compt"
     LPRINT TAB(12); UF$; TAB(23); "Degrees"; TAB(42); UF$; TAB(55); UF$
     LPRINT "______________________________________________________________"
     LPRINT :   LPRINT "The given forces are:"
'
' Enter the direction angles of the unknown forces
     PRINT "Enter the direction angles of the unknown forces:"
     INPUT "-- Direction Angles (Deg.) of Unknown Forces -- P, Q"; AP, AQ
'
' Enter the number of given forces
     PRINT "Enter the number of given forces:"
     INPUT "-- No. of Given Forces"; N
'
' Initialize sum
     RX = 0:     RY = 0
'
' Enter the data of the given forces and compute their components
     PRINT "Input the magnitude and the direction angle (in standard";
     PRINT " postion) of each force:"
'
   FOR I = 1 TO N
     PRINT "-- Mag.("; UF$; ") & Angle (Deg.) of Force No."; :    PRINT I;
     INPUT F, AD
```

Solution to Computer Program Assignment C3-1 Continued

```
' Compute the force components
    A = AD * DR:      FX = F * COS(A):      FY = F * SIN(A)
'
' Print the given forces
    LPRINT "F"; USING "#"; I;
    LPRINT TAB(4); USING "########.##"; F;
    LPRINT TAB(21); USING "####.##"; AD;
    LPRINT TAB(34); USING "########.##"; FX;
    LPRINT TAB(47); USING "########.##"; FY
'
' Compute the sum of force components
    RX = RX + FX:     RY = RY + FY
'
  NEXT I
'
' Find the magnitude of the resultant
    R = SQR(RX * RX + RY * RY)
'
' Find the acute angle alpha
    IF ABS(RX / R) < .00001 THEN RX = 0
    IF ABS(RY / R) < .00001 THEN RY = 0
    IF ABS(RX) > 0 THEN A = ATN(RY / RX)
    ALPHA = A * RD
    IF RX = 0 AND RY > 0 THEN ALPHA = 90
    IF RX = 0 AND RY < 0 THEN ALPHA = -90
'
' Find the direction angle of the resultant in the standard position
    IF RX >= 0 THEN THETA = ALPHA
    IF RX < 0 THEN THETA = 180 + ALPHA
    IF THETA > 180 THEN THETA = THETA - 360
'
    LPRINT "-------------------------------------------------------"
    LPRINT "The resultant of the given forces are:"
    LPRINT "R"; TAB(4); USING "########.##"; R;
    LPRINT TAB(21); USING "####.##"; THETA;
    LPRINT TAB(34); USING "########.##"; RX;
    LPRINT TAB(47); USING "########.##"; RY
    LPRINT "-------------------------------------------------------"
'
' Compute the unknown forces P & Q
    APR = AP * DR:     SAP = SIN(APR):     CAP = COS(APR)
    AQR = AQ * DR:     SAQ = SIN(AQR):     CAQ = COS(AQR)
    D = CAP * SAQ - SAP * CAQ
    DP = RY * CAQ - RX * SAQ: DQ = RX * SAP - RY * CAP
    P = DP / D:    Q = DQ / D:    PX = P * CAP:  PY = P * SAP
    QX = Q * CAQ:  QY = Q * SAQ
'
    LPRINT "The two unknown forces are:"
    LPRINT "P"; TAB(4); USING "########.##"; P;
    LPRINT TAB(21); USING "####.##"; AP;
    LPRINT TAB(34); USING "########.##"; PX;
    LPRINT TAB(47); USING "########.##"; PY
    LPRINT "Q"; TAB(4); USING "########.##"; Q;
    LPRINT TAB(21); USING "####.##"; AQ;
    LPRINT TAB(34); USING "########.##"; QX;
    LPRINT TAB(47); USING "########.##"; QY
    LPRINT "_______________________________________________________"
END
```


C3-1 (a)

```
EQUILIBRIUM OF A CONCURRENT COPLANAR FORCE SYSTEM

Solution to EXAMPLE 3-4

___________________________________________________________________
Force  Magnitude  Direction Angle      x-compt        y-compt
           N          Degrees             N              N
___________________________________________________________________
The given forces are:
F1       491.00         -90.00           -0.00        -491.00
-------------------------------------------------------------------
The resultant of the given forces are:
R        491.00         -90.00            0.00        -491.00
-------------------------------------------------------------------
The two unknown forces are:
P        254.16         135.00         -179.72         179.72
Q        359.44          60.00          179.72         311.28
___________________________________________________________________
```

C3-1 (b)

```
EQUILIBRIUM OF A CONCURRENT COPLANAR FORCE SYSTEM

Solution to Problem 3-17

___________________________________________________________
Force  Magnitude  Direction Angle    x-compt      y-compt
           N          Degrees           N            N
___________________________________________________________
The given forces are:
F1      5000.00         -15.00       4829.63     -1294.10
-----------------------------------------------------------
The resultant of the given forces are:
R       5000.00         -15.00       4829.63     -1294.10
-----------------------------------------------------------
The two unknown forces are:
P       4111.41         125.00      -2358.21      3367.87
Q       3226.21         220.00      -2471.42     -2073.77
___________________________________________________________
```

C3-1 (c)

```
EQUILIBRIUM OF A CONCURRENT COPLANAR FORCE SYSTEM

Solution to Problem 3-30

___________________________________________________________________
Force  Magnitude  Direction Angle      x-compt        y-compt
           N          Degrees             N              N
___________________________________________________________________
The given forces are:
F1     20000.00           0.00        20000.00           0.00
F2      6000.00         180.00        -6000.00          -0.00
-------------------------------------------------------------------
The resultant of the given forces are:
R      14000.00           0.00        14000.00           0.00
-------------------------------------------------------------------
The two unknown forces are:
P      17998.05         230.00       -11568.92      -13787.31
Q      14000.00         100.00        -2431.08       13787.31
___________________________________________________________________
```

Solution to Computer Program Assignment C3-2

```
' * C03-2 *   Reactions on an overhanging beam or a cantilever beam
'
' Clear screen
    CLS
'
' Display the introductory remarks
    PRINT "Want to see the introductory remarks?"
    INPUT "-- Y / N "; RMK$
    IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100 PRINT "REMARKS: (1) This program can be used to compute the reactions"
    PRINT "             for an overhanging beam or a cantilever beam."
    PRINT "         (2) The loads on the beam include a concentrated"
    PRINT "             load, a uniform load, and a couple."
    PRINT "         (3) The input data and the computed results are"
    PRINT "             printed on the printer immediately after"
    PRINT "             data are entered. Make sure your printer is on"
    PRINT "             before entering data."
'
' Input problem I.D.
200 PRINT "Enter problem designation:"
    INPUT "-- Prob. I.D. "; ID$
'
' Specify the type of beam
    PRINT "Specify the type of beam:"
    PRINT "        1 -- Overhanging beam"
    PRINT "        2 -- Cantilever beam"
    INPUT "-- Type of beam -- 1 / 2"; TB
'
' Select system of units
    PRINT "Select the system of units:"
    PRINT "        1 -- SI Units"
    PRINT "        2 -- US customary units"
    INPUT "-- 1 / 2"; U
    IF U = 2 GOTO 300
    UF$ = "N": UL$ = "m": UM$ = "N-m": UW$ = "N/m"
    GOTO 400
300 UF$ = "lb": UL$ = "ft": UM$ = "lb-ft": UW$ = "lb/ft"
'
' Print problem I.D. and heading
400 IF TB = 1 THEN LPRINT "REACTIONS ON AN OVERHANGING BEAM"
    IF TB = 2 THEN LPRINT "REACTIONS ON A CANTILEVER BEAM"
    LPRINT :   LPRINT "Solution to "; ID$
    LPRINT "___________________________________________"
    LPRINT :   LPRINT "The input data are:"
'
' Enter the given data
    PRINT "-- Span length: -- L("; UL$; ")"; :   INPUT L
    IF TB = 1 THEN PRINT "-- Length of overhang: -- a ("; UL$; ")";
    IF TB = 1 THEN INPUT A
    PRINT "-- Mag. & location of conc. load: -- P ("; UF$; "),";
    PRINT "b ("; UL$; ")"; :   INPUT P, B
    PRINT "-- Inten. & location of unif. load: -- w ("; UW$; "),";
    PRINT "c1 ("; UL$; "), c2 ("; UL$; ")";
    INPUT W, c1, c2
    PRINT "-- Couple: -- M ("; UM$; ")"; :   INPUT M
'
' Compute the reactions
    RB = (P * B + W * (c2 - c1) * (c2 + c1) / 2 + M) / L
    RA = P + W * (c2 - c1) - RB
    VA = P + W * (c2 - c1)
    MA = P * B + W * (c2 - c1) * (c2 + c1) / 2 + M
```


Solution to Computer Program Assignment C3-2 Continued

```
' Print the input data
    LPRINT "Span Length L ="; USING "####.###"; L; :   LPRINT " "; UL$
    IF TB = 1 THEN LPRINT "Length of Overhang a ="; USING "####.###"; A;
    IF TB = 1 THEN LPRINT " "; UL$
    LPRINT "Concentrated Load P ="; USING "#####.##"; P;
    LPRINT " "; UF$;
    LPRINT TAB(14); "Located at b ="; USING "####.###"; B;
    LPRINT " "; UL$
    LPRINT "Uniform Load w ="; USING "#####.##"; W; :   LPRINT " "; UW$
    LPRINT TAB(14); "From c1 ="; USING "###.###"; c1;
    LPRINT " "; UL$;
    LPRINT " to c2 ="; USING "####.###"; c2; :   LPRINT " "; UL$
    LPRINT "Couple M ="; USING "########.##"; M; :   LPRINT " "; UM$
    LPRINT "___________________________________________"
'
' Print the computed reactions
    LPRINT :   LPRINT "The computed reactions are:"
    IF TB = 2 GOTO 500
    LPRINT "RA ="; USING "######.##"; RA; : LPRINT " "; UF$;
    LPRINT "     RB ="; USING "######.##"; RB; : LPRINT " "; UF$
    GOTO 600
500 LPRINT "VA ="; USING "######.##"; VA; :   LPRINT " "; UF$;
    LPRINT "     MA ="; USING "######.##"; MA; :   LPRINT " "; UM$
600 LPRINT "___________________________________________"
END
```

C3-2 (a)

```
REACTIONS ON AN OVERHANGING BEAM

Solution to Problem 3-44
________________________________________________

The input data are:
Span Length L =   3.000 m
Length of Overhang a =   1.000 m
Concentrated Load P =    0.00 N
                Located at b =   0.000 m
Uniform Load w = 6000.00 N/m
                From c1 =  1.000 m to c2 =   4.000 m
Couple M =         0.00 N-m
________________________________________________

The computed reactions are:
RA =  3000.00 N      RB = 15000.00 N
________________________________________________
```

C3-2 (b)

```
REACTIONS ON A CANTILEVER BEAM

Solution to Problem 3-48
________________________________________________

The input data are:
Span Length L =   6.000 m
Concentrated Load P =10000.00 N
                Located at b =   6.000 m
Uniform Load w = 2000.00 N/m
                From c1 =  0.000 m to c2 =   6.000 m
Couple M =     4000.00 N-m
________________________________________________

The computed reactions are:
VA = 22000.00 N      MA =100000.00 N-m
________________________________________________
```

Solution to Computer Program Assignment C4-1

```
' * C04-1 *   Analysis of a simple truss subjected to a specific load
'
' Clear screen, compute pi and the conversion factors
      CLS :   PI = 4 * ATN(1):    DR = PI / 180:  RD = 180. / PI
'
' Display the introductory remarks
      PRINT "Want to see the introductory remarks?"
      INPUT "-- Y / N "; RMK$
      IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100   PRINT "REMARKS: (1) This program analyze a simple truss for a"
      PRINT "             specified load.  You need to input the magnitude"
      PRINT "             and direction of the load."
      PRINT "         (2) The input data and the computed results are"
      PRINT "             printed on the printer immediately after"
      PRINT "             data are entered. Make sure your printer is on"
      PRINT "             before entering data."
'
' Input problem I.D.
200   PRINT "Enter problem designation:"
      INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
      PRINT "Select the system of units:"
      PRINT "      1 -- SI Units"
      PRINT "      2 -- US Customary Units"
      INPUT "-- 1 / 2"; U
      IF U = 1 THEN UF$ = "N"
      IF U = 2 THEN UF$ = "lb"
'
' Input the magnitude and direction of the load
      PRINT "Enter the magnitude and direction of the load:"
      PRINT "-- F ("; UF$; "), angle (degree)"; :   INPUT F, AD
'
' Print problem I.D. and heading
      LPRINT "FORCES IN THE MEMBERS OF A SIMPLE TRUSS"
      LPRINT "DUE TO A SPECIFIC LOAD"
      LPRINT :   LPRINT "Solution to "; ID$
      LPRINT
      LPRINT "The load is:  F ="; F; UF$; ", Angle ="; AD; "Degrees"
      LPRINT "____________________________________________________"
      LPRINT
      LPRINT "Member  Horiz. Compt.  Vert. Compt.  Member Force"
      LPRINT TAB(14); UF$; TAB(28); UF$; TAB(43); UF$
      LPRINT "____________________________________________________"
      LPRINT
'
      AR = AD * DR:  S = SIN(AR):    C = COS(AR)
      HAB = 4 * F * S + 4 * F * C: VAB = 3 / 4 * HAB: FAB = 1.25 * HAB
      HAC = -3 * F * S - 4 * F * C: VAC = HAC: FAC = SQR(2) * HAC
      HBC = 0:    VBC = 3 * F * S + 4 * F * C: FBC = VBC
      IF HAB > 0 THEN AB$ = "(T)"
      IF HAB < 0 THEN AB$ = "(C)"
      IF HAC > 0 THEN AC$ = "(T)"
      IF HAC < 0 THEN AC$ = "(C)"
      IF FBC > 0 THEN BC$ = "(T)"
      IF FBC < 0 THEN BC$ = "(C)"
      LPRINT "  AB"; USING "###########.##"; HAB; VAB; FAB; :   LPRINT AB$
      LPRINT "  AC"; USING "###########.##"; HAC; VAC; FAC; :   LPRINT AC$
      LPRINT "  BC"; USING "###########.##"; HBC; VBC; FBC; :   LPRINT BC$
'
      LPRINT "____________________________________________________"
END
```


C4-1 (a)

FORCES IN THE MEMBERS OF A SIMPLE TRUSS
DUE TO A SPECIFIC LOAD

Solution to Problem C4-1(a)

The load is: F = 1000 lb, Angle = 50 Degrees

Member	Horiz. Compt. lb	Vert. Compt. lb	Member Force lb
AB	5635.33	4226.50	7044.16(T)
AC	-4869.28	-4869.28	-6886.21(C)
BC	0.00	4869.28	4869.28(T)

C4-1 (b)

FORCES IN THE MEMBERS OF A SIMPLE TRUSS
DUE TO A SPECIFIC LOAD

Solution to Problem C4-1(b)

The load is: F = 5000 N, Angle = 60 Degrees

Member	Horiz. Compt. N	Vert. Compt. N	Member Force N
AB	27320.51	20490.38	34150.63(T)
AC	-22990.38	-22990.38	-32513.31(C)
BC	0.00	22990.38	22990.38(T)

Solution to Computer Program Assignment C4-2

```
' * C04-2 *   Analysis of a simple truss
'
' Clear screen, compute pi and the conversion factors
    CLS :   PI = 4 * ATN(1):    DR = PI / 180:  RD = 180 / PI
'
' Display the introductory remarks
    PRINT "Want to see the introductory remarks?"
    INPUT "-- Y / N "; RMK$
    IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100 PRINT "REMARKS: (1) This program analyze a simple truss for a load at"
    PRINT "             different inclination. You need to input the"
    PRINT "             magnitude of the load."
    PRINT "         (2) The input data and the computed results are"
    PRINT "             printed on the printer immediately after"
    PRINT "             data are entered. Make sure your printer is on"
    PRINT "             before entering data."
'
' Input problem I.D.
200 PRINT "Enter problem designation:"
    INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
    PRINT "Select the system of units:"
    PRINT "      1 -- SI Units"
    PRINT "      2 -- US Customary Units"
    INPUT "-- 1 / 2"; U
    IF U = 1 THEN UF$ = "N"
    IF U = 2 THEN UF$ = "lb"
'
' Input the magnitude of the force
    PRINT "Enter the magnitude of the force:"
    PRINT "-- F ("; UF$; ")"; :   INPUT F
'
' Print problem I.D. and heading
    LPRINT "FORCES IN THE MEMBERS OF A SIMPLE TRUSS"
    LPRINT "FOR A GIVEN LOAD AT DIFFERENT INCLINATIONS"
    LPRINT :   LPRINT "Solution to "; ID$
    LPRINT :   LPRINT "The load is F ="; F; " "; UF$
    LPRINT "______________________________________________"
    LPRINT :   LPRINT " Angle    Member AB     Member AC     Member BC"
    LPRINT "Degree"; TAB(15); UF$; TAB(29); UF$; TAB(44); UF$
    LPRINT "______________________________________________"
    LPRINT
'
  FOR I = 1 TO 19
    A = (I - 1) * 5:  AD = A * DR:  S = SIN(AD):    C = COS(AD)
    HAB = 4 * F * S + 4 * F * C: VAB = 3 / 4 * HAB: FAB = 1.25 * HAB
    HAC = -3 * F * S - 4 * F * C: VAC = HAC: FAC = SQR(2) * HAC
    FBC = 3 * F * S + 4 * F * C
    IF HAB > 0 THEN AB$ = "(T)"
    IF HAB < 0 THEN AB$ = "(C)"
    IF HAC > 0 THEN AC$ = "(T)"
    IF HAC < 0 THEN AC$ = "(C)"
    IF FBC > 0 THEN BC$ = "(T)"
    IF FBC < 0 THEN BC$ = "(C)"
    LPRINT USING "####"; A; :   LPRINT USING "#########.##"; FAB;
    LPRINT AB$;
    LPRINT USING "#########.##"; FAC; :   LPRINT AC$;
    LPRINT USING "#########.##"; FBC; :   LPRINT BC$
'
  NEXT I
    LPRINT "______________________________________________"
END
```


C4-2 (a)

FORCES IN THE MEMBERS OF A SIMPLE TRUSS
FOR A GIVEN LOAD AT DIFFERENT INCLINATIONS

Solution to Problem C4-2 (a)

The load is F = 1000 lb

Angle Degree	Member AB lb	Member AC lb	Member BC lb
0	5000.00(T)	-5656.85(C)	4000.00(T)
5	5416.75(T)	-6005.10(C)	4246.25(T)
10	5792.28(T)	-6307.64(C)	4460.18(T)
15	6123.72(T)	-6562.18(C)	4640.16(T)
20	6408.56(T)	-6766.77(C)	4784.83(T)
25	6644.63(T)	-6919.87(C)	4893.09(T)
30	6830.13(T)	-7020.30(C)	4964.10(T)
35	6963.64(T)	-7067.30(C)	4997.34(T)
40	7044.16(T)	-7060.52(C)	4992.54(T)
45	7071.07(T)	-7000.00(C)	4949.75(T)
50	7044.16(T)	-6886.21(C)	4869.28(T)
55	6963.64(T)	-6720.01(C)	4751.76(T)
60	6830.13(T)	-6502.66(C)	4598.08(T)
65	6644.63(T)	-6235.83(C)	4409.40(T)
70	6408.56(T)	-5921.54(C)	4187.16(T)
75	6123.72(T)	-5562.18(C)	3933.05(T)
80	5792.28(T)	-5160.49(C)	3649.02(T)
85	5416.75(T)	-4719.52(C)	3337.21(T)
90	5000.00(T)	-4242.64(C)	3000.00(T)

C4-2 (b)

FORCES IN THE MEMBERS OF A SIMPLE TRUSS
FOR A GIVEN LOAD AT DIFFERENT INCLINATIONS

Solution to Problem C4-2 (b)

The load is F = 5000 N

Angle Degree	Member AB N	Member AC N	Member BC N
0	25000.00(T)	-28284.27(C)	20000.00(T)
5	27083.76(T)	-30025.49(C)	21231.23(T)
10	28961.40(T)	-31538.20(C)	22300.88(T)
15	30618.62(T)	-32810.89(C)	23200.80(T)
20	32042.82(T)	-33833.86(C)	23924.15(T)
25	33223.15(T)	-34599.34(C)	24465.43(T)
30	34150.63(T)	-35101.50(C)	24820.51(T)
35	34818.21(T)	-35336.51(C)	24986.69(T)
40	35220.80(T)	-35302.59(C)	24962.70(T)
45	35355.34(T)	-35000.00(C)	24748.74(T)
50	35220.80(T)	-34431.04(C)	24346.42(T)
55	34818.21(T)	-33600.03(C)	23758.81(T)
60	34150.63(T)	-32513.31(C)	22990.38(T)
65	33223.15(T)	-31179.14(C)	22046.98(T)
70	32042.82(T)	-29607.68(C)	20935.79(T)
75	30618.62(T)	-27810.89(C)	19665.27(T)
80	28961.40(T)	-25802.44(C)	18245.08(T)
85	27083.76(T)	-23597.62(C)	16686.04(T)
90	25000.00(T)	-21213.20(C)	15000.00(T)

Solution to Computer Program Assignment C5-1

```
' * C05-1 *    Analysis of square-threaded screws
'
' Clear screen, compute pi and the conversion factors
      CLS :   PI = 4 * ATN(1):    DR = PI / 180:  RD = 180 / PI
'
' Display the introductory remarks
      PRINT "Want to see the introductory remarks?"
      INPUT "-- Y / N "; RMK$
      IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100   PRINT "REMARKS: (1) This program can be used to compute the torques"
      PRINT "             required to raise and to lower a square-threaded"
      PRINT "             screw under load."
      PRINT "         (2) The input data include the mean radius, the"
      PRINT "             number of thread, the static friction coefficient,"
      PRINT "             the load and the arm length."
      PRINT "         (3) The input data and the computed results are"
      PRINT "             printed on the printer immediately after"
      PRINT "             data are entered. Make sure your printer is on"
      PRINT "             before entering data."
'
' Input problem I.D.
200   PRINT "Enter problem designation:"
      INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
      PRINT "Select the system of units:"
      PRINT "       1 -- SI Units"
      PRINT "       2 -- US customary units"
      INPUT "-- 1 / 2"; U
      IF U = 2 GOTO 300
      UF$ = "N": UL1$ = "m": UL2$ = "mm": UM$ = "N-m"
      GOTO 400
300   UF$ = "lb": UL1$ = "ft": UL2$ = "in.": UM$ = "lb-ft"
'
' Print problem I.D. and heading
400   LPRINT "ANALYSIS OF SQUARE-THREADED SCREWS"
      LPRINT :   LPRINT "Solution to "; ID$
      LPRINT "________________________________________________"
'
' Enter the data of the screw
      PRINT "Input the data of the screw:"
      PRINT "-- r ("; UL2$; ")"; " n, p ("; UL2$; ")"; :    INPUT R, N, P
      PRINT "-- mu, W ("; UF$; ")"; " a ("; UL2$; ")"; :    INPUT MU, W, A
      IF U = 2 GOTO 500
      R1 = R / 1000: P1 = P / 1000: A1 = A / 1000
      GOTO 600
500   R1 = R / 12: P1 = P / 12: A1 = A / 12
'
' Print the input data
600   LPRINT :   LPRINT "The input data:":    LPRINT
      LPRINT "The mean radius:"; TAB(35); "r ="; USING "##.##"; R;
      LPRINT " "; UL2$
      LPRINT "The number of thread:"; TAB(35); "n ="; USING "##"; N
      LPRINT "The pitch of the thread:"; TAB(35); "p =";
      LPRINT USING "##.##"; P; :    LPRINT " "; UL2$
      LPRINT "The static friction coefficient:"; TAB(34); "mu =";
      LPRINT USING "##.##"; MU
      LPRINT "The load to be lifted:"; TAB(35); "W =";
      LPRINT USING "######.##"; W; :    LPRINT " "; UF$
      LPRINT "The arm length of the handle:"; TAB(35); "a =";
      LPRINT USING "###.##"; A;
      LPRINT " "; UL2$
```


Solution to Computer Program Assignment C5-1 Continued

```
' Compute the lead angle
     LAR = ATN(N * P1 / 2 / PI / R1): LAD = LAR * RD
' Compute the static friction angle
     PHIR = ATN(MU):      PHID = PHIR * RD
' Compute the loading torque and the corresponding force
     M = W * R1 * TAN(PHIR + LAR):     F = M / A1
' Compute the releasing torque and the corresponding force
     MP = W * R1 * TAN(PHIR - LAR):    FP = MP / A1
     IF MP > 0 THEN SL = 1
     IF MP < 0 THEN SL = 2
' Compute the mechanical advantage
     MA = W / F
'
     LPRINT "________________________________________________"
     LPRINT :   LPRINT "The computed results:":    LPRINT
     LPRINT "The lead angle:"; TAB(31); "theta ="; USING "##.##"; LAD;
     LPRINT " Degree"
     LPRINT "The static friction angle:"; TAB(33); "phi =";
     LPRINT USING "##.##"; PHID; :    LPRINT " Degree"
     LPRINT "The loading torque:"; TAB(35); "M =";
     LPRINT USING "######.##"; M; :    LPRINT " "; UM$
     LPRINT "  The corresponding forces:"; TAB(35); "F =";
     LPRINT USING "######.##"; F; :    LPRINT " "; UF$
     LPRINT "The mechanical advantage:"; TAB(34); "MA =";
     LPRINT USING "####.##"; MA
     IF SL = 2 GOTO 700
     LPRINT "The releasing torque:"; TAB(34); "M' ="; USING "######.##"; MP;
     LPRINT " "; UM$
     LPRINT "  The corresponding forces:"; TAB(34); "F' =";
     LPRINT USING "######.##"; FP;
     LPRINT " "; UF$
     GOTO 800
700  LPRINT "The screw is not self locking, the load will be"
     LPRINT "  released once the loading torque is released."
800  LPRINT "________________________________________________"
END
```

C5-1 (a)

```
ANALYSIS OF SQUARE-THREADED SCREWS

Solution to EXAMPLE 5-7
________________________________________________

The input data:

The mean radius:                   r = 1.00 in.
The number of thread:              n = 2
The pitch of the thread:           p = 0.20 in.
The static friction coefficient:  mu = 0.10
The load to be lifted:             W =  2000.00 lb
The arm length of the handle:      a = 12.00 in.
________________________________________________

The computed results:

The lead angle:                theta = 3.64 Degree
The static friction angle:       phi = 5.71 Degree
The loading torque:                M =     27.45 lb-ft
  The corresponding forces:        F =     27.45 lb
The mechanical advantage:         MA =  72.86
The releasing torque:             M' =      6.02 lb-ft
  The corresponding forces:       F' =      6.02 lb
________________________________________________
```

C5-1 (b)

```
ANALYSIS OF SQUARE-THREADED SCREWS

Solution to Problem 5-28
____________________________________________________

The input data:

The mean radius:                      r =25.00 mm
The number of thread:                 n = 1
The pitch of the thread:              p =10.00 mm
The static friction coefficient:  mu = 0.10
The load to be lifted:                W = 19620.00 N
The arm length of the handle:         a =800.00 mm
____________________________________________________

The computed results:

The lead angle:                   theta = 3.64 Degree
The static friction angle:          phi = 5.71 Degree
The loading torque:                   M =     80.79 N-m
  The corresponding forces:           F =    100.99 N
The mechanical advantage:            MA = 194.28
The releasing torque:                M' =     17.71 N-m
  The corresponding forces:          F' =     22.14 N
____________________________________________________
```

C5-1 (c)

```
ANALYSIS OF SQUARE-THREADED SCREWS

Solution to Problem 5-30
____________________________________________________

The input data:

The mean radius:                      r = 0.50 in.
The number of thread:                 n = 1
The pitch of the thread:              p = 0.25 in.
The static friction coefficient:  mu = 0.14
The load to be lifted:                W = 20000.00 lb
The arm length of the handle:         a =  4.00 in.
____________________________________________________

The computed results:

The lead angle:                   theta = 4.55 Degree
The static friction angle:          phi = 8.00 Degree
The loading torque:                   M =    185.47 lb-ft
  The corresponding forces:           F =    556.41 lb
The mechanical advantage:            MA =  35.94
The releasing torque:                M' =     50.21 lb-ft
  The corresponding forces:          F' =    150.62 lb
____________________________________________________
```


Solution to Computer Program Assignment C5-2

```
' * C05-2 *    Problems involving belt friction
'
' Clear screen, compute pi and the conversion factors
     CLS :    PI = 4 * ATN(1):     DR = PI / 180:   RD = 180 / PI
'
' Display the introductory remarks
     PRINT "Want to see the introductory remarks?"
     INPUT "-- Y / N "; RMK$
     IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100  PRINT "REMARKS: (1) This program can be used to compute the maximum"
     PRINT "              and minimum weight that can be held by a given"
     PRINT "              force applied to a cable wraping around a post."
     PRINT "          (2) The input data include the static friction"
     PRINT "              coefficient and the magnitude of the applied"
     PRINT "              force."
     PRINT "          (3) The input data and the computed results are"
     PRINT "              printed on the printer immediately after"
     PRINT "              data are entered. Make sure your printer is on"
     PRINT "              before entering data."
'
' Input problem I.D.
200  PRINT "Enter problem designation:"
     INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
     PRINT "Select the system of units:"
     PRINT "        1 -- SI Units"
     PRINT "        2 -- US customary units"
     INPUT "-- 1 / 2"; U
     IF U = 1 THEN UF$ = "N"
     IF U = 2 THEN UF$ = "lb"
'
' Print problem I.D. and heading
     LPRINT "BELT FRICTION PROBLEM"
     LPRINT :   LPRINT "Solution to "; ID$:    LPRINT
'
' Enter data
     PRINT "Input data:"
     PRINT "Static friction coefficient -- mu"; :    INPUT MU
     PRINT "The pulling force -- P"; :    INPUT P
'
' Print the input data
     LPRINT "The static friction coefficient: mu ="; USING "##.##"; MU
     LPRINT "The pulling force P ="; USING "####.#"; P;
     LPRINT " "; UF$
'
' Print heading
     LPRINT "______________________________________________________"
     LPRINT
     LPRINT "No. of   Contact   Contact   Maximum   Minimum"
     LPRINT " Wrap     angle     angle     weight    weight"
     LPRINT TAB(11); "radian"; TAB(21); "degree";
     LPRINT TAB(33); UF$; TAB(43); UF$
     LPRINT "______________________________________________________"
     LPRINT
'
' Compute and print max. & min. W
'
     NW = 1
   FOR I = 1 TO 9
```

Solution to Computer Program Assignment C5-2 Continued

```
        BETAR = NW * 2 * PI:     BETAD = BETAR * RD
        K = EXP(MU * BETAR):  WMIN = P / K:     WMAX = P * K
        LPRINT USING "##.##"; NW;
        LPRINT USING "#######.##"; BETAR;
        LPRINT USING "#########"; BETAD;
        LPRINT USING "#########.#"; WMAX;
        LPRINT USING "#######.##"; WMIN
        NW = NW + .25
     NEXT I
        LPRINT "______________________________________________"
END
```

C5-2 (a) BELT FRICTION PROBLEM

Solution to Problem C5-2 (a)

The static friction coefficient: mu = 0.20
The pulling force P = 25.0 lb

No. of Wrap	Contact angle radian	Contact angle degree	Maximum weight lb	Minimum weight lb
1.00	6.28	360	87.8	7.12
1.25	7.85	450	120.3	5.20
1.50	9.42	540	164.7	3.80
1.75	11.00	630	225.4	2.77
2.00	12.57	720	308.6	2.03
2.25	14.14	810	422.6	1.48
2.50	15.71	900	578.5	1.08
2.75	17.28	990	792.1	0.79
3.00	18.85	1080	1084.4	0.58

C5-2 (b) BELT FRICTION PROBLEM

Solution to Problem C5-2 (b)

The static friction coefficient: mu = 0.30
The pulling force P = 180.0 N

No. of Wrap	Contact angle radian	Contact angle degree	Maximum weight N	Minimum weight N
1.00	6.28	360	1185.5	27.33
1.25	7.85	450	1899.1	17.06
1.50	9.42	540	3042.4	10.65
1.75	11.00	630	4873.8	6.65
2.00	12.57	720	7807.7	4.15
2.25	14.14	810	12507.8	2.59
2.50	15.71	900	20037.2	1.62
2.75	17.28	990	32099.2	1.01
3.00	18.85	1080	51422.1	0.63

Solution to Computer Program Assignment C6-1

```
' * C06-1 *    Components of a spatial force passing through two points
'
' Clear screen, compute pi and the conversion factors
      CLS :    PI = 4 * ATN(1):      DR = PI / 180:  RD = 180 / PI
'
' Display the introductory remarks
      PRINT "Want to see the introductory remarks?"
      INPUT "-- Y / N "; RMK$
      IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100   PRINT "REMARKS: (1) This program can be used to compute the"
      PRINT "             rectangular components of a spatial force"
      PRINT "             passing through two known points."
      PRINT "         (2) You need to input the magnitude of the force and"
      PRINT "             the coordinates of two points A and B. The force"
      PRINT "             acts from A to B."
      PRINT "         (3) The input data and the computed results are"
      PRINT "             printed on the line printer immediately after"
      PRINT "             data are entered. Make sure your printer is on"
      PRINT "             before entering data."
'
' Input problem I.D.
200   PRINT "Enter problem designation:"
      INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
      PRINT "Select the system of units:"
      PRINT "        1 -- SI Units"
      PRINT "        2 -- US customary units"
      INPUT "-- 1 / 2"; U
      IF U = 1 THEN UF$ = "N"
      IF U = 1 THEN UL$ = "m"
      IF U = 2 THEN UF$ = "lb"
      IF U = 2 THEN UL$ = "ft"
'
' Input the magnitude of the force
      PRINT "Enter the magnitude of the force:"
      PRINT "-- F("; UF$; ")"; :    INPUT F
'
' Input the coordinates of point A
      PRINT "Enter the coordinates of point A:"
      PRINT "-- XA("; UL$; "), YA("; UL$; "), ZA("; UL$; ")";
      INPUT XA, YA, ZA
'
' Input the coordinates of point B
      PRINT "Enter the coordinates of point B:"
      PRINT "-- XB("; UL$; "), YB("; UL$; "), ZB("; UL$; ")";
      INPUT XB, YB, ZB
'
' Print problem I.D.
      LPRINT "COMPONENTS OF A SPATIAL FORCE ACTING ALONG TWO GIVEN POINTS"
      LPRINT :   LPRINT "Solution to "; ID$:    LPRINT
'
' Print the magnitude of the force
      LPRINT "Magnitude of the force :  P ="; USING "######.#"; F;
      LPRINT " "; UF$
      LPRINT TAB(13); "The force acts in the direction from A to B"
      LPRINT
'
' Print the coordinates of the initial point A
      LPRINT "Coordinates of point A:";    LPRINT TAB(13); "(";
      LPRINT USING "###.###"; XA; :   LPRINT " "; UL$; ",";
      LPRINT USING "###.###"; YA; :   LPRINT " "; UL$; ",";
      LPRINT USING "###.###"; ZA; :   LPRINT " "; UL$; " )":    LPRINT
```

Solution to Computer Program Assignment C6-1 Continued

```
' Print the coordinates of the end point B
    LPRINT "Coordinates of point B:":    LPRINT TAB(13); "(";
    LPRINT USING "###.###"; XB; :   LPRINT " "; UL$; ",";
    LPRINT USING "###.###"; YB; :   LPRINT " "; UL$; ",";
    LPRINT USING "###.###"; ZB; :   LPRINT " "; UL$; " )":    LPRINT
'
' Compute the force components
    DX = XB - XA:    DY = YB - YA:    DZ = ZB - ZA
    D = SQR(DX * DX + DY * DY + DZ * DZ)
    FX = F / D * DX:   FY = F / D * DY:   FZ = F / D * DZ
'
' Print the components of the force
    LPRINT "The rectangular components of the force are:"
    LPRINT TAB(13); "(";
    LPRINT USING "######.#"; FX; :   LPRINT " "; UF$; ",";
    LPRINT USING "######.#"; FY; :   LPRINT " "; UF$; ",";
    LPRINT USING "######.#"; FZ; :   LPRINT " "; UF$; " )"
END
```

C6-1 (a)

```
COMPONENTS OF A SPATIAL FORCE ACTING ALONG TWO GIVEN POINTS

Solution to EXAMPLE 6-3

Magnitude of the force :  P =   210.0 lb
             The force acts in the direction from A to B

Coordinates of point A:
             (  0.000 ft,   6.000 ft,  -5.000 ft )

Coordinates of point B:
             (  2.000 ft,   0.000 ft,  -2.000 ft )

The rectangular components of the force are:
             (     60.0 lb,   -180.0 lb,      90.0 lb )
```

C6-1 (b)

```
COMPONENTS OF A SPATIAL FORCE ACTING ALONG TWO GIVEN POINTS

Solution to Problem 6-11

Magnitude of the force :  P =   300.0 lb
             The force acts in the direction from A to B

Coordinates of point A:
             (  4.000 ft,   0.000 ft,   8.000 ft )

Coordinates of point B:
             (  0.000 ft,   5.000 ft,   2.000 ft )

The rectangular components of the force are:
             (  -136.8 lb,    170.9 lb,   -205.1 lb )
```

C6-1 (c)

```
COMPONENTS OF A SPATIAL FORCE ACTING ALONG TWO GIVEN POINTS

Solution to Problem 6-12

Magnitude of the force :  P =  5000.0 N
             The force acts in the direction from A to B

Coordinates of point A:
             (  4.000 m,   0.000 m,   0.000 m )

Coordinates of point B:
             (  0.000 m,   5.000 m,   5.000 m )

The rectangular components of the force are:
             ( -2461.8 N,   3077.3 N,   3077.3 N )
```


Solution to Computer Program Assignment C6-2

```
' * C06-2 *    Resultant of a concurrent spatial force system
'
' Clear screen, compute pi and the conversion factors
    CLS :    PI = 4 * ATN(1):    DR = PI / 180:   RD = 180 / PI
'
' Display the introductory remarks
    PRINT "Want to see the introductory remarks?"
    INPUT "-- Y / N "; RMK$
    IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100 PRINT "REMARKS: (1) This program can be used to compute the resultant"
    PRINT "             of a concurrent spatial force system."
    PRINT "         (2) You need to input the coordinates of the"
    PRINT "             concurrent point, the magnitude of each force and"
    PRINT "             the coordinates of a point on the line of action"
    PRINT "             of each force."
    PRINT "         (3) The input data and the computed results are"
    PRINT "             printed on the printer immediately after"
    PRINT "             data are entered. Make sure your printer is on"
    PRINT "             before entering data."
'
' Input problem I.D.
200 PRINT "Enter problem designation:"
    INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
    PRINT "Select the system of units:"
    PRINT "         1 -- SI Units"
    PRINT "         2 -- US customary units"
    INPUT "-- 1 / 2"; U
    IF U = 1 THEN UF$ = "N"
    IF U = 1 THEN UL$ = "m"
    IF U = 2 THEN UF$ = "lb"
    IF U = 2 THEN UL$ = "ft"
'
' Input the coordinates of the concurrent point A
    PRINT "Enter the coordinates of the concurrent point A:"
    PRINT "-- XA("; UL$; "), YA("; UL$; "), ZA("; UL$; ")";
    INPUT XA, YA, ZA
'
' Print problem I.D. and heading
    LPRINT "RESULTANT OF A CONCURRENT SPATIAL FORCE SYSTEM"
    LPRINT :    LPRINT "Solution to "; ID$:    LPRINT
    LPRINT "Coordinates of the concurrent point A:"
    LPRINT TAB(13); "(";
    LPRINT USING "###.###"; XA; :   LPRINT " "; UL$; ",";
    LPRINT USING "###.###"; YA; :   LPRINT " "; UL$; ",";
    LPRINT USING "###.###"; ZA; :   LPRINT " "; UL$; " )"
    LPRINT "______________________________________________________________"
    LPRINT :   LPRINT "Force Mag("; UF$; ")"; TAB(15); "xB("; UL$; ")";
    LPRINT TAB(22); "yB("; UL$; ")"; TAB(29); "zB("; UL$; ")";
    LPRINT TAB(37); "Fx("; UF$; ")"; TAB(45); "Fy("; UF$; ")";
    LPRINT TAB(53); "Fz("; UF$; ")"
    LPRINT "______________________________________________________________"
    LPRINT
'
' Enter the number of given forces
    PRINT "Enter the number of given forces:"
    INPUT "-- No. of Forces"; N
'
' Initialize sum
    RX = 0:    RY = 0:    RZ = 0
'
' Enter the data of the given forces and compute their components
    PRINT "Input the magnitude and the coordinates of point B of each";
    PRINT " force:"
```

Solution to Computer Program Assignment C6-2 Continued

```
 FOR I = 1 TO N
   PRINT "-- F"; USING "#"; I; :   PRINT "("; UF$; "),";
   PRINT " XB("; UL$; "), YB("; UL$; "), ZB("; UL$; ")";
   INPUT F, XB, YB, ZB
'
' Compute the force components
   DX = XB - XA:    DY = YB - YA:    DZ = ZB - ZA
   D = SQR(DX * DX + DY * DY + DZ * DZ)
   FX = F / D * DX:  FY = F / D * DY:  FZ = F / D * DZ
'
' Print the given forces
   LPRINT " F"; USING "#"; I; :   LPRINT USING "#######.#"; F;
   LPRINT USING "###.###"; XB; YB; ZB;
   LPRINT USING "######.#"; FX; FY; FZ
'
' Compute the sum of force components
   RX = RX + FX:    RY = RY + FY:    RZ = RZ + FZ
'
 NEXT I
'
' Find the magnitude of the resultant
   R = SQR(RX * RX + RY * RY + RZ * RZ)
'
' Print the magnitude and the components of the resultant
   LPRINT "______________________________________________________________"
   LPRINT
   LPRINT " R"; TAB(7); "Mag("; UF$; ")"; TAB(37); "Rx"; "("; UF$; ")";
   LPRINT TAB(45); "Ry"; "("; UF$; ")";
   LPRINT TAB(53); "Rz"; "("; UF$; ")"
   LPRINT TAB(5); USING "######.#"; R;
   LPRINT TAB(34); USING "######.#"; RX; RY; RZ
'
' Find the direction angles
   THETAX = ATN(SQR(1 - (RX / R) ^ 2) / ABS(RX / R)) * RD
   THETAY = ATN(SQR(1 - (RY / R) ^ 2) / ABS(RY / R)) * RD
   THETAZ = ATN(SQR(1 - (RZ / R) ^ 2) / ABS(RZ / R)) * RD
   IF RX < 0 THEN THETAX = 180 - THETAX
   IF RY < 0 THEN THETAY = 180 - THETAY
   IF RZ < 0 THEN THETAZ = 180 - THETAZ
'
' Print the direction angles of the resultant
    LPRINT :   LPRINT TAB(39); "Direction angles"
    LPRINT TAB(36); "x(Deg.) y(Deg.) z(Deg.)"
    LPRINT TAB(34); USING "#####.##"; THETAX; THETAY; THETAZ
    LPRINT "______________________________________________________________"
END
```


C6-2 (a)

```
RESULTANT OF A CONCURRENT SPATIAL FORCE SYSTEM

Solution to EXAMPLE 6-4

Coordinates of the concurrent point A:
            (  0.000 m,  0.400 m,  0.400 m )
____________________________________________________________
Force Mag(N)  xB(m)  yB(m)  zB(m)    Fx(N)    Fy(N)    Fz(N)
____________________________________________________________
 F1   6000.0  0.800  1.200  0.000   4000.0   4000.0  -2000.0
 F2   7000.0  1.200  0.000  1.000   6000.0  -2000.0   3000.0
____________________________________________________________
 R    Mag(N)                         Rx(N)    Ry(N)    Rz(N)
     10247.0                       10000.0   2000.0   1000.0

                                      Direction angles
                                  x(Deg.) y(Deg.) z(Deg.)
                                    12.60   78.74   84.40
____________________________________________________________
```

C6-2 (b)

```
RESULTANT OF A CONCURRENT SPATIAL FORCE SYSTEM

Solution to Problem 6-16

Coordinates of the concurrent point A:
            (  0.000 ft,  4.000 ft,  3.000 ft )
____________________________________________________________
Force Mag(lb) xB(ft) yB(ft) zB(ft)  Fx(lb)   Fy(lb)   Fz(lb)
____________________________________________________________
 F1    600.0  4.000  0.000  1.000    400.0   -400.0   -200.0
 F2    450.0  4.000  0.000 10.000    200.0   -200.0    350.0
____________________________________________________________
 R    Mag(lb)                       Rx(lb)   Ry(lb)   Rz(lb)
       861.7                         600.0   -600.0    150.0

                                      Direction angles
                                  x(Deg.) y(Deg.) z(Deg.)
                                    45.87  134.13   79.98
____________________________________________________________
```

C6-2 (c)

```
RESULTANT OF A CONCURRENT SPATIAL FORCE SYSTEM

Solution to Problem 6-18

Coordinates of the concurrent point A:
            (  1.200 m,  0.500 m,  0.000 m )
____________________________________________________________
Force Mag(N)  xB(m)  yB(m)  zB(m)    Fx(N)    Fy(N)    Fz(N)
____________________________________________________________
 F1  20000.0  1.600  0.800  0.000 16000.0 12000.0       0.0
 F2  14000.0  0.000  0.900  0.600-12000.0  4000.0    6000.0
 F3  13000.0  0.000  0.800 -0.400-12000.0  3000.0   -4000.0
____________________________________________________________
 R    Mag(N)                         Rx(N)    Ry(N)    Rz(N)
     20712.3                       -8000.0  19000.0   2000.0

                                      Direction angles
                                  x(Deg.) y(Deg.) z(Deg.)
                                   112.72   23.46   84.46
____________________________________________________________
```

Solution to Computer Program Assignment C7-1

```
' * C07-1 *   Location of the centroid of a composite area
'
' Clear screen, compute pi and the conversion factors
     CLS :   PI = 4 * ATN(1):    DR = PI / 180:  RD = 180 / PI
'
' Display the introductory remarks
     PRINT "Want to see the introductory remarks?"
     INPUT "-- Y / N "; RMK$
     IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100  PRINT "REMARKS: (1) This program can be used to compute the centroid"
     PRINT "             of a composite area."
     PRINT "         (2) You need to input each component area and the"
     PRINT "             coordinates of its centroid."
     PRINT "         (3) The input data and the computed results are"
     PRINT "             printed on the printer immediately after"
     PRINT "             data are entered. Make sure your printer is on"
     PRINT "             before entering data."
'
' Input problem I.D.
200  PRINT "Enter problem designation:"
     INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
     PRINT "Select the system of units:"
     PRINT "       1 -- SI Units"
     PRINT "       2 -- US customary units"
     INPUT "-- 1 / 2"; U
     IF U = 1 THEN UL$ = "m"
     IF U = 2 THEN UL$ = "in"
'
' Print problem I.D. and heading
     LPRINT "CENTROID OF A COMPOSITE AREA"
     LPRINT :   LPRINT "Solution to "; ID$
     LPRINT "____________________________________________________________"
     LPRINT :   LPRINT "Area"; TAB(10); "A("; UL$; "^2)";
     LPRINT TAB(21); "x("; UL$; ")"; TAB(31); "y("; UL$; ")";
     LPRINT TAB(40); "Ax("; UL$; "^3)"; TAB(51); "Ay("; UL$; "^3)"
     LPRINT "____________________________________________________________"
     LPRINT
'
' Enter the number of given forces
     PRINT "Enter the total number of component areas:"
     INPUT "-- No. of area"; N
'
' Initialize sum
     SA = 0:    SAX = 0:    SAY = 0
'
' Enter area and coordinates of centroid
     PRINT "Input value of area and the coordinates of its centroid:"
'
   FOR I = 1 TO N
     PRINT "-- A"; USING "#"; I;
     PRINT "("; UL$; "^2),"; " x"; USING "#"; I;
     PRINT "("; UL$; "),"; " y"; USING "#"; I;
     PRINT "("; UL$; "),"; :   INPUT A, X, Y
'
' Compute the first moment of each area and find the sums of the areas
' and the moments
     AX = A * X:    AY = A * Y
     SA = SA + A:    SAX = SAX + AX:    SAY = SAY + AY
'
```


Solution to Computer Program Assignment C7-1 Continued

```
' Print the area and distance
     LPRINT " A"; USING "#"; I; :   LPRINT USING "#######.####"; A;
     LPRINT USING "#####.####"; X; Y;
     LPRINT USING "######.####"; SAX; SAY
'
   NEXT I
'
' Find the location of the centroid
     X = SAX / SA:    Y = SAY / SA
'
' Print the location of the centroid
     LPRINT "____________________________________________________________"
     LPRINT
     LPRINT "Sum"; USING "#######.####"; SA;
     LPRINT TAB(36); USING "######.####"; SAX;
     LPRINT TAB(47); USING "######.####"; SAY
     LPRINT "____________________________________________________________"
     LPRINT :   LPRINT "Location of the centroid:"
     LPRINT "X = Sum(Ax) / Sum(A) ="; USING "####.####"; SAX;
     LPRINT " /";
     LPRINT USING "####.####"; SA; :   LPRINT " ="; USING "####.###"; X;
     LPRINT " "; UL$
     LPRINT "Y = Sum(Ay) / Sum(A) ="; USING "####.####"; SAY;
     LPRINT " /";
     LPRINT USING "####.####"; SA; :   LPRINT " ="; USING "####.###"; Y;
     LPRINT " "; UL$
END
```

C7-1 (a)

CENTROID OF A COMPOSITE AREA

Solution to EXAMPLE 7-4

Area	A(in^2)	x(in)	y(in)	Ax(in^3)	Ay(in^3)
A1	18.0000	4.0000	2.0000	72.0000	36.0000
A2	36.0000	9.0000	3.0000	396.0000	144.0000
A3	-6.3000	9.0000	0.8490	339.3000	138.6513
Sum	47.7000			339.3000	138.6513

Location of the centroid:
X = Sum(Ax) / Sum(A) = 339.3000 / 47.7000 = 7.113 in
Y = Sum(Ay) / Sum(A) = 138.6513 / 47.7000 = 2.907 in

C7-1 (b)

CENTROID OF A COMPOSITE AREA

Solution to Problem 7-16

Area	A(m^2)	x(m)	y(m)	Ax(m^3)	Ay(m^3)
A1	0.6500	0.5000	0.3250	0.3250	0.2112
A2	0.3318	1.2760	0.2760	0.7484	0.3028
A3	-0.0707	0.5000	0.3250	0.7130	0.2798
Sum	0.9111			0.7130	0.2798

Location of the centroid:
X = Sum(Ax) / Sum(A) = 0.7130 / 0.9111 = 0.783 m
Y = Sum(Ay) / Sum(A) = 0.2798 / 0.9111 = 0.307 m

C7-1 (c)

CENTROID OF A COMPOSITE AREA

Solution to Problem 7-19

Area	A(in^2)	x(in)	y(in)	Ax(in^3)	Ay(in^3)
A1	72.0000	0.0000	3.0000	0.0000	216.0000
A2	18.0000	0.0000	7.5000	0.0000	351.0000
A3	7.0700	-4.2700	7.2700	-30.1889	402.3989
A4	7.0700	4.2700	7.2700	0.0000	453.7978
A5	-14.1400	0.0000	1.2700	0.0000	435.8400
Sum	90.0000			0.0000	435.8400

Location of the centroid:
X = Sum(Ax) / Sum(A) = 0.0000 / 90.0000 = 0.000 in
Y = Sum(Ay) / Sum(A) = 435.8400 / 90.0000 = 4.843 in

Solution to Computer Program Assignment C7-2

```
' * C07-2 *   Analysis of parabolic cables
'
' Clear screen, compute pi and the conversion factors
     CLS :   PI = 4 * ATN(1):    DR = PI / 180:  RD = 180 / PI
'
' Display the introductory remarks
     PRINT "Want to see the introductory remarks?"
     INPUT "-- Y / N "; RMK$
     IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100  PRINT "REMARKS: (1) This program can be used to analyze a parabolic"
     PRINT "             cable subjected to a load uniformly distributed"
     PRINT "             along its horizontal length."
     PRINT "         (2) The input data include the horizontal span"
     PRINT "             length, the uniform load, and the sags of the"
     PRINT "             supports."
     PRINT "         (3) The input data and the computed results are"
     PRINT "             printed on the printer immediately after"
     PRINT "             data are entered. Make sure your printer is on"
     PRINT "             before entering data."
'
' Input problem I.D.
200  PRINT "Enter problem designation:"
     INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
     PRINT "Select the system of units:"
     PRINT "       1 -- SI Units"
     PRINT "       2 -- US customary units"
     INPUT "-- 1 / 2"; U
     IF U = 2 GOTO 300
     UF$ = "N": UL$ = "m": UW$ = "N/m"
     GOTO 400
300  UF$ = "lb": UL$ = "ft": UW$ = "lb/ft"
'
' Print problem I.D. and heading
400  LPRINT "ANALYSIS OF PARABOLIC CABLES"
     LPRINT :   LPRINT "Solution to "; ID$
     LPRINT "____________________________________________________________"
'
' Enter the data of the screw
     PRINT "Input the data of the cable:"
     PRINT "-- L ("; UL$; "), q("; UW$; ")"; :   INPUT l, Q
     PRINT "-- yA("; UL$; ")"; " yB("; UL$; ")"; :   INPUT YA, YB
'
' Print the input data
      LPRINT :   LPRINT "The input data:":    LPRINT
      LPRINT "The horizontal span length:"; TAB(39); "L =";
      LPRINT USING "#####.##"; l; :   LPRINT " "; UL$
      LPRINT "The uniform load:"; TAB(39); "q ="; USING "#####.##"; Q;
      LPRINT " "; UW$
      LPRINT "The sag of the left support A:"; TAB(38); "yA =";
      LPRINT USING "#####.##"; YA; :    LPRINT " "; UL$
      LPRINT "The sag of the right support B:"; TAB(38); "yB =";
      LPRINT USING "#####.##"; YB; :    LPRINT " "; UL$
'
' Compute the minimum tension
      T0 = Q * l * l / 2 / (SQR(YA) + SQR(YB)) ^ 2
' Compute the maximum tension
      XA = SQR(2 * T0 * YA / Q)
      XB = SQR(2 * T0 * YB / Q)
      IF YA > YB OR YA = YB THEN TM = SQR(T0 ^ 2 + (Q * XA) ^ 2)
      IF YA < YB THEN TM = SQR(T0 ^ 2 + (Q * XB) ^ 2)
      IF YA > YB OR YA = YB THEN THETA = ATN(Q * XA / T0) * RD
      IF YA < YB THEN THETA = ATN(Q * XB / T0) * RD
```

Solution to Computer Program Assignment C7-2 Continued

```
' Compute the cable length
     YXA = YA / XA:  YXB = YB / XB
     S1 = XA * (1 + 2 * YXA ^ 2 / 3 - .4 * YXA ^ 4)
     S2 = XB * (1 + 2 * YXB ^ 2 / 3 - .4 * YXB ^ 4)
     S = S1 + S2
'
     LPRINT "____________________________________________________"
     LPRINT :   LPRINT "The computed results:":    LPRINT
     LPRINT "The horiz. length of left support:"; TAB(38); "xA =";
     LPRINT USING "#######.##"; XA; :    LPRINT " "; UL$
     LPRINT "The horiz. length of right support:"; TAB(38); "xB =";
     LPRINT USING "#######.##"; XB; :    LPRINT " "; UL$
     LPRINT "The minimum tension:"; TAB(38); "TO =";
     LPRINT USING "#######.##"; TO; :    LPRINT " "; UF$
     LPRINT "The maximum tension:"; TAB(36); "Tmax =";
     LPRINT USING "#######.##"; TM; :    LPRINT " "; UF$
     LPRINT "The direction of maximum tension:"; TAB(35); "Theta =";
     LPRINT USING "#######.##"; THETA; :    LPRINT " Deg"
     LPRINT "The cable length:"; TAB(39); "s ="; USING "#######.##"; S:
     LPRINT " "; UL$
     LPRINT "____________________________________________________"
END
```

C7-2 (a)

```
ANALYSIS OF PARABOLIC CABLES

Solution to EXAMPLE 7-8
______________________________________________________

The input data:

The horizontal span length:              L =  400.00 ft
The uniform load:                        q =  100.00 lb/ft
The sag of the left support A:          yA =  110.00 ft
The sag of the right support B:         yB =   65.00 ft
______________________________________________________

The computed results:

The horiz. length of left support:     xA =     226.15 ft
The horiz. length of right support:    xB =     173.85 ft
The minimum tension:                   TO =   23248.02 lb
The maximum tension:                 Tmax =   32433.42 lb
The direction of maximum tension:   Theta =      44.21 Deg
The cable length:                       s =     445.45 ft
______________________________________________________
```


C7-2 (b)

```
ANALYSIS OF PARABOLIC CABLES

Solution to Problem 7-34
______________________________________________________

The input data:

The horizontal span length:              L =  200.00 m
The uniform load:                        q = 1000.00 N/m
The sag of the left support A:          yA =   40.00 m
The sag of the right support B:         yB =   40.00 m
______________________________________________________

The computed results:

The horiz. length of left support:     xA =     100.00 m
The horiz. length of right support:    xB =     100.00 m
The minimum tension:                   TO =  125000.00 N
The maximum tension:                 Tmax =  160078.11 N
The direction of maximum tension:   Theta =      38.66 Deg
The cable length:                       s =     219.29 m
______________________________________________________
```

C7-2 (c)

```
ANALYSIS OF PARABOLIC CABLES

Solution to Problem 7-39
______________________________________________________

The input data:

The horizontal span length:              L =  100.00 m
The uniform load:                        q = 4000.00 N/m
The sag of the left support A:          yA =    2.00 m
The sag of the right support B:         yB =    6.00 m
______________________________________________________

The computed results:

The horiz. length of left support:     xA =      36.60 m
The horiz. length of right support:    xB =      63.40 m
The minimum tension:                   TO =1339746.00 N
The maximum tension:                 Tmax =1363534.75 N
The direction of maximum tension:   Theta =      10.72 Deg
The cable length:                       s =     100.45 m
______________________________________________________
```

Solution to Computer Program Assignment C8-1

```
' * C08-1 *   Moment of inertia of a composite area about a given axis
'
' Clear screen, compute pi and the conversion factors
      CLS :   PI = 4 * ATN(1):    DR = PI / 180:  RD = 180 / PI
'
' Display the introductory remarks
      PRINT "Want to see the introductory remarks?"
      INPUT "-- Y / N "; RMK$
      IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100   PRINT "REMARKS: (1) This program can be used to compute the moment of"
      PRINT "             inertia of a composite area about a given axis."
      PRINT "         (2) For each component area, you need to input the"
      PRINT "             area, the distance from its centroid to the"
      PRINT "             reference axis, and its moment of inertia about"
      PRINT "             its own centroidal axis."
      PRINT "         (3) The input data and the computed results are"
      PRINT "             printed on the printer immediately after"
      PRINT "             data are entered. Make sure your printer is on"
      PRINT "             before entering data."
'
' Input problem I.D.
200   PRINT "Enter problem designation:"
      INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
      PRINT "Select the system of units:"
      PRINT "      1 -- SI Units"
      PRINT "      2 -- US customary units"
      INPUT "-- 1 / 2"; U
      IF U = 1 THEN UL$ = "m"
      IF U = 2 THEN UL$ = "in"
'
' Print problem I.D. and heading
      LPRINT "MOMENT OF INERTIA OF A COMPOSITE AREA ABOUT A GIVEN AXIS"
      LPRINT :   LPRINT "Solution to "; ID$
      LPRINT "____________________________________________________________"
      LPRINT
      IF U$ = "SI" OR U$ = "si" GOTO 400
      LPRINT "Area"; TAB(9); "A("; UL$; "^2)"; TAB(21); "y("; UL$; ")";
      LPRINT TAB(31); "I("; UL$; "^4)";
      LPRINT TAB(41); "Ay^2("; UL$; "^4)"
      GOTO 420
400   LPRINT "Area"; TAB(12); "A"; TAB(23); "y"; TAB(33); "I";
      LPRINT TAB(44); "Ay^2"
      LPRINT TAB(8); "/1000 m^2"; TAB(23); "m"; TAB(29); "/1000 m^4";
      LPRINT TAB(41); "/1000 m^4"
420   LPRINT "____________________________________________________________"
      LPRINT
'
' Enter the total number of component areas
      PRINT "Enter the total number of component areas:"
      INPUT "-- No. of areas, N"; N
'
' Initialize sum
      SA = 0:    SI = 0:    SAY2 = 0
'
'Input data
      PRINT "Input value of area and the coordinates of its centroid:"
'
    FOR K = 1 TO N
      PRINT "-- A"; USING "#"; K; :   PRINT "("; UL$; "^2),"; " y";
```


Solution to Computer Program Assignment C8-1 Continued

```
      PRINT USING "#"; K;
      PRINT "("; UL$; "),"; " I"; USING "#"; K;
      PRINT "("; UL$; "^4),"; :    INPUT A, Y, I
'
' Compute the first and second moments of each area and find the summations
      AY2 = A * Y * Y:    SA = SA + A:    SI = SI + I:    SAY2 = SAY2 + AY2
'
' Print the input data and the second moments
      IF U$ = "SI" OR U$ = "si" GOTO 660
      LPRINT " A"; USING "#"; K;
      LPRINT USING "#######.###"; A; Y; I; AY2
      GOTO 690
660   AP = A * 1000:  IP = I * 1000:  AY2P = AY2 * 1000
      LPRINT " A"; USING "#"; K;
      LPRINT USING "#######.###"; AP; Y; IP; AY2P
'
690    NEXT K
'
' Find the the centroid, the moments of inertia, and the radii of gyration
      IX = SI + SAY2:    RX = SQR(IX / SA)
'
' Print the computed results
      LPRINT "____________________________________________________________"
      LPRINT
      IF U$ = "SI" OR U$ = "si" GOTO 810
      LPRINT "Sum"; USING "#######.###"; SA;
      LPRINT TAB(26); USING "#######.###"; SI; SAY2
      GOTO 840
810   SAP = SA * 1000:  SIP = SI * 1000:  SAY2P = SAY2 * 1000
      LPRINT "Sum"; USING "#######.###"; SAP;
      LPRINT TAB(26); USING "#######.###"; SIP; SAY2P
840   LPRINT "____________________________________________________________"
      LPRINT "Moment of inertia about the x-axis:"
      IF IX < 1 GOTO 890
      LPRINT TAB(19); "Ix ="; USING "#####.###"; IX;
      LPRINT " "; UL$; "^4"
      GOTO 900
890   LPRINT TAB(19); "Ix ="; USING "##.#####"; IX;
      LPRINT " "; UL$; "^4"
900   LPRINT "Radius of gyration about the x-axis:"
      LPRINT TAB(19); "rx ="; USING "####.###"; RX;
      LPRINT " "; UL$
  END
```

C8-1 (a)

MOMENT OF INERTIA OF A COMPOSITE AREA ABOUT A GIVEN AXIS

Solution to EXAMPLE 8-4

Area	A(m^2)	y(m)	I(m^4)	Ay^2(m^4)
A1	0.700	0.350	0.029	0.086
A2	-0.060	0.600	-0.000	-0.022
A3	-0.063	0.085	-0.000	-0.000
Sum	0.577		0.028	0.064

Moment of inertia about the x-axis:
Ix = 0.09180 m^4
Radius of gyration about the x-axis:
rx = 0.399 m

C8-1 (b)

MOMENT OF INERTIA OF A COMPOSITE AREA ABOUT A GIVEN AXIS

Solution to Problem 8-18

Area	A(m^2)	y(m)	I(m^4)	Ay^2(m^4)
A1	72.000	6.000	864.000	2592.000
A2	-14.140	6.000	-10.600	-509.040
Sum	57.860		853.400	2082.960

Moment of inertia about the x-axis:
Ix = 2936.360 m^4
Radius of gyration about the x-axis:
rx = 7.124 m

C8-1 (c)

MOMENT OF INERTIA OF A COMPOSITE AREA ABOUT A GIVEN AXIS

Solution to Problem 8-22

Area	A(in^2)	y(in)	I(in^4)	Ay^2(in^4)
A1	72.000	6.000	864.000	2592.000
A2	-12.570	6.000	-12.560	-452.520
Sum	59.430		851.440	2139.480

Moment of inertia about the x-axis:
Ix = 2990.920 in^4
Radius of gyration about the x-axis:
rx = 7.094 in

Solution to Computer Program Assignment C8-2

```
' * C08-2 *    Centroidal moment of inertia of a composite area
'
' Clear screen, compute pi and the conversion factors
      CLS :   PI = 4 * ATN(1):    DR = PI / 180:  RD = 180 / PI
'
' Display the introductory remarks
      PRINT "Want to see the introductory remarks?"
      INPUT "-- Y / N "; RMK$
      IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100   PRINT "REMARKS: (1) This program can be used to compute the moment of"
      PRINT "             inertia of a composite area about the centroidal"
      PRINT "             axis."
      PRINT "         (2) For each component area, you need to input the"
      PRINT "             area, the distance from its centroid to the"
      PRINT "             reference axis, and its moment of inertial about"
      PRINT "             its own centroidal axis."
      PRINT "         (3) The input data and the computed results are"
      PRINT "             printed on the printer immediately after"
      PRINT "             data are entered. Make sure your printer is on"
      PRINT "             before entering data."
'
' Input problem I.D.
200   PRINT "Enter problem designation:"
      INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
      PRINT "Select the system of units:"
      PRINT "       1 -- SI Units"
      PRINT "       2 -- US customary units"
      INPUT "-- 1 / 2"; U
      IF U = 1 THEN UL$ = "m"
      IF U = 2 THEN UL$ = "in"
'
' Print problem I.D. and heading
      LPRINT "MOMENT OF INERTIA OF A COMPOSITE AREA"
      LPRINT "ABOUT THE HORIZONTAL CENTROIDAL AXIS"
      LPRINT :   LPRINT "Solution to "; ID$
      LPRINT "____________________________________________________________"
      LPRINT
      IF U$ = "SI" OR U$ = "si" GOTO 420
      LPRINT "Area"; TAB(9); "A("; UL$; "^2)"; TAB(21); "y("; UL$; ")";
      LPRINT TAB(31); "I("; UL$; "^4)";
      LPRINT TAB(41); "Ay("; UL$; "^3)"; TAB(51); "Ay^2("; UL$; "^4)"
      GOTO 440
420   LPRINT "Area"; TAB(12); "A"; TAB(23); "y"; TAB(34); "I";
      LPRINT TAB(44); "Ay"; TAB(54); "Ay^2"
      LPRINT TAB(8); "/1000 m^2"; TAB(23); "m"; TAB(30); "/1000 m^4";
      LPRINT TAB(41); "/1000 m^3"; TAB(52); "/1000 m^4"
440   LPRINT "____________________________________________________________"
      LPRINT
'
' Enter the total number of component areas
      PRINT "Enter the total number of component areas:"
      INPUT "-- No. of areas, N"; N
'
' Initialize sum
      SA = 0:    SI = 0:    SAY = 0:    SAY2 = 0
'
'Input data
```

Solution to Computer Program Assignment C8-2 Continued

```
      PRINT "Input value of area and the coordinates of its centroid:"
    '
     FOR K = 1 TO N
      PRINT "-- A"; USING "#"; K;
      PRINT "("; UL$; "^2),"; " y"; USING "#"; K;
      PRINT "("; UL$; "),"; " I"; USING "#"; K;
      PRINT "("; UL$; "^4),"; :   INPUT A, Y, I
    '
    ' Compute the first and second moments of each area and find the summations
      AY = A * Y:    AY2 = AY * Y:    SA = SA + A
      SI = SI + I:    SAY = SAY + AY:    SAY2 = SAY2 + AY2
    '
    ' Print the input data and the first and second moments
      IF U$ = "SI" OR U$ = "si" GOTO 690
      LPRINT " A"; USING "#"; K;
      LPRINT USING "#######.###"; A; Y; I; AY; AY2
      GOTO 720
690   AP = A * 1000:  IP = I * 1000:  AYP = AY * 1000:  AY2P = AY2 * 1000
      LPRINT " A"; USING "#"; K;
      LPRINT USING "#######.###"; AP; Y; IP; AYP; AY2P
    '
720  NEXT K
    '
    ' Find the the centroid, the moments of inertia, and the radii of gyration
      Y = SAY / SA:    IX = SI + SAY2:    IXB = IX - SA * Y * Y
      RX = SQR(IX / SA):     RXB = SQR(IXB / SA)
    '
    ' Print the computed results
      LPRINT "____________________________________________________________"
      LPRINT
      IF U = 2 GOTO 850
      LPRINT "Sum"; USING "#######.###"; SA;
      LPRINT TAB(26); USING "#######.###"; SI; SAY; SAY2
      GOTO 880
850   SAP = SA * 1000:  SIP = SI * 1000
      SAYP = SAY * 1000:  SAY2P = SAY2 * 1000
      LPRINT "Sum"; USING "#######.###"; SAP;
      LPRINT TAB(26); USING "#######.###"; SIP; SAYP; SAY2P
880   LPRINT "____________________________________________________________"
      LPRINT :   LPRINT "Location of the centroidal axis (from the ";
      LPRINT "reference axis):"
      LPRINT TAB(20); "Y ="; USING "#####.###"; Y; :   LPRINT " "; UL$
      LPRINT "Moment of inertia about the x-axis:"
      IF IX < 1 GOTO 950
      LPRINT TAB(19); "Ix ="; USING "#####.###"; IX;
      LPRINT " "; UL$; "^4"
      GOTO 960
950   LPRINT TAB(19); "Ix ="; USING "##.######"; IX;
      LPRINT " "; UL$; "^4"
960   LPRINT "Radius of gyration about the x-axis:"
      LPRINT TAB(19); "rx ="; USING "#####.###"; RX; :   LPRINT " "; UL$
      LPRINT "Moment of inertia about the centroidal x bar axis:"
      IF IX < 1 GOTO 1020
      LPRINT TAB(18); "Ixb ="; USING "#####.###"; IXB;
      LPRINT " "; UL$; "^4"
      GOTO 1030
1020  LPRINT TAB(18); "Ixb ="; USING "##.######"; IXB;
      LPRINT " "; UL$; "^4"
1030  LPRINT "Radius of gyration about the centroidal x bar axis:"
      LPRINT TAB(18); "rxb ="; USING "#####.###"; RXB;
      LPRINT " "; UL$

   END
```


C8-2 (a)

MOMENT OF INERTIA OF A COMPOSITE AREA
ABOUT THE HORIZONTAL CENTROIDAL AXIS

Solution to EXAMPLE 8-3

Area	A(m^2)	y(m)	I(m^4)	Ay(m^3)	Ay^2(m^4)
A1	3.000	0.500	0.250	1.500	0.750
A2	6.000	4.000	18.000	24.000	96.000
A3	6.000	7.500	0.500	45.000	337.500
Sum	15.000		18.750	70.500	434.250

```
Location of the centroidal axis (from the reference axis):
                   Y =     4.700 m
Moment of inertia about the x-axis:
                  Ix =   453.000 m^4
Radius of gyration about the x-axis:
                  rx =     5.495 m
Moment of inertia about the centroidal x bar axis:
                 Ixb =   121.650 m^4
Radius of gyration about the centroidal x bar axis:
                 rxb =     2.848 m
```

C8-2 (b)

MOMENT OF INERTIA OF A COMPOSITE AREA
ABOUT THE HORIZONTAL CENTROIDAL AXIS

Solution to EXAMPLE 8-7

Area	A(in^2)	y(in)	I(in^4)	Ay(in^3)	Ay^2(in^4)
A1	9.960	0.787	8.130	7.839	6.169
A2	14.700	9.395	800.000	138.107	1297.511
Sum	24660.000		808130.000	145945.016	1303679.750

```
Location of the centroidal axis (from the reference axis):
                   Y =     5.918 in
Moment of inertia about the x-axis:
                  Ix =  2111.810 in^4
Radius of gyration about the x-axis:
                  rx =     9.254 in
Moment of inertia about the centroidal x bar axis:
                 Ixb =  1248.065 in^4
Radius of gyration about the centroidal x bar axis:
                 rxb =     7.114 in
```

C8-2 (c)

```
MOMENT OF INERTIA OF A COMPOSITE AREA
ABOUT THE HORIZONTAL CENTROIDAL AXIS

Solution to Problem 8-31
```

Area	A(in^2)	y(in)	I(in^4)	Ay(in^3)	Ay^2(in^4)
A1	16.100	9.000	804.000	144.900	1304.100
A2	16.100	9.000	804.000	144.900	1304.100
A3	22.500	18.630	2.930	419.175	7809.229
Sum	54700.000		1610930.000	708975.000	%10417430.000

```
Location of the centroidal axis (from the reference axis):
                    Y =   12.961 in
Moment of inertia about the x-axis:
                    Ix =12028.359 in^4
Radius of gyration about the x-axis:
                    rx =   14.829 in
Moment of inertia about the centroidal x bar axis:
                    Ixb =  2839.228 in^4
Radius of gyration about the centroidal x bar axis:
                    rxb =     7.205 in
```


Solution to Computer Program Assignment C9-1

```
    '
    ' * C09-1 *   Normal stresses in an axially loaded member
    '
    ' Clear screen, compute pi and the conversion factors
         CLS :   PI = 4 * ATN(1):    DR = PI / 180:  RD = 180 / PI
    '
    ' Display the introductory remarks
         PRINT "Want to see the introductory remarks?"
         INPUT "-- Y / N "; RMK$
         IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100      PRINT "REMARKS: (1) This program can be used to compute the normal"
         PRINT "             stresses of a stepped circular member subjected"
         PRINT "             to axial forces."
         PRINT "         (2) For each segment of the member, you need to input"
         PRINT "             the diameter and the external load applied to the"
         PRINT "             right side of each segment."
         PRINT "         (3) The input data and the computed results are"
         PRINT "             printed on the printer immediately after"
         PRINT "             data are entered. Make sure your printer is on"
         PRINT "             before entering data."
    '
    ' Input problem I.D.
200      PRINT "Enter problem designation:"
         INPUT "-- Prob. I.D. "; ID$
    '
    ' Select system of units
         PRINT "Select the system of units:"
         PRINT "        1 -- SI Units"
         PRINT "        2 -- US customary units"
         INPUT "-- 1 / 2"; U
         IF U = 2 GOTO 300
         UL$ = "m": UF$ = "N": US$ = "MPa"
         GOTO 400
300      UL$ = "in.": UF$ = "lb": US$ = "psi"
    '
    ' Print problem I.D. and heading
400      LPRINT "NORMAL STRESSES IN AN AXIALLY LOADED MEMBER"
         LPRINT :   LPRINT "Solution to "; ID$
         LPRINT "______________________________________________________________"
         LPRINT
         IF U = 2 GOTO 440
         LPRINT "Segment  Diam.  Ext. Load   Area   Int. Force  Normal Stress"
         LPRINT TAB(10); "d("; UL$; ")"; TAB(20); "P("; UF$; ")";
         LPRINT TAB(28); "A("; UL$; "^2)";
         LPRINT TAB(39); "F("; UF$; ")"; TAB(50); "Sigma("; US$; ")"
         GOTO 480
440      LPRINT "Segment  Diam.  Ext. Load    Area  Int. Force  Normal Stress"
         LPRINT TAB(29); "/1000"
         LPRINT TAB(10); "d("; UL$; ")"; TAB(20); "P("; UF$; ")";
         LPRINT TAB(29); "A("; UL$; "^2)";
         LPRINT TAB(39); "F("; UF$; ")"; TAB(50); "Sigma("; US$; ")"
480      LPRINT "______________________________________________________________"
         LPRINT
    '
    ' Input data
         PRINT "Enter the total number of segments:"
         INPUT "-- No. of segments, N"; N
         PRINT "Input the diameter and the axial load applied to the right ";
         PRINT "end of each segment"
    '
    ' Initialize the sum
         F = 0
```

Solution to Computer Program Assignment C9-1 Continued

```
   '
    FOR I = 1 TO N
      PRINT "-- d"; USING "#"; I; :   PRINT "("; UL$; "),"; " P";
      PRINT USING "#"; I; : PRINT "("; UF$; ")";
      INPUT D, P
   '
   ' Compute the cross-sectional area, the internal force, and the average
   ' stress in each segment
      A = PI * D * D / 4: F = F + P:    S = F / A
      IF S > 0 THEN S$ = "(T)"
      IF S < 0 THEN S$ = "(C)"
   '
   ' Print the input data and the computed results
      IF U = 1 GOTO 750
      LPRINT USING "####"; I; :   LPRINT USING "######.###"; D;
      LPRINT USING "#########.#"; P; :   LPRINT USING "####.###"; A;
      LPRINT USING "#########.#"; F; :   LPRINT USING "#########.#"; S;
      LPRINT " "; S$
      GOTO 810
750   AP = A * 1000:  SP = S / 1000000#
      LPRINT USING "####"; I; :   LPRINT USING "######.###"; D;
      LPRINT USING "#########.#"; P; :   LPRINT USING "#####.###"; AP;
      LPRINT USING "#########.#"; F; :   LPRINT USING "#######.###"; SP;
      LPRINT " "; S$
   '
810  NEXT I
   '
      LPRINT "________________________________________________________________"
 END
```

C9-1 (a) NORMAL STRESSES IN AN AXIALLY LOADED MEMBER

Solution to Problem 9-3

Segment	Diam. d(in.)	Ext. Load P(lb)	Area A(in.^2)	Int. Force F(lb)	Normal Stress Sigma(psi)
1	2.000	-20000.0	3.142	-20000.0	-6366.2 (C)
2	2.000	30000.0	3.142	10000.0	3183.1 (T)
3	2.000	40000.0	3.142	50000.0	15915.5 (T)

C9-1 (b) NORMAL STRESSES IN AN AXIALLY LOADED MEMBER

Solution to Problem 9-5

Segment	Diam. d(m)	Ext. Load P(N)	Area A(m^2)	Int. Force F(N)	Normal Stress Sigma(MPa)
1	0.020	20000.0	0.314	20000.0	63.662 (T)
2	0.040	40000.0	1.257	60000.0	47.746 (T)
3	0.060	-200000.0	2.827	-140000.0	-49.515 (C)

Solution to Computer Program Assignment C9-2

```
   ' * C09-2 *   Analysis of thin-walled pressure vessels
   '
   ' Clear screen
        CLS
   '
   ' Display the introductory remarks
        PRINT "Want to see the introductory remarks?"
        INPUT "-- Y / N "; RMK$
        IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100     PRINT "REMARKS: (1) This program can be used to compute the normal"
        PRINT "             stresses or the allowable internal pressure in a"
        PRINT "             thin-walled cylindrical or spherical pressure"
        PRINT "             vessel."
        PRINT "         (2) The input data include the inside radius, the"
        PRINT "             wall thickness, the internal pressure or the"
        PRINT "             allowable normal stress."
        PRINT "         (3) The input data and the computed results are"
        PRINT "             printed on the printer immediately after"
        PRINT "             data are entered. Make sure your printer is on"
        PRINT "             before entering data."
   '
   ' Input problem I.D.
200     PRINT "Enter problem designation:"
        INPUT "-- Prob. I.D. "; ID$
   '
   ' Specify the type of vessel
        PRINT "Type of vessel:"
        PRINT "       1 -- Cylindrical vessel"
        PRINT "       2 -- Spherical vessel"
        INPUT "-- Type of vessel -- 1 / 2"; TV
   '
   ' Specify the type of problem
        PRINT "Specify the type of problem:"
        PRINT "       1 -- to find the normal stresses"
        PRINT "       2 -- to find the allowable internal pressure"
        INPUT "-- Type of problem -- 1 / 2"; TP
   '
   ' Select system of units
        PRINT "Select the system of units:"
        PRINT "       1 -- SI Units"
        PRINT "       2 -- US customary units"
        INPUT "-- 1 / 2"; U
        IF U = 2 GOTO 300
        UL$ = "m": US$ = "MPa": UP$ = "MPa"
        GOTO 400
300     UL$ = "in.": US$ = "psi": UP$ = "psi"
   '
   ' Print problem I.D. and heading
400     IF TV = 1 THEN LPRINT "ANALYSIS OF THIN-WALLED CYLINDRICAL PRESSURE";
        IF TV = 1 THEN LPRINT " VESSEL"
        IF TV = 2 THEN LPRINT "ANALYSIS OF THIN-WALLED SPHERICAL PRESSURE";
        IF TV = 2 THEN LPRINT " VESSEL"
560     LPRINT :   LPRINT "Solution to "; ID$
        LPRINT "______________________________________________"
        LPRINT :   LPRINT "The input data are:"
   '
   ' Enter and print data
        PRINT "-- Inside radius and thickness: -- ri("; UL$; "), t(";
        PRINT UL$; ")"; : INPUT RI, T
        LPRINT "  Inside radius"; TAB(31); "ri =";
```

Solution to Computer Program Assignment C9-2 Continued

```
        LPRINT USING "#######.###"; RI; :   LPRINT " "; UL$
        LPRINT "  Wall thickness"; TAB(31); " t =";
        LPRINT USING "#######.###"; T; :   LPRINT " "; UL$
        IF TP = 1 GOTO 660
        IF TP = 2 GOTO 690
660     PRINT "-- Internal pressure -- p("; UP$; ")"; :   INPUT P
        LPRINT "  Internal pressure"; TAB(31); " p =";
        LPRINT USING "#######.###"; P; :   LPRINT " "; UP$
        GOTO 730
690     PRINT "-- Alowable normal stress -- sallow("; US$; ")";
        INPUT SALLOW
        LPRINT "  Allowable normal stress"; TAB(27); "Sallow =";
        LPRINT USING "#######.###"; SALLOW; :   LPRINT " "; US$

    Compute and print stresses
730     LPRINT :   LPRINT "The computed results are:"
        IF TP = 2 GOTO 830
        IF TV = 2 GOTO 800
        SC = P * RI / T:    SL = SC / 2
        LPRINT "  The circumferential stress"; TAB(31); "Sc =";
        LPRINT USING "#######.###"; SC; :   LPRINT " "; US$
        LPRINT "  The longitudinal stress"; TAB(31); "Sl =";
        LPRINT USING "#######.###"; SL; :   LPRINT " "; US$
        GOTO 860
800     S = P * RI / 2 / T
        LPRINT "  The normal stress"; TAB(28); "sigma =";
        LPRINT USING "#######.###"; S; :   LPRINT " "; US$
        GOTO 860
830     IF TV = 1 THEN P = T * SALLOW / RI
        IF TV = 2 THEN P = 2 * T * SALLOW / RI
        LPRINT "  Allowable internal pressure"; TAB(31); " p =";
        LPRINT USING "#######.###"; P; :   LPRINT " "; UP$
860     LPRINT "_______________________________________________"
    END
```


C9-2 (a)

```
ANALYSIS OF THIN-WALLED CYLINDRICAL PRESSURE VESSEL

Solution to EXAMPLE 9-11
_______________________________________________
The input data are:
  Inside radius                   ri =     10.000 in.
  Wall thickness                   t =      0.500 in.
  Internal pressure                p =    300.000 psi

The computed results are:
  The circumferential stress      Sc =   6000.000 psi
  The longitudinal stress         Sl =   3000.000 psi
_______________________________________________
```

C9-2 (b)

```
ANALYSIS OF THIN-WALLED CYLINDRICAL PRESSURE VESSEL

Solution to Problem 9-36
_______________________________________________
The input data are:
  Inside radius                   ri =      8.000 in.
  Wall thickness                   t =      0.125 in.
  Allowable normal stress Sallow =   8000.000 psi

The computed results are:
  Allowable internal pressure  p =    125.000 psi
_______________________________________________
```

C9-2 (c)

```
ANALYSIS OF THIN-WALLED SPHERICAL PRESSURE VESSEL

Solution to Problem 9-38
_______________________________________________
The input data are:
  Inside radius                   ri =      4.000 in.
  Wall thickness                   t =      0.250 in.
  Allowable normal stress Sallow =   6000.000 psi

The computed results are:
  Allowable internal pressure  p =    750.000 psi
_______________________________________________
```

Solution to Computer Program Assignment C10-1

```
      ' * C10-1 *   Stresses and Deformatiom in an axially loaded member
      '
      ' Clear screen
            CLS
      '
      ' Display the introductory remarks
            PRINT "Want to see the introductory remarks?"
            INPUT "-- Y / N "; RMK$
            IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100         PRINT "REMARKS: (1) This program can be used to compute the normal"
            PRINT "             stresses and the axial deformations of a stepped"
            PRINT "             circular member subjected to axial forces."
            PRINT "         (2) You need to input the modulus of elasticity, the"
            PRINT "             cross-sectional area, the length, and the"
            PRINT "             external load applied to the right side of each"
            PRINT "             segment."
            PRINT "         (3) The input data and the computed results are"
            PRINT "             printed on the printer immediately after"
            PRINT "             data are entered. Make sure your printer is on"
            PRINT "             before entering data."
      '
      ' Input problem I.D.
200         PRINT "Enter problem designation:"
            INPUT "-- Prob. I.D. "; ID$
      '
      ' Select system of units
            PRINT "Select the system of units:"
            PRINT "       1 -- SI Units"
            PRINT "       2 -- US customary units"
            INPUT "-- 1 / 2"; U
            IF U = 2 GOTO 300
            UL$ = "m": UF$ = "N": US1$ = "MPa": US2$ = "GPa"
            GOTO 400
300         UL$ = "in.": UF$ = "lb": US1$ = "psi": US2$ = "ksi"
      '
      ' Print problem I.D. and heading
400         LPRINT "STRESSES AND DEFORMATIOMS IN AN AXIALLY LOADED MEMBER"
            LPRINT :   LPRINT "Solution to "; ID$
            LPRINT "____________________________________________________________";
            LPRINT "______________"
            LPRINT
            IF U = 1 GOTO 500
            LPRINT "Segment  Area   Length   Ext. Load  Int. Force   N. Stress  ";
            LPRINT "Axial Deform."
            LPRINT TAB(9); "A("; UL$; "^2)"; TAB(18); "L("; UL$; ")";
            LPRINT TAB(28); "P("; UF$; ")";
            LPRINT TAB(40); "F("; UF$; ")"; TAB(50); "Sigma("; US1$; ")";
            LPRINT TAB(63); "Delta("; UL$; ")"
            GOTO 540
500         LPRINT "Segment  Area   Length   Ext. Load  Int. Force   N. Stress  ";
            LPRINT "Axial Deform."
            LPRINT TAB(10); "/1000"
            LPRINT TAB(9); "A("; UL$; "^2)"; TAB(18); "L("; UL$; ")";
            LPRINT TAB(29); "P("; UF$; ")";
            LPRINT TAB(40); "F("; UF$; ")"; TAB(50); "Sigma("; US1$; ")";
            LPRINT TAB(63); "Delta("; UL$; ")"
540         LPRINT "____________________________________________________________";
            LPRINT "______________"
            LPRINT
      '
      ' Input data
```


Solution to Computer Program Assignment C10-1 Continued

```
            PRINT "Enter the modulus of elasticity of the material:"
            PRINT "-- E("; US2$; ")"; :   INPUT E
            IF U = 1 THEN EP = E * 1E+09
            IF U = 2 THEN EP = E * 1000
            PRINT "Enter the total number of segments:"
            INPUT "-- No. of segments, N"; N
            PRINT "Input area, length, and load:"
      '
      ' Initialize the sum
            F = 0:    TDFM = 0
      '
          FOR I = 1 TO N
            PRINT "-- A"; USING "#"; I; :   PRINT "("; UL$; "^2),"; " L";
            PRINT USING "#"; I;
            PRINT "("; UL$; "),"; " P"; USING "#"; I;
            PRINT "("; UF$; ")";
            INPUT A, L, P
      '
      ' Compute the internal force, the normal stress, and the axial deformation
      ' in each segment
            F = F + P:    S = F / A:  DFM = S * L / EP:    TDFM = TDFM + DFM
            IF S > 0 THEN S$ = "(T)"
            IF S < 0 THEN S$ = "(C)"
      '
      ' Print the input data and the computed results
            IF U = 1 GOTO 860
            LPRINT USING "####"; I; :   LPRINT USING "######.###"; A;
            LPRINT USING "#####.##"; L; :   LPRINT USING "#########.#"; P;
            LPRINT USING "#########.#"; F; :   LPRINT USING "#########.#"; S;
            LPRINT " "; S$; USING "#####.#####"; DFM
            GOTO 910
860         LPRINT USING "####"; I; :   LPRINT USING "######.###"; A * 1000;
            LPRINT USING "###.####"; L; :   LPRINT USING "#########.#"; P;
            LPRINT USING "#########.#"; F;
            LPRINT USING "#######.###"; S / 1000000!;
            LPRINT " "; S$; USING "#####.#####"; DFM
      '
910       NEXT I
            LPRINT "____________________________________________________________";
            LPRINT "______________"
            LPRINT :   LPRINT "Modulus of elasticity   ="; USING "########"; E;
            LPRINT " "; US2$
            IF TDFM > 0 THEN EC$ = "(elongation)"
            IF TDFM < 0 THEN EC$ = "(contraction)"
            LPRINT "Total axial deformation ="; USING "##.#####"; TDFM;
            LPRINT " "; UL$; " "; EC$
      END
```

C10-1 (a)

STRESSES AND DEFORMATIOMS IN AN AXIALLY LOADED MEMBER

Solution to EXAMPLE 10-2

Segment	Area A(in.^2)	Length L(in.)	Ext. Load P(lb)	Int. Force F(lb)	N. Stress Sigma(psi)	Axial Deform. Delta(in.)
1	0.500	36.00	8000.0	8000.0	16000.0 (T)	0.01920
2	0.500	60.00	-6000.0	2000.0	4000.0 (T)	0.00800
3	0.500	48.00	-7000.0	-5000.0	-10000.0 (C)	-0.01600

Modulus of elasticity = 30000 ksi
Total axial deformation = 0.01120 in. (elongation)

C10-1 (b)

STRESSES AND DEFORMATIOMS IN AN AXIALLY LOADED MEMBER

Solution to Problem 10-8

Segment	Area /1000 A(m^2)	Length L(m)	Ext. Load P(N)	Int. Force F(N)	N. Stress Sigma(MPa)	Axial Deform. Delta(m)
1	0.707	1.5000	20000.0	20000.0	28.289 (T)	0.00061
2	0.707	2.0000	10000.0	30000.0	42.433 (T)	0.00121

Modulus of elasticity = 70 GPa
Total axial deformation = 0.00182 m (elongation)

C10-1 (c)

STRESSES AND DEFORMATIOMS IN AN AXIALLY LOADED MEMBER

Solution to Problem 10-9

Segment	Area A(in.^2)	Length L(in.)	Ext. Load P(lb)	Int. Force F(lb)	N. Stress Sigma(psi)	Axial Deform. Delta(in.)
1	2.000	48.00	20000.0	20000.0	10000.0 (T)	0.02824
2	2.000	24.00	-30000.0	-10000.0	-5000.0 (C)	-0.00706
3	2.000	36.00	40000.0	30000.0	15000.0 (T)	0.03176

Modulus of elasticity = 17000 ksi
Total axial deformation = 0.05294 in. (elongation)

C10-1 (d)

STRESSES AND DEFORMATIOMS IN AN AXIALLY LOADED MEMBER

Solution to Problem 10-11

Segment	Area /1000 A(m^2)	Length L(m)	Ext. Load P(N)	Int. Force F(N)	N. Stress Sigma(MPa)	Axial Deform. Delta(m)
1	0.491	1.7000	50000.0	50000.0	101.833 (T)	0.00082
2	1.260	1.8000	-175000.0	-125000.0	-99.206 (C)	-0.00085
3	0.707	1.2000	225000.0	100000.0	141.443 (T)	0.00081

Modulus of elasticity = 210 GPa
Total axial deformation = 0.00078 m (elongation)

Solution to Computer Program Assignment C10-2

```
' * C10-2 *   Reactions on an axially loaded member with fixed ends
'
' Clear screen
     CLS
'
' Display the introductory remarks
     PRINT "Want to see the introductory remarks?"
     INPUT "-- Y / N "; RMK$
     IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100  PRINT "REMARKS: (1) This program can be used to compute the fixed-end"
     PRINT "             reactions in an axially loaded member."
     PRINT "         (2) You need to input the cross-sectional area, the"
     PRINT "             length, and the external load applied to the"
     PRINT "             right side of each segment."
     PRINT "         (3) The input data and the computed results are"
     PRINT "             printed on the printer immediately after"
     PRINT "             data are entered. Make sure your printer is on"
     PRINT "             before entering data."
'
' Input problem I.D.
200  PRINT "Enter problem designation:"
     INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
     PRINT "Select the system of units:"
     PRINT "      1 -- SI Units"
     PRINT "      2 -- US customary units"
     INPUT "-- 1 / 2"; U
     IF U = 2 GOTO 300
     UL$ = "m": UF1$ = "N": UF2$ = "kN": US1$ = "MPa": US2$ = "GPa"
     GOTO 400
300  UL$ = "in.": UF1$ = "lb": UF2$ = "kips": US1$ = "psi": US2$ = "ksi"
'
' Print problem I.D. and heading
400  LPRINT "REACTIONS ON AN AXIALLY-LOADED MEMBER WITH FIXED ENDS"
     LPRINT :   LPRINT "Solution to "; ID$
     LPRINT "____________________________________________________________"
     LPRINT
     IF U = 1 GOTO 510
     LPRINT "Segment  Area   Length   Ext. Load  Int. Force   N. Stress"
     LPRINT TAB(9); "A("; UL$; "^2)"; TAB(18); "L("; UL$; ")";
     LPRINT TAB(28); "P("; UF1$; ")";
     LPRINT TAB(40); "F("; UF1$; ")"; TAB(50); "Sigma("; US1$; ")"
     GOTO 550
510  LPRINT "Segment  Area   Length   Ext. Load  Int. Force   N. Stress"
     LPRINT TAB(10); "/1000"
     LPRINT TAB(9); "A("; UL$; "^2)"; TAB(18); "L("; UL$; ")";
     LPRINT TAB(29); "P("; UF1$; ")";
     LPRINT TAB(40); "F("; UF1$; ")"; TAB(50); "Sigma("; US1$; ")"
550  LPRINT "____________________________________________________________"
     LPRINT
'
' Input data
     PRINT "Enter the total number of segments:"
     INPUT "-- No. of segments, N"; N
     PRINT "Input area, length, and load:"
'
' Initialize the sum
      P(1) = 0:    F = 0:    SB = 0:    ST = 0
'
```

Solution to Computer Program Assignment C10-2 Continued

```
      FOR I = 1 TO N
        PRINT "-- A"; USING "#"; I; :   PRINT "("; UL$; "^2),"; " L";
        PRINT USING "#"; I; : PRINT "("; UL$; ")";
        IF I = 1 GOTO 710
        PRINT ", P"; USING "#"; I; :   PRINT "("; UF1$; ")";
        INPUT A(I), L(I), P(I):    GOTO 720
710     INPUT A(I), L(I)
720     F = F + P(I):    SB = SB + L(I) / A(I):  ST = ST + F * L(I) / A(I)
      NEXT I
    '
    ' Compute the fixed end reactions
        RB = -ST / SB:   RA = -F - RB
    '
    ' Initialize the sum
        P(1) = RB:    F = 0
    '
    ' Compute the internal force, the normal stress, and the axial deformation
    '
      FOR I = 1 TO N
        F = F + P(I):    S = F / A(I)
        IF S > 0 THEN S$ = "(T)"
        IF S < 0 THEN S$ = "(C)"
    '
    ' Print the input data and the computed results
        IF U = 1 GOTO 950
        LPRINT USING "####"; I; :   LPRINT USING "######.###"; A(I);
        LPRINT USING "#####.##"; L(I); :   LPRINT USING "#########.#"; P(I);
        LPRINT USING "#########.#"; F; :   LPRINT USING "#########.#"; S;
        LPRINT " "; S$
        GOTO 1000
950     LPRINT USING "####"; I; :   LPRINT USING "######.###"; A(I) * 1000;
        LPRINT USING "###.####"; L(I); :   LPRINT USING "#########.#"; P(I);
        LPRINT USING "#########.#"; F;
        LPRINT USING "#######.###"; S / 1000000!;
        LPRINT " "; S$

1000  NEXT I
        LPRINT "_______________________________________________________________"
        LPRINT :   LPRINT "Reactions at the fixed ends are:"
        IF RA > 0 THEN DIRA$ = "(to the right)"
        IF RA < 0 THEN DIRA$ = "(to the left)"
        IF RB > 0 THEN DIRB$ = "(to the right)"
        IF RB < 0 THEN DIRB$ = "(to the left)"
        LPRINT TAB(20); "RA ="; USING "#####.###"; RA / 1000;
        LPRINT " "; UF2$; " "; DIRA$
        LPRINT TAB(20); "RB ="; USING "#####.###"; RB / 1000;
        LPRINT " "; UF2$; " "; DIRB$
    END
```


C10-2 (a)

```
REACTIONS ON AN AXIALLY-LOADED MEMBER WITH FIXED ENDS

Solution to Problem 10-15
```

Segment	Area A(in.^2)	Length L(in.)	Ext. Load P(lb)	Int. Force F(lb)	N. Stress Sigma(psi)
1	3.000	8.00	-38461.5	-38461.5	-12820.5 (C)
2	6.000	10.00	100000.0	61538.5	10256.4 (T)

```
Reactions at the fixed ends are:
                   RA =  -61.538 kips (to the left)
                   RB =  -38.462 kips (to the left)
```

C10-2 (b)

```
REACTIONS ON AN AXIALLY-LOADED MEMBER WITH FIXED ENDS

Solution to Problem 10-16
```

Segment	Area /1000 A(m^2)	Length L(m)	Ext. Load P(N)	Int. Force F(N)	N. Stress Sigma(MPa)
1	1.000	0.3000	101818.2	101818.2	101.818 (T)
2	1.000	0.6000	-160000.0	-58181.8	-58.182 (C)
3	1.000	0.2000	80000.0	21818.2	21.818 (T)

```
Reactions at the fixed ends are:
                   RA =  -21.818 kN (to the left)
                   RB =  101.818 kN (to the right)
```

Solution to Computer Program Assignment C12-1

```
' * C12-1 *   Shear stress & angle of twist in circular shaft
'
' Clear screen, compute pi and the degree-radian conversion factors
     CLS :   PI = 4 * ATN(1):    DR = PI / 180:  RD = 1 / DR
'
' Display the introductory remarks
     PRINT "Want to see the introductory remarks?"
     INPUT "-- Y / N ": RMK$
     IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100  PRINT "REMARKS: (1) This program can be used to compute the maximum"
     PRINT "             shear stresses and the angles of twist of a"
     PRINT "             stepped circular shaft."
     PRINT "         (2) You need to input the modulus of rigidity, the"
     PRINT "             outside and inside diameter, the length, and the"
     PRINT "             external torque applied to the right side of each"
     PRINT "             segment."
     PRINT "         (3) The input data and the computed results are"
     PRINT "             printed on the printer immediately after"
     PRINT "             data are entered. Make sure the printer is on"
     PRINT "             before entering data."
'
' Input problem I.D.
200  PRINT "Enter problem designation:"
     INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
     PRINT "Select the system of units:"
     PRINT "       1 -- SI Units"
     PRINT "       2 -- US customary units"
     INPUT "-- 1 / 2"; U
     IF U = 2 GOTO 300
     UL$ = "m": UF$ = "N": UM$ = "N-m": US1$ = "MPa": US2$ = "GPa"
     GOTO 400
300  UL$ = "in.": UF$ = "lb": UM$ = "lb-in.": US1$ = "psi": US2$ = "ksi"
'
' Print problem I.D. and heading
400  LPRINT "SHEAR STRESS AND ANGLE OF TWIST IN CIRCULAR SHAFT"
     LPRINT :   LPRINT "Solution to "; ID$
     LPRINT "____________________________________________________________";
     LPRINT "______________"
     LPRINT
     IF U = 1 GOTO 530
     LPRINT "Segment O.Diam  I.Diam  Length Ext.Torque Int.Torque ";
     LPRINT "S.Stress Twist Angle"
     LPRINT TAB(9); "do("; UL$; ")"; TAB(17); "di("; UL$; ")";
     LPRINT TAB(25); "L("; UL$; ")";
     LPRINT TAB(33); "T("; UM$; ")"; TAB(43); "TI("; UM$; ")";
     LPRINT TAB(54); "Tau("; US1$; ")";
     LPRINT TAB(64); "Phi(Deg.)"
     GOTO 570
530  LPRINT "Segment O.Diam  I.Diam  Length Ext.Torque Int.Torque ";
     LPRINT "S.Stress Twist Angle"
     LPRINT TAB(10); "do("; UL$; ")"; TAB(18); "di("; UL$; ")";
     LPRINT TAB(26); "L("; UL$; ")";
     LPRINT TAB(34); "T("; UM$; ")"; TAB(45); "TI("; UM$; ")";
     LPRINT TAB(54); "Tau("; US1$; ")";
     LPRINT TAB(64); "Phi(Deg.)"
570  LPRINT "____________________________________________________________";
     LPRINT "______________"
     LPRINT
```


Solution to Computer Program Assignment C12-1 Continued

```
' Input data
     PRINT "Enter the modulus of rigidity of the material:"
     PRINT "-- G("; US2$; ")"; :   INPUT G
     IF U = 1 THEN GP = G * 1E+09
     IF U = 2 THEN GP = G * 1000
     PRINT "Enter the total number of segments:"
     INPUT "-- No. of segments, N"; N
     PRINT "Input outside and inside diameters, length, and load:"
'
' Initialize the sum
     TI = 0:    TAOT = 0
'
    FOR I = 1 TO N
     PRINT "-- do"; USING "#"; I; :   PRINT "("; UL$; ")."; " di";
     PRINT USING "#"; I; : PRINT "("; UL$; "),";
     PRINT " L"; USING "#"; I; :   PRINT "("; UL$; "),"; " T";
     PRINT USING "#"; I; : PRINT "("; UM$; ")";
     INPUT OD, DI, L, T
'
' Compute the internal torque, the maximum shear stress, and the angle of
' twist
     TI = TI + T:    J = PI / 32 * (OD ^ 4 - DI ^ 4): SS = TI * OD / 2 / J
     AOT = TI * L / GP / J:    TAOT = TAOT + AOT
'
' Print the input data and the results
     IF U = 1 GOTO 880
     LPRINT USING "####"; I; :   LPRINT USING "#####.###"; OD; DI;
     LPRINT USING "#####.##"; L; :   LPRINT USING "########"; T;
     LPRINT USING "###########"; TI; :   LPRINT USING "##########"; SS;
     LPRINT USING "######.####"; AOT * RD
     GOTO 930
880  LPRINT USING "####"; I; :   LPRINT USING "####.####"; OD; DI;
     LPRINT USING "####.###"; L; :   LPRINT USING "######.##"; T;
     LPRINT USING "#######.###"; TI;
     LPRINT USING "######.###"; SS / 1000000#;
     LPRINT USING "######.####"; AOT * RD
'
930   NEXT I
     LPRINT "____________________________________________________________";
     LPRINT "______________"
     LPRINT :   LPRINT "Modulus of rigidity  ="; USING "########"; G;
     LPRINT " "; US2$
     LPRINT "Total angle of twist ="; USING "###.####"; TAOT * RD;
     LPRINT " Degrees"
  END
```

C12-1 (a) SHEAR STRESS AND ANGLE OF TWIST IN CIRCULAR SHAFT

Solution to EXAMPLE 12-13

Segment	O.Diam do(in.)	I.Diam di(in.)	Length L(in.)	Ext.Torque T(lb-in.)	Int.Torque TI(lb-in.)	S.Stress Tau(psi)	Twist Angle Phi(Deg.)
1	2.000	0.000	84.00	-6300	-6300	-4011	-1.6086
2	2.000	0.000	72.00	-5250	-11550	-7353	-2.5278
3	2.000	0.000	60.00	21000	9450	6016	1.7235

Modulus of rigidity = 12000 ksi
Total angle of twist = -2.4129 Degrees

C12-1 (b) SHEAR STRESS AND ANGLE OF TWIST IN CIRCULAR SHAFT

Solution to Problem 12-38

Segment	O.Diam do(m)	I.Diam di(m)	Length L(m)	Ext.Torque T(N-m)	Int.Torque TI(N-m)	S.Stress Tau(MPa)	Twist Angle Phi(Deg.)
1	0.0250	0.0000	1.000	-20.00	-20.000	-6.519	-0.3557
2	0.0500	0.0000	1.500	-60.00	-80.000	-3.259	-0.1334

Modulus of rigidity = 84 GPa
Total angle of twist = -0.4891 Degrees

C12-1 (c) SHEAR STRESS AND ANGLE OF TWIST IN CIRCULAR SHAFT

Solution to Problem 12-39

Segment	O.Diam do(m)	I.Diam di(m)	Length L(m)	Ext.Torque T(N-m)	Int.Torque TI(N-m)	S.Stress Tau(MPa)	Twist Angle Phi(Deg.)
1	0.0700	0.0000	2.000	3000.00	3000.000	44.545	1.7362
2	0.0850	0.0000	2.000	3000.00	6000.000	49.758	1.5972
3	0.0750	0.0000	2.000	-11000.00	-5000.000	-60.361	-2.1958

Modulus of rigidity = 84 GPa
Total angle of twist = 1.1375 Degrees

Solution to Computer Program Assignment C12-2

```
' * C12-2 * Size of solid or hollow circular shaft for strength & stiffness
'
' Clear screen, compute pi and the conversion factors
      CLS :   PI = 4 * ATN(1):    DR = PI / 180:  RD = 180 / PI
'
' Display the introductory remarks
      PRINT "Want to see the introductory remarks?"
      INPUT "-- Y / N "; RMK$
      IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 220
100   PRINT "REMARKS: (1) This program can be used to compute the required"
      PRINT "             size of a solid or hollow circular shaft to"
      PRINT "             satisfy the requirements on strength and"
      PRINT "             stiffness."
      PRINT "         (2) The input data for the shaft include the power"
      PRINT "             transmitted, the the angular velocity, the shear"
      PRINT "             modulus, the allowable shear stress, the"
      PRINT "             allowable angle of twist per unit length, and"
      PRINT "             the ratio of diameters for a hollow shaft."
      PRINT "         (3) The input data and the computed results are"
      PRINT "             printed on the printer immediately after"
      PRINT "             data are entered. Make sure the printer is on";
      PRINT "             before entering data."
'
' Input problem I.D.
220   PRINT "Enter problem designation:"
      INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
      PRINT "Select the system of units:"
      PRINT "        1 -- SI Units"
      PRINT "        2 -- US customary units"
      INPUT "-- 1 / 2"; U
      IF U = 2 GOTO 300
      UF$ = "N": UL1$ = "m": UL2$ = "mm"
      UM$ = "N-m": UP$ = "kW": US$ = "MPa": UG$ = "GPa"
      GOTO 400
300   UF$ = "lb": UL1$ = "ft": UL2$ = "in."
      UM$ = "lb-in.": UP$ = "hp.": US$ = "psi": UG$ = "ksi"
'
' Print problem I.D. and heading
400   LPRINT "THE SIZE OF A SOLID OR HOLLOW CIRCULAR SHAFT"
      LPRINT :   LPRINT "Solution to "; ID$
      LPRINT "____________________________________________________________"
'
' Enter the data of the shaft
      PRINT "Select the type of shaft:"
      PRINT "        1 -- Solid circular shaft"
      PRINT "        2 -- Hollow circular shaft"
      INPUT "-- 1 / 2"; SHAFT
      PRINT "Input data of the shaft:"
      PRINT "The power transmitted -- P ("; UP$; "),"; :   INPUT P
      PRINT "The angular velocity -- n (rpm)"; :   INPUT N
      PRINT "The shear modulus -- G ("; UG$; "),"; :   INPUT G
      PRINT "The allowable shear stress -- tau ("; US$; ")"; :   INPUT TAU
      PRINT "The allowable angle of twist -- phi (degree/"; UL1$; ")";
      INPUT PHI
      IF SHAFT = 1 THEN K = 0
      IF SHAFT = 2 THEN PRINT "The ratio of diameters di/do -- k";
      IF SHAFT = 2 THEN INPUT K
'
' Print the input data
```

Solution to Computer Program Assignment C12-2 Continued

```
        LPRINT :   LPRINT "The input data:":    LPRINT
        LPRINT "The power transmitted:"; TAB(35); "P =";
        LPRINT USING "#####.##"; P: :   LPRINT " "; UP$
        LPRINT "The angular velocity:"; TAB(35); "n =";
        LPRINT USING "########"; N: :    LPRINT " rpm"
        LPRINT "The shear modulus:"; TAB(35); "G =";
        LPRINT USING "########"; G: :    LPRINT " "; UG$
        LPRINT "The allowable shear stress:"; TAB(33); "tau =";
        LPRINT USING "#####.##"; TAU;
        LPRINT " "; US$
        LPRINT "The allowable angle of twist:"; TAB(33); "phi =";
        LPRINT USING "####.###"; PHI;
        LPRINT " (Deg./"; UL1$; ")"
        IF SHAFT = 2 THEN LPRINT "The ratio k = di/do:"; TAB(35); "k ="; USING
"####.###"; K
    '
    ' Compute the torque
        IF U = 1 THEN TAU = TAU * 1000000!
        IF U = 1 THEN T = 9550 * P / N
        IF U = 2 THEN T = 63000! * P / N
    '
    ' Compute the required diameter for strength
        D1 = (16 * T / PI / TAU / (1 - K ^ 4)) ^ (1 / 3)
        IF U = 1 THEN D1 = D1 * 1000
    '
    ' Compute the required diameter for stiffness
        PHI = PHI * DR
        IF U = 1 THEN L = 1
        IF U = 1 THEN G = G * 1E+09
        IF U = 1 THEN TAU = TAU * 1000000!
        IF U = 2 THEN L = 12
        IF U = 2 THEN G = G * 1000
        D2 = (32 * T * L / PI / G / (1 - K ^ 4) / PHI) ^ (.25)
        IF U = 1 THEN D2 = D2 * 1000
        D = D1
        IF D2 > D1 THEN D = D2
    '
        LPRINT "____________________________________________________"
        LPRINT :   LPRINT "The computed results:":    LPRINT
        IF SHAFT = 2 GOTO 1090
        LPRINT "The required diameter for strength:  d1 =";
        LPRINT USING "###.###"; D1;
        LPRINT " "; UL2$
        LPRINT "The required diameter for stiffness: d2 =";
        LPRINT USING "###.###"; D2;
        LPRINT " "; UL2$
        LPRINT "The required diameter for the shaft:  d =";
        LPRINT USING "###.###"; D;
        LPRINT " "; UL2$
        GOTO 1160
1090    LPRINT "The required outside diameter for strength:"
        LPRINT TAB(14); "do1 ="; USING "###.###"; D1;
        LPRINT " "; UL2$
        LPRINT "The required outside diameter for stiffness:"
        LPRINT TAB(14); "do2 ="; USING "###.###"; D2;
        LPRINT " "; UL2$
        LPRINT "The required diameters for the hollow shaft:"
        LPRINT TAB(14); "do  ="; USING "###.###"; D;
        LPRINT " "; UL2$;
        LPRINT ",  di ="; USING "###.###"; D * K; : LPRINT " "; UL2$
1160    LPRINT "____________________________________________________"
    END
```


C12-2 (a)

```
THE SIZE OF A SOLID OR HOLLOW CIRCULAR SHAFT

Solution to EXAMPLE 12-14
____________________________________________________

The input data:

The power transmitted:                P =  150.00 hp.
The angular velocity:                 n =     300 rpm
The shear modulus:                    G =   12000 ksi
The allowable shear stress:         tau = 8000.00 psi
The allowable angle of twist:       phi =   0.300 (Deg./ft)
____________________________________________________

The computed results:

The required diameter for strength:  d1 =  2.717 in.
The required diameter for stiffness: d2 =  2.798 in.
The required diameter for the shaft:  d =  2.798 in.
____________________________________________________
```

C12-2 (b)

```
THE SIZE OF A SOLID OR HOLLOW CIRCULAR SHAFT

Solution to Problem 12-45
____________________________________________________

The input data:

The power transmitted:                P =  186.00 kW
The angular velocity:                 n =     450 rpm
The shear modulus:                    G =      84 GPa
The allowable shear stress:         tau =   70.00 MPa
The allowable angle of twist:       phi =   1.000 (Deg./m)
____________________________________________________

The computed results:

The required diameter for strength:  d1 = 65.977 mm
The required diameter for stiffness: d2 = 72.366 mm
The required diameter for the shaft:  d = 72.366 mm
____________________________________________________
```

C12-2 (c)

```
THE SIZE OF A SOLID OR HOLLOW CIRCULAR SHAFT

Solution to Problem 12-46
____________________________________________________

The input data:

The power transmitted:                P =  100.00 hp.
The angular velocity:                 n =     250 rpm
The shear modulus:                    G =   12000 ksi
The allowable shear stress:         tau = 8000.00 psi
The allowable angle of twist:       phi =   0.240 (Deg./ft)
The ratio k = di/do:                  k =   0.800
____________________________________________________

The computed results:

The required outside diameter for strength:
             do1 =  3.006 in.
The required outside diameter for stiffness:
             do2 =  3.192 in.
The required diameters for the hollow shaft:
             do  =  3.192 in.,  di =  2.553 in.
____________________________________________________
```

Solution to Computer Program Assignment C13-1

```
' * C13-1 *   Shear and moment in a simple beam sujected to a uniform load
'
' Clear screen
      CLS
'
' Display the introductory remarks
      PRINT "Want to see the introductory remarks?"
      INPUT "-- Y / N "; RMK$
      IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100   PRINT "REMARKS: (1) This program can be used to compute the shear"
      PRINT "             force and the bending moment in a simple beam"
      PRINT "             subjected to a uniform load."
      PRINT "         (2) The input data include the span length and the"
      PRINT "             intensity of the uniform load."
      PRINT "         (3) The input data and the computed results are"
      PRINT "             printed on the printer immediately after"
      PRINT "             data are entered. Make sure the printer is on"
      PRINT "             before entering data."
'
' Input problem I.D.
200   PRINT "Enter problem designation:"
      INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
      PRINT "Select the system of units:"
      PRINT "       1 -- SI Units"
      PRINT "       2 -- US customary units"
      INPUT "--1 / 2"; U
      IF U = 2 GOTO 300
      UF$ = "N": UL$ = "m": UM$ = "N-m": UW$ = "N/m"
      GOTO 400
300   UF$ = "lb": UL$ = "ft": UM$ = "lb-ft": UW$ = "lb/ft"
'
' Print problem I.D.
400   LPRINT :   LPRINT "SHEAR AND MOMENT FOR A SIMPLE BEAM"
      LPRINT "SUBJECTED TO A UNIFORM LOAD"
      LPRINT :   LPRINT "Solution to "; ID$
'
' Enter the given data
      PRINT "Span length: -- L ("; UL$; ")"; :   INPUT L
      PRINT "Intensity of the uniform load: -- w ("; UW$; ")";
      INPUT W
'
' Print the input data
      LPRINT :   LPRINT "The input data are:"
      LPRINT TAB(3); "Span length:   L ="; USING "#####.##"; L;
      LPRINT " "; UL$
      LPRINT TAB(3); "Uniform load:  w ="; USING "#####.##"; W;
      LPRINT " "; UW$
'
' Compute and print the shear force and the bending moments
      LPRINT :  LPRINT "The shear and moment along the beam:"
      LPRINT "____________________________________________"
      LPRINT
      LPRINT "Point   x ("; UL$; ")      V ("; UF$; ")     M("; UM$; ")"
      LPRINT "____________________________________________"
      LPRINT :   LPRINT "  0"; USING "######.###"; 0;
      LPRINT USING "##########.#"; W * L / 2; 0
    FOR I = 1 TO 9
      X = I / 10 * L:  V = W * L / 2 - W * X:
      M = W * L * X / 2 - W * X * X / 2
```


Solution to Computer Program Assignment C13-1 Continued

```
      LPRINT USING "###"; I; :   LPRINT USING "######.###"; X;
      LPRINT USING "##########.#"; V; M
    NEXT I
      LPRINT " 10"; USING "######.###"; L;
      LPRINT USING "##########.#"; -W * L / 2; 0
      LPRINT "____________________________________________"
  END
```

C13-1 (a)

SHEAR AND MOMENT FOR A SIMPLE BEAM
SUBJECTED TO A UNIFORM LOAD

Solution to EXAMPLE 13-7

The input data are:
  Span length: L =   8.00 ft
  Uniform load: w = 3000.00 lb/ft

The shear and moment along the beam:

Point	x (ft)	V (lb)	M(lb-ft)
0	0.000	12000.0	0.0
1	0.800	9600.0	8640.0
2	1.600	7200.0	15360.0
3	2.400	4800.0	20160.0
4	3.200	2400.0	23040.0
5	4.000	0.0	24000.0
6	4.800	-2400.0	23040.0
7	5.600	-4800.0	20160.0
8	6.400	-7200.0	15360.0
9	7.200	-9600.0	8640.0
10	8.000	-12000.0	0.0

Solution to Computer Program Assignment C13-2

```
'
' * C13-2 *   Shear and moment in a simple beams sujected to concen. loads
'
' Clear screen
     CLS
'
' Display the introductory remarks
     PRINT "Want to see the introductory remarks?"
     INPUT "-- Y / N "; RMK$
     IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100  PRINT "REMARKS: (1) This program can be used to compute the shear"
     PRINT "             force and the bending moment in a simple beam"
     PRINT "             subjected to concentrated loads."
     PRINT "         (2) The input data include the span length, the"
     PRINT "             magnitude and location of each concentrated load."
     PRINT "         (3) The input data and the computed results are"
     PRINT "             printed on the printer immediately after"
     PRINT "             data are entered. Make sure your printer is on"
     PRINT "             before entering data."
'
' Input problem I.D.
200  PRINT "Enter problem designation:"
     INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
     PRINT "Select the system of units:"
     PRINT "      1 -- SI Units"
     PRINT "      2 -- US customary units"
     INPUT "-- 1 / 2"; U
     IF U = 2 GOTO 300
     UF$ = "N": UL$ = "m": UM$ = "N-m"
     GOTO 400
300  UF$ = "lb": UL$ = "ft": UM$ = "lb-ft"
'
' Print problem I.D.
400  LPRINT :   LPRINT "SHEAR AND MOMENT IN A SIMPLE BEAM"
     LPRINT "SUBJECTED TO CONCENTRATED LOADS"
     LPRINT :   LPRINT "Solution to "; ID$
'
' Enter the given data
     PRINT "Span length: -- L ("; UL$; ")"; :   INPUT L
     PRINT "No. of concentrated loads: -- N"; :   INPUT N
   FOR I = 1 TO N
     PRINT "Mag. & location of conc. load: -- P"; USING "#"; I;
     PRINT " ("; UF$; "),";
     PRINT " a"; USING "#"; I; :   PRINT " ("; UL$; ")";
     INPUT P(I), A(I)
   NEXT I
'
' Print the input data
     LPRINT :   LPRINT "The input data are:"
     LPRINT TAB(3); "Span length:   L ="; USING "#####.##"; L;
     LPRINT " "; UL$
     LPRINT TAB(3); "Tle concentrated loads:"
   FOR I = 1 TO N
     LPRINT TAB(3); "P"; USING "#"; I;
     LPRINT " ="; USING "#######.#"; P(I); :   LPRINT " "; UF$;
     LPRINT " at a"; USING "#"; I;
     LPRINT " ="; USING "####.###"; A(I); :   LPRINT " "; UL$
   NEXT I
'
' Compute the reactions
```


Solution to Computer Program Assignment C13-2 Continued

```
   SP = 0:    SM = 0
 FOR I = 1 TO N
   SP = SP + P(I):    SM = SM + P(I) * A(I)
 NEXT I
   RB = SM / L:    RA = SP - RB
'
' Compute the shear forces
   V(1) = RA
 FOR I = 2 TO N + 1
   V(I) = V(I - 1) - P(I - 1)
 NEXT I
'
' Compute the bending moments
   M(1) = V(1) * A(1)
 FOR I = 2 TO N
   M(I) = M(I - 1) + V(I) * (A(I) - A(I - 1))
 NEXT I
'
' Print shear forces
   LPRINT :   LPRINT "The shear forces are:"
   LPRINT "______________________________"
   LPRINT :   LPRINT " Region         V ("; UF$; ")"
   LPRINT "______________________________":    LPRINT
 FOR I = 1 TO N + 1
   IF I = 1 THEN LPRINT "   A-"; USING "#"; I;
   LPRINT USING "#########.##"; V(I);
   IF I = 1 THEN LPRINT " "; UF$
   IF I > 1 AND I < N + 1 THEN LPRINT USING "####"; I - 1;
   IF I > 1 AND I < N + 1 THEN LPRINT "-"; USING "#"; I;
   IF I > 1 AND I < N + 1 THEN LPRINT USING "#########.##"; V(I);
   IF I > 1 AND I < N + 1 THEN LPRINT " "; UF$
   IF I = N + 1 THEN LPRINT USING "####"; I - 1;
   IF I = N + 1 THEN LPRINT "-B"; USING "#########.##"; V(I);
   IF I = N + 1 THEN LPRINT " "; UF$
 NEXT I
   LPRINT "______________________________": LPRINT
'
' Print bending moment
   LPRINT :   LPRINT "The bending moments are:"
   LPRINT "______________________________"
   LPRINT :   LPRINT " point          M ("; UM$; ")"
   LPRINT "______________________________":    LPRINT
   LPRINT "   A"; TAB(15); "0.00"; " "; UM$
 FOR I = 1 TO N
   LPRINT USING "####"; I; :   LPRINT USING "###########.##"; M(I);
   LPRINT " "; UM$
 NEXT I
   LPRINT "   B"; TAB(15); "0.00"; " "; UM$
   LPRINT "______________________________"
END
```

C13-2 (a)

```
SHEAR AND MOMENT IN A SIMPLE BEAM
SUBJECTED TO CONCENTRATED LOADS

Solution to EXAMPLE 13-6

The input data are:
  Span length:   L =     4.00 m
  Tle concentrated loads:
  P1 =  40000.0 N at a1 =    3.000 m

The shear forces are:
```

Region	V (N)
A-1	10000.00 N
1-B	-30000.00 N

The bending moments are:

point	M (N-m)
A	0.00 N-m
1	30000.00 N-m
B	0.00 N-m

C13-2 (b)

```
SHEAR AND MOMENT IN A SIMPLE BEAM
SUBJECTED TO CONCENTRATED LOADS

Solution to Problem 13-44

The input data are:
  Span length:   L =     6.00 ft
  Tle concentrated loads:
  P1 =    200.0 lb at a1 =    1.000 ft
  P2 =    400.0 lb at a2 =    3.000 ft
  P3 =    200.0 lb at a3 =    5.000 ft

The shear forces are:
```

Region	V (lb)
A-1	400.00 lb
1-2	200.00 lb
2-3	-200.00 lb
3-B	-400.00 lb

The bending moments are:

point	M (lb-ft)
A	0.00 lb-ft
1	400.00 lb-ft
2	800.00 lb-ft
3	400.00 lb-ft
B	0.00 lb-ft

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Solution to Computer Program Assignment C14-1

```
' * C14-1 *   Maximum stresses in a simple or a cantilever beam
'
' Clear screen and compute pi
      CLS :   PI = 4 * ATN(1)
'
' Display the introductory remarks
      PRINT "Want to see the introductory remarks?"
      INPUT "-- Y / N "; RMK$
      IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 210
100   PRINT "REMARKS: (1) This program can be used to compute the maximum"
      PRINT "             normal and shear stresses in a simple OR"
      PRINT "             cantilever beam."
      PRINT "         (2) The input data for the beam include the span"
      PRINT "             length, the concentrated and uniform loads."
      PRINT "         (3) The user has the option to enter data for either"
      PRINT "             a rectangular section, a circular section, a"
      PRINT "             W-shape, or an S-shape."
      PRINT "         (4) The input data and the computed results are"
      PRINT "             printed on the printer immediately after"
      PRINT "             data are entered. Make sure your printer is on"
      PRINT "             before entering data."
'
' Input problem I.D.
210   PRINT "Enter problem designation:"
      INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
      PRINT "Select the system of units:"
      PRINT "      1 -- SI units"
      PRINT "      2 -- US customary units"
      INPUT "-- 1 / 2"; U
      IF U = 2 GOTO 330
      UF$ = "N": UL1$ = "m": UL2$ = "mm": UM$ = "kN-m"
      UV$ = "kN": UW$ = "N/m": US$ = "MPa": USM$ = "m^3"
      GOTO 360
330   UF$ = "lb": UL1$ = "ft": UL2$ = "in.": UM$ = "kip-in"
      UV$ = "kip": UW$ = "lb/ft": US$ = "ksi": USM$ = "in^3"
'
360   PRINT "Type of beam:"
      PRINT "      1 -- simple beam"
      PRINT "      2 -- cantilever beam"
      PRINT "-- 1 / 2"; :   INPUT TB
'
' Print problem I.D.
      IF TB = 1 THEN LPRINT "MAXIMUM STRESSES IN A SIMPLE BEAM"
      IF TB = 2 THEN LPRINT "MAXIMUM STRESSES IN A CANTILEVER BEAM"
      LPRINT :   LPRINT "Solution to "; ID$
'
' Enter data for the beam
      PRINT "Span length:":    PRINT "-- L ("; UL1$; ")"; :   INPUT L
      IF TB = 2 GOTO 520
      PRINT "Concentrated loads:":    PRINT "-- P1 ("; UF$; ")";
      INPUT P1
      PRINT "-- P2 ("; UF$; "), a ("; UL1$; ")"; :   INPUT P2, a
      GOTO 530
520   PRINT "The concentrated load:"
      PRINT "-- P ("; UF$; "), a ("; UL1$; ")"; :   INPUT P, a
530   PRINT "Uniform load:":   PRINT "-- w ("; UW$; ")"; :   INPUT W
'
' Print the input data for the beam
      LPRINT :   LPRINT "The input data:"
```

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Solution to Computer Program Assignment C14-1 Continued

```
        LPRINT TAB(3); "Span length:"; TAB(28); "L =";
        LPRINT USING "######.###"; L; :   LPRINT " "; UL1$
        IF TB = 2 GOTO 630
        LPRINT TAB(3); "The concentrated loads:"; TAB(27); "P1 =";
        LPRINT USING "########.#"; P1; :   LPRINT " "; UF$;
        LPRINT TAB(27); "P2 ="; USING "########.#"; P2; :   LPRINT " "; UF$
        GOTO 650
630     LPRINT TAB(3); "The concentrated loads:"; TAB(28); "P =";
        LPRINT USING "########.#"; P; :   LPRINT " "; UF$;
650     LPRINT TAB(28); "a ="; USING "######.###"; a; :   LPRINT " "; UL1$
        LPRINT TAB(3); "Uniform load:"; TAB(28); "w =";
        LPRINT USING "#######.##"; W; :   LPRINT " "; UW$
   '
   ' Maximum shear and moment for simple beam
        IF TB = 2 GOTO 740
        VMAX = P1 / 2 + P2 + W * L / 2
        MMAX = P1 * L / 4 + P2 * a + W * L * L / 8
        GOTO 760
   '
   ' Maximum shear and moment for cantilever beam
740     VMAX = P + W * L:  MMAX = P * a + P2 * a + W * L * L / 2
   '
760     PRINT "Type of section:"
        PRINT "        1 -- rectangular section"
        PRINT "        2 -- circular section"
        PRINT "        3 -- W-shape or S-shape"
        PRINT "-- 1 / 2 / 3"; :   INPUT TS
        IF TS = 2 GOTO 940
        IF TS = 3 GOTO 1010
   '
   ' Rectangular section
        PRINT "For rectangular section:":   PRINT "-- b ("; UL2$; "), ";
        PRINT "h ("; UL2$; ")"; :   INPUT B, H
        LPRINT TAB(3); "For rectangular section:";
        LPRINT TAB(28); "b ="; USING "######.###"; B; :   LPRINT " "; UL2$
        LPRINT TAB(28); "h ="; USING "######.###"; H; :   LPRINT " "; UL2$
        S = B * H * H / 6: AREA = B * H
        GOTO 1140
   '
   ' Circular section
940     PRINT "For circular section:":   PRINT "-- d ("; UL2$; ")";
        INPUT D
        LPRINT TAB(3); "For circular section:";
        LPRINT TAB(28); "d ="; USING "######.###"; D; :   LPRINT " "; UL2$
        S = PI * D ^ 3 / 32: AREA = PI * D * D / 4
        GOTO 1140
   '
   ' W-shape or S-shape section
1010    PRINT "For W-shape or S-shape section:"
        PRINT "-- designation"; :   INPUT SD$
        PRINT "-- d ("; UL2$; "), tw ("; UL2$; "), Sx ("; USM$; ")";
        INPUT D, TW, S
        LPRINT TAB(3); "For "; SD$; " section:";
        LPRINT TAB(28); "d ="; USING "######.###"; D; :   LPRINT " "; UL2$
        LPRINT TAB(27); "tw ="; USING "######.###"; TW; :   LPRINT " "; UL2$
        IF U = 1 THEN LPRINT TAB(27); "Sx ="; USING "######.###"; 1000 * S;
        IF U = 1 THEN LPRINT " E-3 "; USM$
        IF U = 2 THEN LPRINT TAB(27); "Sx ="; USING "######.###"; S;
        IF U = 2 THEN LPRINT " "; USM$
        AREA = D * TW
        IF U = 1 THEN S = S * 1E+09
```


Solution to Computer Program Assignment C14-1 Continued

```
   '
1140    IF TS = 1 THEN K = 1.5
        IF TS = 2 THEN K = 4 / 3
        IF TS = 3 THEN K = 1
        IF TB = 1 THEN B$ = "simple beam"
        IF TB = 2 THEN B$ = "cantilever beam"
   '
   ' Compute the maximum normal and shear stresses
        SIGMA = MMAX / S:   TAU = K * VMAX / AREA
   '
   ' Unit conversion
        IF U = 1 GOTO 1270
        MMAX = MMAX * 12 / 1000:  VMAX = VMAX / 1000
        SIGMA = SIGMA * 12 / 1000:  TAU = TAU / 1000:   GOTO 1310
1270    MMAX = MMAX / 1000:  VMAX = VMAX / 1000
        SIGMA = SIGMA * 1000
   '
   ' Print the computed results
1310    LPRINT :   LPRINT "The computed results for the "; B$; ":"
        LPRINT TAB(3); "Max. moment:"; TAB(28); "M =";
        LPRINT USING "######.###"; MMAX; :   LPRINT " "; UM$
        LPRINT TAB(3); "Max. shear force:"; TAB(28); "V =";
        LPRINT USING "######.###"; VMAX; :   LPRINT " "; UV$
        LPRINT TAB(3); "Max. normal stress:";
        LPRINT TAB(24); "sigma ="; USING "######.###"; SIGMA;
        LPRINT " "; US$
        LPRINT TAB(3); "Max. shear stress:";
        LPRINT TAB(26); "tau ="; USING "######.###"; TAU;
        LPRINT " "; US$
   END
```

C14-1 (a)

```
MAXIMUM STRESSES IN A SIMPLE BEAM

Solution to Problem 14-1

The input data:
  Span length:                      L =    10.000 ft
  The concentrated loads:          P1 =     800.0 lb
                                   P2 =       0.0 lb
                                    a =     0.000 ft
  Uniform load:                     w =      0.00 lb/ft
  For rectangular section:          b =     4.000 in.
                                    h =     6.000 in.

The computed results for the simple beam:
  Max. moment:                      M =    24.000 kip-in
  Max. shear force:                 V =     0.400 kip
  Max. normal stress:           sigma =     1.000 ksi
  Max. shear stress:              tau =     0.025 ksi
```

C14-1 (b)

```
MAXIMUM STRESSES IN A SIMPLE BEAM

Solution to Problem 14-5

The input data:
  Span length:                      L =    24.000 ft
  The concentrated loads:          P1 =       0.0 lb
                                   P2 =   20000.0 lb
                                    a =     8.000 ft
  Uniform load:                     w =      0.00 lb/ft
  For W16x50 section:               d =    16.260 in.
                                   tw =     0.380 in.
                                   Sx =    81.000 in^3

The computed results for the simple beam:
  Max. moment:                      M =  1920.000 kip-in
  Max. shear force:                 V =    20.000 kip
  Max. normal stress:           sigma =    23.704 ksi
  Max. shear stress:              tau =     3.237 ksi
```

C14-1 (c)

```
MAXIMUM STRESSES IN A SIMPLE BEAM

Solution to Problem 14-6

The input data:
  Span length:                      L =     8.000 m
  The concentrated loads:          P1 =       0.0 N
                                   P2 =   90000.0 N
                                    a =     2.667 m
  Uniform load:                     w =      0.00 N/m
  For W410x0.73 section:            d =   413.000 mm
                                   tw =     9.650 mm
                                   Sx =     1.330 E-3 m^3

The computed results for the simple beam:
  Max. moment:                      M =   240.030 kN-m
  Max. shear force:                 V =    90.000 kN
  Max. normal stress:           sigma =   180.474 MPa
  Max. shear stress:              tau =    22.582 MPa
```


C14-1 (d)

```
MAXIMUM STRESSES IN A SIMPLE BEAM

Solution to Problem 14-8

The input data:
  Span length:                      L =    15.000 ft
  The concentrated loads:          P1 =       0.0 lb
                                   P2 =       0.0 lb
                                    a =     0.000 ft
  Uniform load:                     w =    200.00 lb/ft
  For circular section:             d =    10.000 in.

The computed results for the simple beam:
  Max. moment:                      M =    67.500 kip-in
  Max. shear force:                 V =     1.500 kip
  Max. normal stress:           sigma =     0.688 ksi
  Max. shear stress:              tau =     0.025 ksi
```

C14-1 (e)

```
MAXIMUM STRESSES IN A CANTILEVER BEAM

Solution to EXAMPLE 14-2

The input data:
  Span length:                      L =     4.000 m
  The concentrated loads:           P =    2000.0 N
                                    a =     4.000 m
  Uniform load:                     w =      0.00 N/m
  For circular section:             d =   100.000 mm

The computed results for the cantilever beam:
  Max. moment:                      M =     8.000 kN-m
  Max. shear force:                 V =     2.000 kN
  Max. normal stress:           sigma =    81.487 MPa
  Max. shear stress:              tau =     0.340 MPa
```

C14-1(f)

```
MAXIMUM STRESSES IN A CANTILEVER BEAM

Solution to Problem 14-3

The input data:
  Span length:                      L =     3.000 m
  The concentrated loads:           P =    5000.0 N
                                    a =     3.000 m
  Uniform load:                     w =      0.00 N/m
  For circular section:             d =   100.000 mm

The computed results for the cantilever beam:
  Max. moment:                      M =    15.000 kN-m
  Max. shear force:                 V =     5.000 kN
  Max. normal stress:           sigma =   152.789 MPa
  Max. shear stress:              tau =     0.849 MPa
```

Solution to Computer Program Assignment C14-2

```
' * C14-2 *     Allowable concentrated or uniform load
'               for a simple or a cantilever beam
'
' Clear screen and compute pi
      CLS :   PI = 4 * ATN(1)
'
' Display the introductory remarks
      PRINT "Want to see the introductory remarks?"
      INPUT "-- Y / N "; RMK$
      IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 210
100   PRINT "REMARKS: (1) This program can be used to compute the"
      PRINT "             allowable concentrated or uniform load for a"
      PRINT "             simple or a cantilever beam."
      PRINT "         (2) The input data for the beam include the span"
      PRINT "             length, the allowable flexural and shear"
      PRINT "             stresses."
      PRINT "         (3) The user has the option to enter data for either"
      PRINT "             a rectangular section, a circular section, a"
      PRINT "             W-shape, or an S-shape."
      PRINT "         (4) The input data and the computed results are"
      PRINT "             printed on the line printer immediately after"
      PRINT "             data are entered.  Make sure your printer is on"
      PRINT "             before entering data."
'
' Input problem I.D.
210   PRINT "Enter problem designation:"
      INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
      PRINT "Select either one of the following systems of units:"
      PRINT "       1 -- SI units"
      PRINT "       2 -- US customary units"
      INPUT "-- 1 / 2"; U
      IF U = 2 GOTO 330
      UF$ = "kN":     UL1$ = "m":     UL2$ = "mm"
      UW$ = "kN/m":     US$ = "MPa":     USM$ = "m^3"
      GOTO 370
330   UF$ = "kip":     UL1$ = "ft":     UL2$ = "in."
      UW$ = "kip/ft":     US$ = "ksi":     USM$ = "in^3"
'
      PRINT "Type of beam:"
370   PRINT "       1 -- simple beam"
      PRINT "       2 -- cantilever beam"
      PRINT "-- 1 / 2"; :   INPUT TB
'
      PRINT "Type of loading:"
      PRINT "       1 -- concentrated load"
      PRINT "       2 -- uniform load"
      PRINT "-- 1 / 2"; :   INPUT TL
'
' Print problem I.D.
      IF TB = 2 GOTO 510
      IF TL = 1 THEN LPRINT "ALLOWABLE CONCENTRATED LOAD FOR A SIMPLE BEAM"
      IF TL = 2 THEN LPRINT "ALLOWABLE UNIFORM LOAD FOR A SIMPLE BEAM"
      GOTO 540
510   IF TL = 1 THEN LPRINT "ALLOWABLE CONCENTRATED LOAD FOR A CANTILEVER";
      IF TL = 1 THEN LPRINT " BEAM"
      IF TL = 2 THEN LPRINT "ALLOWABLE UNIFORM LOAD FOR A CANTILEVER BEAM"
'
540   LPRINT :   LPRINT "Solution to "; ID$
```


Solution to Computer Program Assignment C14-2 Continued

```
'
' Enter data for the beam
      PRINT "Span length:":    PRINT "-- L ("; UL1$; ")"; :    INPUT L
      PRINT "Allowable flexural stress:":    PRINT "-- sigma ("; US$; ")"; :
INPUT SIGMA
      PRINT "Allowable shear stress:":    PRINT "-- tau ("; US$; ")";
      INPUT TAU
'
' Print the input data for the beam
      LPRINT :   LPRINT "The input data:":    LPRINT
      LPRINT TAB(3); "Span length:"; TAB(34); "L =";
      LPRINT USING "######.###"; L; :   LPRINT " "; UL1$
      LPRINT TAB(3); "Allowable flexural stress:"; TAB(30); "sigma =";
      LPRINT USING "######.###"; SIGMA; :   LPRINT " "; US$
      LPRINT TAB(3); "Allowable shear stress:"; TAB(32); "tau =";
      LPRINT USING "######.###"; TAU; :   LPRINT " "; US$
'
      PRINT "Type of section:"
      PRINT "      1 -- rectangular section"
      PRINT "      2 -- circular section"
      PRINT "      3 -- W-shape or S-shape"
      PRINT "-- 1 / 2 / 3"; :   INPUT TS
      IF TS = 2 GOTO 850
      IF TS = 3 GOTO 920
'
' Rectangular section
      PRINT "For rectangular section:":    PRINT "-- b ("; UL2$; "), ";
      PRINT "h ("; UL2$; ")"; :   INPUT B, H
      LPRINT TAB(3); "For rectangular section:";
      LPRINT TAB(34); "b ="; USING "######.###"; B; :   LPRINT " "; UL2$
      LPRINT TAB(34); "h ="; USING "######.###"; H; :   LPRINT " "; UL2$
      S = B * H * H / 6: A = B * H
      GOTO 1020
'
' Circular section
850   PRINT "For circular section:":    PRINT "-- d ("; UL2$; ")";
      INPUT D
      LPRINT TAB(3); "For circular section:";
      LPRINT TAB(34); "d ="; USING "######.###"; D; :   LPRINT " "; UL2$
      S = PI * D ^ 3 / 32: A = PI * D * D / 4
      GOTO 1020
'
' W-shape or S-shape section
920   PRINT "For W-shape or S-shape section:"
      PRINT "-- designation"; :   INPUT SD$
      PRINT "-- d ("; UL2$; "), tw ("; UL2$; "), Sx ("; USM$; ")";
      INPUT D, TW, S
      LPRINT TAB(3); "For "; SD$; " section:";
      LPRINT TAB(34); "d ="; USING "######.###"; D; :   LPRINT " "; UL2$
      LPRINT TAB(33); "tw ="; USING "######.###"; TW; :   LPRINT " "; UL2$
      LPRINT TAB(33); "Sx ="; USING "######.###"; S; :   LPRINT " "; USM$
      A = D * TW
      IF U = 1 THEN S = S * 1E+09
'
1020  IF TS = 1 THEN K = 1.5
      IF TS = 2 THEN K = 4 / 3
      IF TS = 3 THEN K = 1
      IF TB = 1 THEN B$ = "simple beam"
      IF TB = 2 THEN B$ = "cantilever beam"
      IF TL = 1 AND TB = 1 GOTO 1140
```

Solution to Computer Program Assignment C14-2 Continued

```
     IF TL = 1 AND TB = 2 GOTO 1180
     IF TL = 2 AND TB = 1 GOTO 1220
     IF TL = 2 AND TB = 2 GOTO 1260
'
' Allowable concentrated load for a simple beam
1140 P1 = 4 * S * SIGMA / L: P2 = 2 * A * TAU / K
     GOTO 1290
'
' Allowable concentrated load for a cantilever beam
1180 P1 = S * SIGMA / L: P2 = A * TAU / K
     GOTO 1290
'
' Allowable uniform load for a simple beam
1220 W1 = 8 * S * SIGMA / L / L: W2 = 2 * A * TAU / K / L
     GOTO 1290
'
' Allowable uniform load for a cantilever beam
1260 W1 = 2 * S * SIGMA / L / L: W2 = A * TAU / K / L
'
' Unit conversion
1290 IF U = 1 GOTO 1310
     P1 = P1 / 12:  W1 = W1 / 12:  GOTO 1340
1310 P1 = P1 / 1000000!:  P2 = P2 / 1000:  W1 = W1 / 1000000!
     W2 = W2 / 1000
'
' Print results
1340 LPRINT :  LPRINT "The computed results:":   LPRINT
     IF TL = 2 GOTO 1520
'
' Allowable concentrated load
     LPRINT TAB(3); "The maximum concentrated load based on"
     LPRINT TAB(5); "allowable flexural stress:";
     LPRINT TAB(33); "P1 ="; USING "######.###"; P1;
     LPRINT " "; UF$
     LPRINT TAB(3); "The maximum concentrated load based on"
     LPRINT TAB(5); "allowable shear stress:";
     LPRINT TAB(33); "P2 ="; USING "######.###"; P2; :   LPRINT " "; UF$
     P = P1
     IF P2 < P1 THEN P = P2
     LPRINT TAB(3); "The maximum concentrated load"
     LPRINT TAB(5); "for the "; B$; ":";
     LPRINT TAB(33); "P  ="; USING "######.###"; P; :   LPRINT " "; UF$
     GOTO 1630
'
' Allowable uniform load
1520 LPRINT TAB(3); "The maximum uniform load based on"
     LPRINT TAB(5); "allowable flexural stress:";
     LPRINT TAB(33); "w1 ="; USING "######.###"; W1; :   LPRINT " "; UW$
     LPRINT TAB(3); "The maximum uniform load based on"
     LPRINT TAB(5); "allowable shear stress:";
     LPRINT TAB(33); "w2 ="; USING "######.###"; W2; :   LPRINT " "; UW$
     W = W1
     IF W2 < W1 THEN W = W2
     LPRINT TAB(3); "The maximum uniform load"
     LPRINT TAB(5); "for the "; B$; ":";
     LPRINT TAB(33); "w  ="; USING "######.###"; W; :   LPRINT " "; UW$
1630 END
```


C14-2 (a)

```
ALLOWABLE CONCENTRATED LOAD FOR A SIMPLE BEAM

Solution to Problem 14-19

The input data:

   Span length:                         L =     10.000 ft
   Allowable flexural stress: sigma =     33.000 ksi
   Allowable shear stress:           tau =     14.500 ksi
   For W14x82 section:                 d =     14.310 in.
                                      tw =      0.510 in.
                                      Sx =    123.000 in^3

The computed results:

   The maximum concentrated load based on
     allowable flexural stress:  P1 =    135.300 kip
   The maximum concentrated load based on
     allowable shear stress:     P2 =    211.645 kip
   The maximum concentrated load
     for the simple beam:        P  =    135.300 kip
```

C14-2 (b)

```
ALLOWABLE UNIFORM LOAD FOR A CANTILEVER BEAM

Solution to Problem 14-20

The input data:

   Span length:                         L =      8.000 ft
   Allowable flexural stress: sigma =     24.000 ksi
   Allowable shear stress:           tau =     14.500 ksi
   For S15x50 section:                 d =     15.000 in.
                                      tw =      0.550 in.
                                      Sx =     64.800 in^3

The computed results:

   The maximum uniform load based on
     allowable flexural stress:  w1 =      4.050 kip/ft
   The maximum uniform load based on
     allowable shear stress:     w2 =     14.953 kip/ft
   The maximum uniform load
     for the cantilever beam:    w  =      4.050 kip/ft
```

C14-2 (c)

```
ALLOWABLE CONCENTRATED LOAD FOR A CANTILEVER BEAM

Solution to Problem 14-24

The input data:

  Span length:                      L =      1.000 m
  Allowable flexural stress: sigma =     10.000 MPa
  Allowable shear stress:         tau =      0.800 MPa
  For rectangular section:          b =    100.000 mm
                                    h =    150.000 mm

The computed results:

  The maximum concentrated load based on
    allowable flexural stress:  P1 =      3.750 kN
  The maximum concentrated load based on
    allowable shear stress:     P2 =      8.000 kN
  The maximum concentrated load
    for the cantilever beam:    P  =      3.750 kN
```

C14-2 (d)

```
ALLOWABLE UNIFORM LOAD FOR A SIMPLE BEAM

Solution to Problem 14-25

The input data:

  Span length:                      L =     10.000 ft
  Allowable flexural stress: sigma =      1.900 ksi
  Allowable shear stress:         tau =      0.145 ksi
  For rectangular section:          b =      3.000 in.
                                    h =     12.000 in.

The computed results:

  The maximum uniform load based on
    allowable flexural stress:  w1 =      0.912 kip/ft
  The maximum uniform load based on
    allowable shear stress:     w2 =      0.696 kip/ft
  The maximum uniform load
    for the simple beam:        w  =      0.696 kip/ft
```


Solution to Computer Program Assignment C15-1

```
' * C15-1 *    Size for a simply supported timber beam
'              subjected to uniform load
'
' Clear screen and compute pi
     CLS :   PI = 4 * ATN(1)
'
' Display the introductory remarks
     PRINT "Want to see the introductory remarks?"
     INPUT "-- Y / N "; RMK$
     IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 190
100  PRINT "REMARKS: (1) This program can be used to select rectangular"
     PRINT "             timber sections for a simple span subjected to"
     PRINT "             a uniform load."
     PRINT "         (2) The input data for the beam include the span"
     PRINT "             length, the uniform load, and the allowable"
     PRINT "             normal and shear stresses."
     PRINT "         (3) The input data and the computed results are"
     PRINT "             printed on the printer immediately after"
     PRINT "             data are entered. Make sure your printer is on"
     PRINT "             before entering data."
'
' Input problem I.D.
190  PRINT "Enter problem designation:"
     INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
     PRINT "Select the system of units:"
     PRINT "       1 -- SI units"
     PRINT "       2 -- US customary units"
     INPUT "-- 1 / 2"; U
     IF U = 2 GOTO 310
     UF$ = "N": UL1$ = "m": UL2$ = "mm": UM$ = "kN-m"
     UV$ = "kN": UW$ = "N/m": US$ = "kPa": USM$ = "m^3"
     GOTO 340
310  UF$ = "lb": UL1$ = "ft": UL2$ = "in.": UM$ = "kip-in"
     UV$ = "kip": UW$ = "lb/ft": US$ = "psi": USM$ = "in^3"
'
340 ' Print problem I.D.
     LPRINT "SIZE FOR A SIMPLY SUPPORTED TIMBER BEAM"
     LPRINT "SUBJECTED TO A UNIFORM LOAD"
     LPRINT :   LPRINT "Solution to "; ID$
'
' Enter data for the beam
     PRINT "Span length:":    PRINT "-- L ("; UL1$; ")"; :   INPUT L
     PRINT "Uniform load:":   PRINT "-- w ("; UW$; ")"; :   INPUT W
     PRINT "Allowable normal stress:"
     PRINT "-- sigma ("; US$; ")"; :   INPUT SIGMA
     PRINT "Allowable shear stress:"
     PRINT "-- tau ("; US$; ")"; :   INPUT TAU
     PRINT "The maximum aspect ratio r = h/b:"
     PRINT "-- r"; :    INPUT R
'
' Print the input data for the beam
     LPRINT :   LPRINT "The input data:"
     LPRINT TAB(3); "The simple span length:";
     LPRINT TAB(32); "L ="; USING "######.###"; L; :   LPRINT " "; UL1$
     LPRINT TAB(3); "Uniform load:"; TAB(32); "w =";
     LPRINT USING "#######.##"; W; :   LPRINT " "; UW$
     LPRINT TAB(3); "Allow. normal stress:"; TAB(28); "sigma =";
     LPRINT USING "#######.##"; SIGMA; :   LPRINT " "; US$
```

Solution to Computer Program Assignment C15-1 Continued

```
      LPRINT TAB(3); "Allow. shear stress:"; TAB(30); "tau =";
      LPRINT USING "#######.##"; TAU; :    LPRINT " "; US$
      LPRINT TAB(3); "Max. aspect ratio, h/b:"; TAB(32); "r =";
      LPRINT USING "#######.##"; R
   '
   ' Convert SI units to US units
       IF U = 2 GOTO 400
       L = L / .3048:  W = W / 14.59:  SIGMA = SIGMA / 6.895
       TAU = TAU / 6.895
   '
   ' Specify the initial width and initial height
400    BB(1) = 2:    BB(2) = 3:    BB(3) = 4:    BB(4) = 6:    BB(5) = 8
       BB(6) = 10:    BB(7) = 12
       HH(1) = 4:    HH(2) = 4:    HH(3) = 4:    HH(4) = 6:    HH(5) = 8
       HH(6) = 10:    HH(7) = 12
   '
   ' Print heading
       LPRINT :   LPRINT "The selected sections are:"
       LPRINT "________________________________________________________";
       LPRINT "__________"
       LPRINT
       LPRINT "Nominal   Dressed   Load plus   S         S       A        A"
       LPRINT " Size      Size     Beam wt.  provided  req'd  provided";
       LPRINT "  req'd"
       IF U = 1 GOTO 500
       LPRINT "in.xin.   in.xin.     lb/ft     in^3      in^3     in^2  ";
       LPRINT "   in^2"
       GOTO 550
500    LPRINT "mm x mm   mm x mm     kN/m    m^3/1000 m^3/1000 m^2/1000 ";
       LPRINT "m^2/1000"
550    LPRINT "________________________________________________________";
       LPRINT "__________"
       LPRINT
   '
   ' Initialize count
       K = 0
   '
   ' Check section for strength criteria
     FOR I = 1 TO 7
       B = BB(I):     BD = B - .5
       HO = HH(I)
      FOR J = 1 TO 7
       H = HO + 2 * (J - 1): HD = H - .5
   '
   ' Keep the aspect ratio h/b to the specified r
       IF H / B > R GOTO 1100
   '
   ' Compute the section modulus and the area using dressed size
       S = BD * HD ^ 2 / 6: A = BD * HD
   '
   ' Compute the weight of beam and the total load
       WEIT = BD * HD * 40 / 144: WT = W + WEIT
   '
   ' Compute the maximum moment and the maximum shear
       MMAX = WT * L ^ 2 / 8: VMAX = WT * L / 2
   '
   ' Compute the req'd section modulus and the req'd area
       SR = MMAX / SIGMA * 12: AR = 1.5 * VMAX / TAU
   '
```


Solution to Computer Program Assignment C15-1 Continued

```
   ' Weigh the strength criteria
        IF S < SR OR A < AR GOTO 1070
        K = K + 1
        AA(K) = A:     BA(K) = B:     HA(K) = H
   '
   ' Print the sections selected
        IF U = 1 GOTO 600
   ' US units
        LPRINT USING "###"; B; :     LPRINT "x"; USING "##"; H;
        LPRINT USING "#####.#"; BD; :     LPRINT "x"; USING "##.#"; HD;
        LPRINT USING "#######.#"; WT;
        LPRINT USING "#######.#"; S; :    LPRINT USING "#######.#"; SR;
        LPRINT USING "#######.#"; A; :    LPRINT USING "#######.#"; AR
        GOTO 1100
   ' SI units
600     GOSUB 2000
        BD = BD * 25.4: HD = HD * 25.4: WT = WT * .01459
        S = S * .01639: SR = SR * .01639: A = A * .6452: AR = AR * .6452
        LPRINT USING "###"; BM; :    LPRINT "x"; USING "###"; HM;
        LPRINT USING "######"; BD; :    LPRINT "x"; USING "###"; HD;
        LPRINT USING "#######.##"; WT;
        LPRINT USING "######.##"; S; :    LPRINT USING "######.##"; SR;
        LPRINT USING "#######.#"; A; :    LPRINT USING "#######.#"; AR
        GOTO 1100
1070   NEXT J
   '
   ' Keep the sections selected no more than four
1100    IF K > 3 GOTO 1140
       NEXT I
   '
   ' Identify the lightest section
1140    N = K
        C = AA(1):     J = 1
       FOR I = 2 TO N
        IF C > AA(I) THEN J = I
        IF C > AA(I) THEN C = AA(I)
       NEXT I
        B = BA(J):     H = HA(J)
        LPRINT "__________________________________________________________";
        LPRINT "________"
   '
   ' Print the lightest section
        LPRINT :    LPRINT "The lightest section is: ";
        IF U = 1 GOTO 1300
        LPRINT USING "##"; B; :    LPRINT " in. x "; USING "##"; H;
        LPRINT " in."
        GOTO 1400
1300    GOSUB 2000
        LPRINT USING "###"; BM; :    LPRINT " mm x "; USING "####"; HM;
        LPRINT " mm"
1400 END
   '
   ' -- SUBROUTINE -- Find the SI designation
2000    BM = ((B * 25.4 + 5) \ 10) * 10:    HM = ((H * 25.4 + 5) \ 10) * 10
        RETURN
     END
```

C15-1 (a)

SIZE FOR A SIMPLY SUPPORTED TIMBER BEAM
SUBJECTED TO A UNIFORM LOAD

Solution to EXAMPLE 15-5

The input data:

The simple span length:	L =	12.000 ft
Uniform load:	w =	500.00 lb/ft
Allow. normal stress:	sigma =	1450.00 psi
Allow. shear stress:	tau =	95.00 psi
Max. aspect ratio, h/b:	r =	3.00

The selected sections are:

Nominal Size in.xin.	Dressed Size in.xin.	Load plus Beam wt. lb/ft	S provided in^3	S req'd in^3	A provided in^2	A req'd in^2
6x10	5.5x 9.5	514.5	82.7	76.6	52.3	48.7
8x10	7.5x 9.5	519.8	112.8	77.4	71.3	49.2
10x10	9.5x 9.5	525.1	142.9	78.2	90.3	49.7
12x12	11.5x11.5	536.7	253.5	80.0	132.3	50.8

The lightest section is: 6 in. x 10 in.

C15-1 (b)

SIZE FOR A SIMPLY SUPPORTED TIMBER BEAM
SUBJECTED TO A UNIFORM LOAD

Solution to Problem 15-11

The input data:

The simple span length:	L =	16.000 ft
Uniform load:	w =	800.00 lb/ft
Allow. normal stress:	sigma =	1350.00 psi
Allow. shear stress:	tau =	80.00 psi
Max. aspect ratio, h/b:	r =	3.00

The selected sections are:

Nominal Size in.xin.	Dressed Size in.xin.	Load plus Beam wt. lb/ft	S provided in^3	S req'd in^3	A provided in^2	A req'd in^2
8x18	7.5x17.5	836.5	382.8	237.9	131.3	125.5
10x14	9.5x13.5	835.6	288.6	237.7	128.3	125.3
12x12	11.5x11.5	836.7	253.5	238.0	132.3	125.5

The lightest section is: 10 in. x 14 in.

C15-1 (c)

SIZE FOR A SIMPLY SUPPORTED TIMBER BEAM
SUBJECTED TO A UNIFORM LOAD

Solution to Problem 15-12

The input data:

The simple span length:	L =	5.000 m
Uniform load:	w =	12000.00 N/m
Allow. normal stress:	sigma =	9310.00 kPa
Allow. shear stress:	tau =	550.00 kPa
Max. aspect ratio, h/b:	r =	3.00

The selected sections are:

Nominal Size mm x mm	Dressed Size mm x mm	Load plus Beam wt. kN/m	S provided m^3/1000	S req'd m^3/1000	A provided m^2/1000	A req'd m^2/1000
200x510	191x495	12.59	7.79	4.23	94.4	85.9
250x410	241x394	12.60	6.23	4.23	95.0	85.9
310x360	292x343	12.63	5.73	4.24	100.2	86.1

The lightest section is: 200 mm x 510 mm

C15-1 (d)

SIZE FOR A SIMPLY SUPPORTED TIMBER BEAM
SUBJECTED TO A UNIFORM LOAD

Solution to Problem 15-19

The input data:

The simple span length:	L =	18.000 ft
Uniform load:	w =	255.00 lb/ft
Allow. normal stress:	sigma =	1600.00 psi
Allow. shear stress:	tau =	90.00 psi
Max. aspect ratio, h/b:	r =	4.00

The selected sections are:

Nominal Size in.xin.	Dressed Size in.xin.	Load plus Beam wt. lb/ft	S provided in^3	S req'd in^3	A provided in^2	A req'd in^2
4x14	3.5x13.5	268.1	106.3	81.4	47.3	40.2
6x10	5.5x 9.5	269.5	82.7	81.9	52.3	40.4
8x10	7.5x 9.5	274.8	112.8	83.5	71.3	41.2
10x10	9.5x 9.5	280.1	142.9	85.1	90.3	42.0

The lightest section is: 4 in. x 14 in.

Solution to Computer Program Assignment C15-2

```
' * C15-2 *   Size for a simply supported timber beam subjected to
'             uniform and concentrated loads
'
' Clear screen and compute pi
      CLS :    PI = 4 * ATN(1)
'
' Display the introductory remarks
      PRINT "Want to see the introductory remarks?"
      INPUT "-- Y / N "; RMK$
      IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100   PRINT "REMARKS: (1) This program can be used to select rectangular"
      PRINT "             timber sections for a simple span subjected to a"
      PRINT "             uniform load, a concentrated load at the"
      PRINT "             mid-span, and two symmetrically placed"
      PRINT "             concentrated loads."
      PRINT "         (2) The input data for the beam include the span"
      PRINT "             length, the uniform load, the concentrated load"
      PRINT "             at the mid-span, the symmetrically placed"
      PRINT "             concentrated loads and the location."
      PRINT "         (3) The input data and the computed results are"
      PRINT "             printed on the printer immediately after"
      PRINT "             data are entered. Make sure the printer is on"
      PRINT "             before entering data."
'
' Input problem I.D.
200   PRINT "Enter problem designation:"
      INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
      PRINT "Select the system of units:"
      PRINT "      1 -- SI units"
      PRINT "      2 -- US customary units"
      INPUT "-- 1 / 2"; U
      IF U = 2 GOTO 340
      UF$ = "N": UL1$ = "m": UL2$ = "mm": UM$ = "kN-m"
      UV$ = "kN": UW$ = "N/m": US$ = "kPa": USM$ = "m^3"
      GOTO 370
340   UF$ = "lb": UL1$ = "ft": UL2$ = "in.": UM$ = "kip-in"
      UV$ = "kip": UW$ = "lb/ft": US$ = "psi": USM$ = "in^3"
'
370 ' Print problem I.D.
      LPRINT "SIZE FOR A SIMPLY SUPPORTED TIMBER BEAM SUBJECTED TO A"
      LPRINT "UNIFORM LOAD AND SYMMETRICALLY PLACED CONCENTRATED LOADS"
      LPRINT :   LPRINT "Solution to "; ID$
'
' Enter data for the beam
      PRINT "Span length:":     PRINT "-- L ("; UL1$; ")"; :    INPUT L
      PRINT "Uniform load:":    PRINT "-- w ("; UW$; ")"; :     INPUT W
      PRINT "Concentrated loads:"
      PRINT "-- P1 ("; UF$; ")"; :    INPUT P1
      PRINT "-- P2 ("; UF$; "), a ("; UL1$; ")"; :    INPUT P2, AD
      PRINT "Allowable normal stress:"
      PRINT "-- sigma ("; US$; ")"; :    INPUT SIGMA
      PRINT "Allowable shear stress:"
      PRINT "-- tau ("; US$; ")"; :    INPUT TAU
      PRINT "The maximum aspect ratio r = h/b:"
      PRINT "-- r"; :    INPUT R
'
' Print the input data for the beam
      LPRINT :    LPRINT "The input data:"
```

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Solution to Computer Program Assignment C15-2 Continued

```
      LPRINT TAB(3); "The simple span length:"; TAB(32); "L =";
      LPRINT USING "######.###"; L; :    LPRINT " "; UL1$
      LPRINT TAB(3); "Uniform load:"; TAB(32); "w =";
      LPRINT USING "#######.##"; W; :    LPRINT " "; UW$
      LPRINT TAB(3); "Concentrated loads:"; TAB(31); "P1 =";
      LPRINT USING "#######.##"; P1; :    LPRINT " "; UF$
      LPRINT TAB(31); "P2 ="; USING "#######.##"; P2; :    LPRINT " "; UF$
      LPRINT TAB(32); "a ="; USING "#######.##"; AD; :    LPRINT " "; UL1$
      LPRINT TAB(3); "Allow. normal stress:"; TAB(28); "sigma =";
      LPRINT USING "#######.##"; SIGMA; :    LPRINT " "; US$
      LPRINT TAB(3); "Allow. shear stress:"; TAB(30); "tau =";
      LPRINT USING "#######.##"; TAU; :    LPRINT " "; US$
      LPRINT TAB(3); "Aspect ratio h/b:"; TAB(32); "r =";
      LPRINT USING "#######.##"; R
'
' Convert SI units to US units
      IF U = 2 GOTO 670
      L = L / .3048:  W = W / 14.59:  P1 = P1 / 4.448:  P2 = P2 / 4.448
      AD = AD / .3048:  SIGMA = SIGMA / 6.895:  TAU = TAU / 6.895
'
' Specify the initial width and initial height
670   BB(1) = 2:    BB(2) = 3:    BB(3) = 4:    BB(4) = 6:    BB(5) = 8
      BB(6) = 10:    BB(7) = 12
      HH(1) = 4:    HH(2) = 4:    HH(3) = 4:    HH(4) = 6:    HH(5) = 8
      HH(6) = 10:    HH(7) = 12
'
' Print heading
      LPRINT :    LPRINT "The selected sections are:"
      LPRINT "_____________________________________________________________";
      LPRINT "________"
      LPRINT
      LPRINT "Nominal   Dressed   Unif.Load    S         S         A         A"
;
      LPRINT " Size      Size     +Beam wt. provided  req'd    provided  ";
      LPRINT "req'd"
      IF U = 1 GOTO 700
      LPRINT "in.xin.   in.xin.     lb/ft     in^3      in^3      in^2     ";
      LPRINT "in^2"
      GOTO 750
700   LPRINT "mm x mm   mm x mm     kN/m    m^3/1000 m^3/1000 m^2/1000 ";
      LPRINT "m^2/1000"
750   LPRINT "_____________________________________________________________";
      LPRINT "________"
      LPRINT
'
' Initialize count
      K = 0
'
' Check section for strength criteria
   FOR I = 1 TO 7
      B = BB(I):    BD = B - .5
      HO = HH(I)
    FOR J = 1 TO 7
      H = HO + 2 * (J - 1): HD = H - .5
'
' Keep the aspect ratio h/b to the specified r
      IF H / B > R GOTO 1190
'
' Compute the section modulus and the area using dressed size
      S = BD * HD ^ 2 / 6: A = BD * HD
'
' Compute the weight of beam and the total load
      WEIT = BD * HD * 40 / 144: WT = W + WEIT
```

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Solution to Computer Program Assignment C15-2 Continued

```
 '
 ' Compute the maximum moment and the maximum shear
      MMAX = P1 * L / 4 + P2 * AD + WT * L ^ 2 / 8
      VMAX = P1 / 2 + P2 + WT * L / 2
 '
 ' Compute the req'd section modulus and the req'd area
      SR = MMAX / SIGMA * 12: AR = 1.5 * VMAX / TAU
 '
 ' Check the strength criteria
      IF S < SR OR A < AR GOTO 1160
      K = K + 1
      AA(K) = A:    BA(K) = B:    HA(K) = H
 '
 ' Print the sections selected
      IF U = 1 GOTO 800
 ' US units
      LPRINT USING "###"; B; :   LPRINT "x"; USING "##"; H;
      LPRINT USING "#####.#"; BD; :   LPRINT "x"; USING "##.#"; HD;
      LPRINT USING "#######.#"; WT;
      LPRINT USING "#######.#"; S; :   LPRINT USING "#######.#"; SR;
      LPRINT USING "#######.#"; A; :   LPRINT USING "#######.#"; AR
      GOTO 1190
 ' SI units
800   GOSUB 2000
      BD = BD * 25.4: HD = HD * 25.4: WT = WT * .1459
      S = S * .01639: SR = SR * .01639: A = A * .6452: AR = AR * .6452
      LPRINT USING "###"; BM; :   LPRINT "x"; USING "###"; HM;
      LPRINT USING "######"; BD; :   LPRINT "x"; USING "###"; HD;
      LPRINT USING "######.##"; WT;
      LPRINT USING "######.##"; S; :   LPRINT USING "######.##"; SR;
      LPRINT USING "#######.#"; A; :   LPRINT USING "#######.#"; AR
      GOTO 1190
1160    NEXT J
 '
 ' Keep the sections selected no more than four
1190  IF K > 3 GOTO 1230
     NEXT I
 '
 ' Identify the lightest section
1230  N = K
      C = AA(1):    J = 1
     FOR I = 2 TO N
      IF C > AA(I) THEN J = I
      IF C > AA(I) THEN C = AA(I)
     NEXT I
      B = BA(J):    H = HA(J)
      LPRINT "________________________________________________________________";
      LPRINT "________"
 '
 ' Print the lightest section
      LPRINT :   LPRINT "The lightest section is: ";
      IF U = 1 GOTO 1300
      LPRINT USING "##"; B; :   LPRINT " in. x "; USING "##"; H;
      LPRINT " in."
      GOTO 1400
1300  GOSUB 2000
      LPRINT USING "###"; BM; :   LPRINT " mm x "; USING "####"; HM;
      LPRINT " mm"
1400 END
 '
 ' -- SUBROUTINE -- Find the SI designation
2000  BM = ((B * 25.4 + 5) \ 10) * 10:   HM = ((H * 25.4 + 5) \ 10) * 10
      RETURN
      END
```


C15-2 (a) SIZE FOR A SIMPLY SUPPORTED TIMBER BEAM SUBJECTED TO A UNIFORM LOAD AND SYMMETRICALLY PLACED CONCENTRATED LOADS

Solution to Problem 15-13

```
The input data:
  The simple span length:        L =      4.000 m
  Uniform load:                  w =       0.00 N/m
  Concentrated loads:           P1 =   45000.00 N
                                P2 =       0.00 N
                                 a =       0.00 m
  Allow. normal stress:      sigma =   13000.00 kPa
  Allow. shear stress:         tau =    1000.00 kPa
  Aspect ratio h/b:              r =       3.00
```

The selected sections are:

Nominal Size mm x mm	Dressed Size mm x mm	Unif.Load +Beam wt. kN/m	S provided m^3/1000	S req'd m^3/1000	A provided m^2/1000	A req'd m^2/1000
150x410	140x394	3.45	3.61	3.52	55.0	34.8
200x360	191x343	4.10	3.73	3.53	65.3	35.0
250x360	241x343	5.20	4.73	3.54	82.7	35.3
310x310	292x292	5.36	4.15	3.54	85.3	35.4

The lightest section is: 150 mm x 410 mm

C15-2 (b) SIZE FOR A SIMPLY SUPPORTED TIMBER BEAM SUBJECTED TO A UNIFORM LOAD AND SYMMETRICALLY PLACED CONCENTRATED LOADS

Solution to Problem 15-16

```
The input data:
  The simple span length:        L =     16.000 ft
  Uniform load:                  w =     300.00 lb/ft
  Concentrated loads:           P1 =    4000.00 lb
                                P2 =       0.00 lb
                                 a =       0.00 ft
  Allow. normal stress:      sigma =    1600.00 psi
  Allow. shear stress:         tau =      90.00 psi
  Aspect ratio h/b:              r =       3.00
```

The selected sections are:

Nominal Size in.xin.	Dressed Size in.xin.	Unif.Load +Beam wt. lb/ft	S provided in^3	S req'd in^3	A provided in^2	A req'd in^2
6x16	5.5x15.5	323.7	220.2	197.7	85.3	76.5
8x14	7.5x13.5	328.1	227.8	198.8	101.3	77.1
10x12	9.5x11.5	330.3	209.4	199.3	109.3	77.4
12x12	11.5x11.5	336.7	253.5	200.8	132.3	78.2

The lightest section is: 6 in. x 16 in.

Solution to Computer Program Assignment C16-1

```
' * C16-1 *   Deflections of a cantilever beam
'
' Clear screen
     CLS
'
' Display the introductory remarks
     PRINT "Want to see the introductory remarks?"
     INPUT "-- Y / N "; RMK$
     IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100  PRINT "REMARKS: (1) This program can be used to compute the deflec-"
     PRINT "             tions at every tenth point of a cantilever beam"
     PRINT "             due to a uniform load and a concentrated load."
     PRINT "         (2) The input data for the beam include the span"
     PRINT "             length, the stiffness EI, the uniform load and"
     PRINT "             its location, the concentrated load and its"
     PRINT "             location."
     PRINT "         (3) The input data and the computed results are"
     PRINT "             printed on the printer immediately after"
     PRINT "             data are entered. Make sure your printer is on"
     PRINT "             before entering data."
'
' Input problem I.D.
200  PRINT "Enter problem designation:"
     INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
     PRINT "Select the system of units:"
     PRINT "      1 -- SI units"
     PRINT "      2 -- US customary units"
     INPUT "-- 1 / 2"; U
     IF U = 2 GOTO 310
     UF$ = "kN": UL1$ = "m": UL2$ = "mm": UW$ = "kN/m": UEI$ = "kN-m^2"
     GOTO 330
310  UF$ = "kip": UL1$ = "ft": UL2$ = "in.": UW$ = "kip/ft"
     UEI$ = "kip-ft^2"
'
' Print problem I.D.
330  LPRINT "DEFLECTIONS OF A CANTILEVER BEAM"
     LPRINT :   LPRINT "Solution to "; ID$
'
' Enter data for the beam
     PRINT "Span length:":    PRINT "-- L ("; UL1$; ")"; :   INPUT L
     PRINT "Stiffness:":   PRINT "-- EI ("; UEI$; ")"; :   INPUT EI
     PRINT "Uniform load:"
     PRINT "-- w ("; UW$; "), b ("; UL1$; "), c ("; UL1$; ")";
     INPUT W, B, C
     PRINT "Concentrated load:"
     PRINT "-- P ("; UF$; "), d ("; UL1$; ")"; :   INPUT P, D
'
' Print the input data for the beam
     LPRINT :   LPRINT "The input data:"
     LPRINT TAB(3); "The cantilever span length:"; TAB(32): "L =";
     LPRINT USING "######.###"; L; :   LPRINT " "; UL1$
     LPRINT TAB(3); "The beam stiffness:"; TAB(31); "EI =";
     LPRINT USING "#######.##"; EI; :   LPRINT " "; UEI$
     IF W = 0 GOTO 540
     LPRINT TAB(3); "Uniform load:"; TAB(32); "w =";
     LPRINT USING "#######.##"; W; :   LPRINT " "; UW$
     LPRINT TAB(32); "b ="; USING "#######.##"; B; :   LPRINT " "; UL1$
     LPRINT TAB(32); "c ="; USING "#######.##"; C; :   LPRINT " "; UL1$
```


Solution to Computer Program Assignment C16-1 Continued

```
     IF P = 0 GOTO 560
540  LPRINT TAB(3); "Concentrated load:"; TAB(32); "P =";
     LPRINT USING "#######.##"; P; :   LPRINT " "; UF$
     LPRINT TAB(32); "d ="; USING "#######.##"; D; :   LPRINT " "; UL1$
560 '
' Print heading
     LPRINT
     LPRINT "Deflections at every tenth points:"
     LPRINT "______________________________________________"
     LPRINT
     LPRINT "Point  Distance   Deflection   Deflection"
     LPRINT "______________________________________________"
     LPRINT
'
' Compute deflection at every tenth point
    FOR I = 0 TO 10
      X = L / 10 * I
      IF X <= C THEN Y1 = W * X ^ 2 * (X ^ 2 - 4 * C * X + 6 * C ^ 2)
      IF X <= C THEN Y1 = Y1 / (24 * EI)
      IF X > C THEN Y1 = W * C ^ 4 / (8 * EI)
      IF X > C THEN Y1 = Y1 + W * C ^ 3 * (X - C) / (6 * EI)
      IF X <= B THEN Y2 = -W * X ^ 2 * (X ^ 2 - 4 * B * X + 6 * B ^ 2)
      IF X <= B THEN Y2 = Y2 / (24 * EI)
      IF X > B THEN Y2 = -W * B ^ 4 / (8 * EI)
      IF X > B THEN Y2 = Y2 - W * B ^ 3 * (X - B) / (6 * EI)
      IF X <= D THEN Y3 = P * X ^ 2 * (3 * D - X) / (6 * EI)
      IF X > D THEN Y3 = P * D ^ 2 * (3 * X - D) / (6 * EI)
      Y = Y1 + Y2 + Y3
      LPRINT USING "##.#"; I / 10:
      LPRINT USING "#####.##"; X;
      LPRINT " "; UL1$; USING "######.#####"; Y;
      IF U = 1 GOTO 800
      LPRINT " "; UL1$; USING "######.###"; Y * 12; : LPRINT " "; UL2$
      GOTO 810
800   LPRINT " "; UL1$; USING "######.###"; Y * 1000; : LPRINT " "; UL2$
810  NEXT I
      LPRINT "______________________________________________"
  END
```

C16-1 (a)

DEFLECTIONS OF A CANTILEVER BEAM

Solution to EXAMPLE 16-5

The input data:

The cantilever span length:	L =	1.200 m
The beam stiffness:	EI =	554.00 kN-m^2
Uniform load:	w =	3.00 kN/m
	b =	0.00 m
	c =	1.20 m
Concentrated load:	P =	4.00 kN
	d =	0.80 m

Deflections at every tenth points:

Point	Distance	Deflection	Deflection
0.0	0.00 m	0.00000 m	0.000 mm
0.1	0.12 m	0.00007 m	0.066 mm
0.2	0.24 m	0.00025 m	0.248 mm
0.3	0.36 m	0.00052 m	0.524 mm
0.4	0.48 m	0.00087 m	0.874 mm
0.5	0.60 m	0.00128 m	1.277 mm
0.6	0.72 m	0.00172 m	1.715 mm
0.7	0.84 m	0.00217 m	2.171 mm
0.8	0.96 m	0.00263 m	2.632 mm
0.9	1.08 m	0.00310 m	3.096 mm
1.0	1.20 m	0.00356 m	3.560 mm

C16-1 (b)

DEFLECTIONS OF A CANTILEVER BEAM

Solution to Problem 16-15

The input data:

The cantilever span length:	L =	8.000 ft
The beam stiffness:	EI =	434.00 kip-ft^2
Uniform load:	w =	0.02 kip/ft
	b =	0.00 ft
	c =	8.00 ft
Concentrated load:	P =	0.10 kip
	d =	4.00 ft

Deflections at every tenth points:

Point	Distance	Deflection	Deflection
0.0	0.00 ft	0.00000 ft	0.000 in.
0.1	0.80 ft	0.00072 ft	0.009 in.
0.2	1.60 ft	0.00267 ft	0.032 in.
0.3	2.40 ft	0.00558 ft	0.067 in.
0.4	3.20 ft	0.00920 ft	0.110 in.
0.5	4.00 ft	0.01327 ft	0.159 in.
0.6	4.80 ft	0.01760 ft	0.211 in.
0.7	5.60 ft	0.02209 ft	0.265 in.
0.8	6.40 ft	0.02665 ft	0.320 in.
0.9	7.20 ft	0.03126 ft	0.375 in.
1.0	8.00 ft	0.03588 ft	0.431 in.

C16-1 (c)

DEFLECTIONS OF A CANTILEVER BEAM

Solution to Problem 16-16

The input data:

The cantilever span length:	L =	3.000 m
The beam stiffness:	EI =	114000.00 kN-m^2
Uniform load:	w =	8.00 kN/m
	b =	0.00 m
	c =	2.00 m
Concentrated load:	P =	20.00 kN
	d =	3.00 m

Deflections at every tenth points:

Point	Distance	Deflection	Deflection
0.0	0.00 m	0.00000 m	0.000 mm
0.1	0.30 m	0.00003 m	0.029 mm
0.2	0.60 m	0.00011 m	0.109 mm
0.3	0.90 m	0.00023 m	0.234 mm
0.4	1.20 m	0.00040 m	0.395 mm
0.5	1.50 m	0.00059 m	0.587 mm
0.6	1.80 m	0.00080 m	0.804 mm
0.7	2.10 m	0.00104 m	1.039 mm
0.8	2.40 m	0.00129 m	1.289 mm
0.9	2.70 m	0.00155 m	1.549 mm
1.0	3.00 m	0.00181 m	1.813 mm

C16-1 (d)

DEFLECTIONS OF A CANTILEVER BEAM

Solution to Problem 16-17

The input data:

The cantilever span length:	L =	4.000 m
The beam stiffness:	EI =	824000.00 kN-m^2
Uniform load:	w =	300.00 kN/m
	b =	2.00 m
	c =	4.00 m

Deflections at every tenth points:

Point	Distance	Deflection	Deflection
0.0	0.00 m	0.00000 m	0.000 mm
0.1	0.40 m	0.00017 m	0.167 mm
0.2	0.80 m	0.00064 m	0.637 mm
0.3	1.20 m	0.00136 m	1.363 mm
0.4	1.60 m	0.00230 m	2.299 mm
0.5	2.00 m	0.00340 m	3.398 mm
0.6	2.40 m	0.00461 m	4.614 mm
0.7	2.80 m	0.00591 m	5.905 mm
0.8	3.20 m	0.00724 m	7.239 mm
0.9	3.60 m	0.00859 m	8.593 mm
1.0	4.00 m	0.00995 m	9.951 mm

Solution to Computer Program Assignment C16-2

```
' * C16-2 *   Deflections of a simple beam
'
' Clear screen
      CLS
'
' Display the introductory remarks
      PRINT "Want to see the introductory remarks?"
      INPUT "-- Y / N "; RMK$
      IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100   PRINT "REMARKS: (1) This program can be used to compute the deflection"
      PRINT "             at every tenth point of a simple beam due to a"
      PRINT "             uniform load and up to two concentrated loads."
      PRINT "         (2) The input data for the beam include the span"
      PRINT "             length, the stiffness EI, the uniform load, the"
      PRINT "             concentrated loads and their locations."
      PRINT "         (3) The input data and the computed results are"
      PRINT "             printed on the printer immediately after"
      PRINT "             data are entered. Make sure your printer is on"
      PRINT "             before entering data."
'
' Input problem I.D.
200   PRINT "Enter problem designation:"
      INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
      PRINT "Select the system of units:"
      PRINT "      1 -- SI units"
      PRINT "      2 -- US customary units"
      INPUT "-- 1 / 2"; U
      IF U = 2 GOTO 300
      UF$ = "kN": UL1$ = "m": UL2$ = "mm": UW$ = "kN/m": UEI$ = "kN-m^2"
      GOTO 320
300   UF$ = "kip": UL1$ = "ft": UL2$ = "in.": UW$ = "kip/ft"
      UEI$ = "kip-ft^2"
'
320 ' Print problem I.D.
      LPRINT "DEFLECTIONS OF A SIMPLE BEAM"
      LPRINT :   LPRINT "Solution to "; ID$
'
' Enter data for the beam
      PRINT "Span length:":    PRINT "-- L ("; UL1$; ")"; :   INPUT L
      PRINT "Stiffness:":    PRINT "-- EI ("; UEI$; ")"; :   INPUT EI
      PRINT "Uniform load:":    PRINT "-- w ("; UW$; ")"; :   INPUT W
      PRINT "Concentrated load:"
      PRINT "-- P1 ("; UF$; "), a1 ("; UL1$; ")"; :   INPUT P1, A1
      IF P1 = 0 THEN GOTO 460
      PRINT "-- P2 ("; UF$; "), a2 ("; UL1$; ")"; :   INPUT P2, A2
'
' Print the input data for the beam
460   LPRINT :   LPRINT "The input data:"
      LPRINT TAB(3); "The simple span length:"; TAB(32); "L =";
      LPRINT USING "######.###"; L; :   LPRINT " "; UL1$
      LPRINT TAB(3); "The beam stiffness:"; TAB(31); "EI =";
      LPRINT USING "#######.##"; EI; :   LPRINT " "; UEI$
      IF W = 0 GOTO 520
      LPRINT TAB(3); "Uniform load:"; TAB(32); "w =";
      LPRINT USING "#######.##"; W; : LPRINT " "; UW$
      IF P1 = 0 GOTO 590
520   LPRINT TAB(3); "Concentrated loads:"; TAB(31); "P1 =";
      LPRINT USING "#######.##"; P1; :   LPRINT " "; UF$
      LPRINT TAB(31); "a1 ="; USING "#######.##"; A1;
```


Solution to Computer Program Assignment C16-2 Continued

```
      LPRINT " "; UL1$
      IF P2 = 0 THEN GOTO 590
      LPRINT TAB(31); "P2 ="; USING "#######.##"; P2; :   LPRINT " "; UF$
      LPRINT TAB(31); "a2 ="; USING "#######.##"; A2; :   LPRINT " "; UL1$
'
' Print heading
590   LPRINT
      LPRINT "Deflections at every tenth points:"
      LPRINT "________________________________________________"
      LPRINT
      LPRINT "Point  Distance    Deflection     Deflection"
      LPRINT "________________________________________________"
      LPRINT
'
' Compute deflection at every tenth point
     FOR I = 0 TO 10
      X = L / 10 * I:  B1 = L - A1:    B2 = L - A2
      Y1 = W * X * (L ^ 3 - 2 * L * X ^ 2 + X ^ 3) / (24 * EI)
      IF X <= A1 THEN Y2 = P1 * B1 * X * (L ^ 2 - X ^ 2 - B1 ^ 2)
      IF X <= A1 THEN Y2 = Y2 / (6 * EI * L)
      IF X > A1 THEN Y2 = (L / B1) * (X - A1) ^ 3
      IF X > A1 THEN Y2 = Y2 + (L ^ 2 - B1 ^ 2) * X - X ^ 3
      IF X > A1 THEN Y2 = P1 * B1 * Y2 / (6 * EI * L)
      IF X <= A2 THEN Y3 = P2 * B2 * X * (L ^ 2 - X ^ 2 - B2 ^ 2)
      IF X <= A2 THEN Y3 = Y3 / (6 * EI * L)
      IF X > A2 THEN Y3 = (L / B2) * (X - A2) ^ 3
      IF X > A2 THEN Y3 = Y3 + (L ^ 2 - B2 ^ 2) * X - X ^ 3
      IF X > A2 THEN Y3 = P2 * B2 * Y3 / (6 * EI * L)
      Y = Y1 + Y2 + Y3
      LPRINT USING "##.#"; I / 10;
      LPRINT USING "#####.##"; X;
      LPRINT " "; UL1$; USING "######.#####"; Y;
      IF U = 1 GOTO 800
      LPRINT " "; UL1$; USING "######.###"; Y * 12; : LPRINT " "; UL2$
      GOTO 810
800   LPRINT " "; UL1$; USING "########.###"; Y * 1000; : LPRINT " "; UL2$
810  NEXT I
      LPRINT "________________________________________________"
'
END
```

C16-2 (a)

DEFLECTIONS OF A SIMPLE BEAM

Solution to EXAMPLE 16-7

The input data:

The simple span length:	L =	2.000 m
The beam stiffness:	EI =	554.00 kN-m^2
Uniform load:	w =	3.00 kN/m
Concentrated loads:	P1 =	8.00 kN
	a1 =	1.20 m

Deflections at every tenth points:

Point	Distance	Deflection	Deflection
0.0	0.00 m	0.00000 m	0.000 mm
0.1	0.20 m	0.00099 m	0.993 mm
0.2	0.40 m	0.00190 m	1.902 mm
0.3	0.60 m	0.00265 m	2.650 mm
0.4	0.80 m	0.00317 m	3.169 mm
0.5	1.00 m	0.00340 m	3.400 mm
0.6	1.20 m	0.00329 m	3.292 mm
0.7	1.40 m	0.00282 m	2.823 mm
0.8	1.60 m	0.00206 m	2.056 mm
0.9	1.80 m	0.00108 m	1.082 mm
1.0	2.00 m	0.00000 m	0.000 mm

C16-2 (b)

DEFLECTIONS OF A SIMPLE BEAM

Solution to Problem 16-19

The input data:

The simple span length:	L =	6.000 m
The beam stiffness:	EI =	3670.00 kN-m^2
Uniform load:	w =	0.19 kN/m
Concentrated loads:	P1 =	5.00 kN
	a1 =	2.00 m
	P2 =	5.00 kN
	a2 =	4.00 m

Deflections at every tenth points:

Point	Distance	Deflection	Deflection
0.0	0.00 m	0.00000 m	0.000 mm
0.1	0.60 m	0.00349 m	3.495 mm
0.2	1.20 m	0.00667 m	6.666 mm
0.3	1.80 m	0.00920 m	9.195 mm
0.4	2.40 m	0.01079 m	10.787 mm
0.5	3.00 m	0.01132 m	11.319 mm
0.6	3.60 m	0.01079 m	10.787 mm
0.7	4.20 m	0.00920 m	9.195 mm
0.8	4.80 m	0.00667 m	6.666 mm
0.9	5.40 m	0.00349 m	3.495 mm
1.0	6.00 m	-0.00000 m	-0.000 mm

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C16-2 (c)

DEFLECTIONS OF A SIMPLE BEAM

Solution to Problem 16-21

The input data:

The simple span length:	L =	10.000 ft
The beam stiffness:	EI =	26000.00 kip-ft^2
Uniform load:	w =	2.00 kip/ft
Concentrated loads:	P1 =	10.00 kip
	a1 =	5.00 ft

Deflections at every tenth points:

Point	Distance	Deflection	Deflection
0.0	0.00 ft	0.00000 ft	0.000 in.
0.1	1.00 ft	0.00552 ft	0.066 in.
0.2	2.00 ft	0.01050 ft	0.126 in.
0.3	3.00 ft	0.01449 ft	0.174 in.
0.4	4.00 ft	0.01710 ft	0.205 in.
0.5	5.00 ft	0.01803 ft	0.216 in.
0.6	6.00 ft	0.01710 ft	0.205 in.
0.7	7.00 ft	0.01449 ft	0.174 in.
0.8	8.00 ft	0.01050 ft	0.126 in.
0.9	9.00 ft	0.00552 ft	0.066 in.
1.0	10.00 ft	-0.00000 ft	-0.000 in.

C16-2 (d)

DEFLECTIONS OF A SIMPLE BEAM

Solution to Problem 16-22

The input data:

The simple span length:	L =	8.000 ft
The beam stiffness:	EI =	190.00 kip-ft^2
Concentrated loads:	P1 =	0.20 kip
	a1 =	4.00 ft
	P2 =	0.40 kip
	a2 =	6.00 ft

Deflections at every tenth points:

Point	Distance	Deflection	Deflection
0.0	0.00 ft	0.00000 ft	0.000 in.
0.1	0.80 ft	0.00749 ft	0.090 in.
0.2	1.60 ft	0.01444 ft	0.173 in.
0.3	2.40 ft	0.02031 ft	0.244 in.
0.4	3.20 ft	0.02457 ft	0.295 in.
0.5	4.00 ft	0.02667 ft	0.320 in.
0.6	4.80 ft	0.02616 ft	0.314 in.
0.7	5.60 ft	0.02296 ft	0.276 in.
0.8	6.40 ft	0.01709 ft	0.205 in.
0.9	7.20 ft	0.00908 ft	0.109 in.
1.0	8.00 ft	0.00000 ft	0.000 in.

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Solution to Computer Program Assignment C17-1

```
'
' * C17-1 * Reaction components on a fixed beam
'
' Clear screen
      CLS
'
' Display the introductory remarks
      PRINT "Want to see the introductory remarks?"
      INPUT "-- Y / N "; RMK$
      IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100   PRINT "REMARKS: (1) This program can be used to compute the reaction"
      PRINT "             components for a fixed beam subjected to a"
      PRINT "             uniform load and up to two concentrated loads."
      PRINT "         (2) The input data for the beam include the span"
      PRINT "             length, the uniform load, the concentrated"
      PRINT "             loads and their locations."
      PRINT "         (3) The input data and the computed results are"
      PRINT "             printed on the printer immediately after"
      PRINT "             data are entered. Make sure your printer is on"
      PRINT "             before entering data."
'
' Input problem I.D.
200   PRINT "Enter problem designation:"
      INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
      PRINT "Select the system of units:"
      PRINT "      1 -- SI units"
      PRINT "      2 -- US customary units"
      INPUT "-- 1 / 2"; U
      IF U = 2 GOTO 310
      UF$ = "kN":    UL$ = "m":    UW$ = "kN/m":    UM$ = "kN-m"
      GOTO 330
310   UF$ = "kip":    UL$ = "ft":    UW$ = "kip/ft":    UM$ = "kip-ft"
'
330 ' Print problem I.D.
      LPRINT "REACTION COMPONENTS ON A FIXED BEAM"
      LPRINT :   LPRINT "Solution to "; ID$
'
' Enter data for the beam
      PRINT "Span length:":    PRINT "-- L ("; UL$; ")"; :   INPUT L
      PRINT "Uniform load:":    PRINT "-- w ("; UW$; ")"; :   INPUT W
      PRINT "Concentrated load:"
      PRINT "-- P1 ("; UF$; "), a1 ("; UL$; ")"; :   INPUT P1, A1
      IF P1 = 0 THEN GOTO 460
      PRINT "-- P2 ("; UF$; "), a2 ("; UL$; ")"; :   INPUT P2, A2
'
' Print the input data for the beam
460   LPRINT :   LPRINT "The input data:"
      LPRINT TAB(3); "The simple span length:"; TAB(32); "L =";
      LPRINT USING "######.###"; L; :   LPRINT " "; UL$
      IF W = 0 GOTO 510
      LPRINT TAB(3); "Uniform load:"; TAB(32); "w =";
      LPRINT USING "#######.##"; W; :   LPRINT " "; UW$
      IF P1 = 0 GOTO 580
510   LPRINT TAB(3); "Concentrated loads:"; TAB(31); "P1 =";
      LPRINT USING "#######.##"; P1; :   LPRINT " "; UF$
      LPRINT TAB(31); "a1 ="; USING "#######.##"; A1; :   LPRINT " "; UL$
      IF P2 = 0 THEN GOTO 580
      LPRINT TAB(31); "P2 ="; USING "#######.##"; P2; :   LPRINT " "; UF$
      LPRINT TAB(31); "a2 ="; USING "#######.##"; A2; :   LPRINT " "; UL$
```


Solution to Computer Program Assignment C17-1 Continued

```
'
' Print heading
580   LPRINT
      LPRINT "The reaction components at the fixed supports are:"
      LPRINT
'
' Compute the reaction components at the fixed supports
      B1 = L - A1:    B2 = L - A2
      MA1 = W * L ^ 2 / 12: MB1 = MA1: RA1 = W * L / 2: RB1 = RA1
      MA2 = P1 * A1 * B1 ^ 2 / L ^ 2: MB2 = P1 * A1 ^ 2 * B1 / L ^ 2
      RA2 = (MA2 - MB2 + P1 * B1) / L:    RB2 = (MB2 - MA2 + P1 * A1) / L
      MA3 = P2 * A2 * B2 ^ 2 / L ^ 2: MB3 = P2 * A2 ^ 2 * B2 / L ^ 2
      RA3 = (MA3 - MB3 + P2 * B2) / L:    RB3 = (MB3 - MA3 + P2 * A2) / L
      MA = MA1 + MA2 + MA3:    MB = MB1 + MB2 + MB3
      RA = RA1 + RA2 + RA3:    RB = RB1 + RB2 + RB3
      DM1$ = "(c.c.w.)":    DM2$ = "(c.c.)":    DR1$ = "(upward)"
      DR2$ = "(downward)"
      LPRINT TAB(31); "MA ="; USING "######.###"; MA; :   LPRINT " "; UM$;
      IF MA > 0 THEN LPRINT TAB(53); DM1$
      IF MA < 0 THEN LPRINT TAB(53); DM2$
      LPRINT TAB(31); "MB ="; USING "######.###"; MB; :   LPRINT " "; UM$;
      IF MB > 0 THEN LPRINT TAB(53); DM2$
      IF MB < 0 THEN LPRINT TAB(53); DM1$
      LPRINT TAB(31); "RA ="; USING "######.###"; RA; :   LPRINT " "; UF$;
      IF RA > 0 THEN LPRINT TAB(53); DR1$
      IF RA < 0 THEN LPRINT TAB(53); DR2$
      LPRINT TAB(31); "RB ="; USING "######.###"; RB; :   LPRINT " "; UF$;
      IF RB > 0 THEN LPRINT TAB(53); DR1$
      IF RB < 0 THEN LPRINT TAB(53); DR2$
END
```

C17-1 (a)

```
REACTION COMPONENTS ON A FIXED BEAM

Solution to Problem 17-13

The input data:
  The simple span length:        L =      6.000 m
  Uniform load:                  w =       2.00 kN/m
  Concentrated loads:           P1 =      10.00 kN
                                a1 =       3.00 m

The reaction components at the fixed supports are:

                                MA =    13.500 kN-m    (c.c.w.)
                                MB =    13.500 kN-m    (c.c.)
                                RA =    11.000 kN      (upward)
                                RB =    11.000 kN      (upward)
```

C17-1 (b)

```
REACTION COMPONENTS ON A FIXED BEAM

Solution to Problem 17-12

The input data:
  The simple span length:        L =    12.000 ft
  Concentrated loads:           P1 =      6.00 kip
                                a1 =      4.00 ft
                                P2 =      6.00 kip
                                a2 =      8.00 ft

The reaction components at the fixed supports are:

                                MA =    16.000 kip-ft (c.c.w.)
                                MB =    16.000 kip-ft (c.c.)
                                RA =     6.000 kip    (upward)
                                RB =     6.000 kip    (upward)
```

C17-1 (c)

```
REACTION COMPONENTS ON A FIXED BEAM

Solution to Problem 17-14

The input data:
  The simple span length:        L =     7.000 m
  Uniform load:                  w =      3.00 kN/m
  Concentrated loads:           P1 =     20.00 kN
                                a1 =      4.00 m

The reaction components at the fixed supports are:

                                MA =    26.944 kN-m    (c.c.w.)
                                MB =    31.842 kN-m    (c.c.)
                                RA =    18.372 kN      (upward)
                                RB =    22.628 kN      (upward)
```


Solution to Computer Program Assignment C17-2

```
' * C17-2 *   Analysis of a two-span continuous beam with overhangs
'
' Clear screen
     CLS
'
' Display the introductory remarks
     PRINT "Want to see the introductory remarks?"
     INPUT "-- Y / N "; RMK$
     IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 210
100  PRINT "REMARKS: (1) This program can be used to compute the support"
     PRINT "             moments and reactions for a two-span continuous"
     PRINT "             beam."
     PRINT "         (2) The input data include the length, the uniform"
     PRINT "             load, the the concentrated load and its location"
     PRINT "             for each span and each overhang."
     PRINT "         (3) The input data and the computed results are"
     PRINT "             printed on the printer immediately after"
     PRINT "             data are entered. Make sure your printer is on"
     PRINT "             before entering data."
'
' Input problem I.D.
210  PRINT "Enter problem designation:"
     INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
     PRINT "Select the system of units:"
     PRINT "      1 -- SI units"
     PRINT "      2 -- US customary units"
     INPUT "-- 1 / 2"; U
     IF U = 2 GOTO 320
     UF$ = "kN": UL$ = "m": UW$ = "kN/m": UM$ = "kN-m"
     GOTO 340
320  UF$ = "kip": UL$ = "ft": UW$ = "kip/ft": UM$ = "kip-ft"
'
340 ' Print problem I.D.
     LPRINT "ANALYSIS OF TWO-SPAN CONTINUOUS BEAM WITH OVERHANGS"
     LPRINT :   LPRINT "Solution to "; ID$
'
' Enter data for the beam
     PRINT "Lengths of overhangs and span lengths:"
     PRINT "-- L0 ("; UL$; ")"; :   INPUT L0
     PRINT "-- L1 ("; UL$; ")"; :   INPUT L1
     PRINT "-- L2 ("; UL$; ")"; :   INPUT L2
     PRINT "-- L3 ("; UL$; ")"; :   INPUT L3
     IF L0 = 0 GOTO 470
     PRINT "Loads on the left overhang:"
     PRINT "-- w0 ("; UW$; ")"; :   INPUT W0
     PRINT "-- P0 ("; UF$; "), a0 ("; UL$; ")"; :   INPUT P0, A0
470  PRINT "Loads on the first span:"
     PRINT "-- w1 ("; UW$; ")"; :   INPUT W1
     PRINT "-- P1 ("; UF$; "), a1 ("; UL$; ")"; :   INPUT P1, A1
     PRINT "Loads on the second span:"
     PRINT "-- w2 ("; UW$; ")"; :   INPUT W2
     PRINT "-- P2 ("; UF$; "), a2 ("; UL$; ")"; :   INPUT P2, A2
     IF L3 = 0 GOTO 560
     PRINT "Loads on the right overhang:"
     PRINT "-- w3 ("; UW$; ")"; :   INPUT W3
     PRINT "-- P3 ("; UF$; "), a3 ("; UL$; ")"; :   INPUT P3, A3
'
' Print the input data for the beam
560  LPRINT :   LPRINT "The input data:"
```

Solution to Computer Program Assignment C17-2 Continued

```
        IF L0 = 0 GOTO 660
        LPRINT TAB(3); "The left overhang:"; TAB(31); "L0 =";
        LPRINT USING "######.###"; L0; :   LPRINT " "; UL$
        IF W0 = 0 GOTO 620
        LPRINT TAB(31); "w0 ="; USING "######.###"; W0; :   LPRINT " "; UW$
620     IF P0 = 0 GOTO 660
        LPRINT TAB(31); "P0 ="; USING "######.###"; P0: :   LPRINT " "; UF$
        LPRINT TAB(31); "a0 ="; USING "######.###"; A0: :   LPRINT " "; UL$
'
660     LPRINT TAB(3); "The first span:"; TAB(31); "L1 =";
        LPRINT USING "######.###"; L1; :   LPRINT " "; UL$
        IF W1 = 0 GOTO 690
        LPRINT TAB(31); "w1 ="; USING "######.###"; W1; :   LPRINT " "; UW$
690     IF P1 = 0 GOTO 730
        LPRINT TAB(31); "P1 ="; USING "######.###"; P1; :   LPRINT " "; UF$
        LPRINT TAB(31); "a1 ="; USING "######.###"; A1; :   LPRINT " "; UL$
'
730     LPRINT TAB(3); "The second span:"; TAB(31); "L2 =";
        LPRINT USING "######.###"; L2; :   LPRINT " "; UL$
        IF W2 = 0 GOTO 760
        LPRINT TAB(31); "w2 ="; USING "######.###"; W2; :   LPRINT " "; UW$
760     IF P2 = 0 GOTO 800
        LPRINT TAB(31); "P2 ="; USING "######.###"; P2; :   LPRINT " "; UF$
        LPRINT TAB(31); "a2 ="; USING "######.###"; A2; :   LPRINT " "; UL$
'
800     IF L3 = 0 GOTO 890
        LPRINT TAB(3); "The right overhang:"; TAB(31); "L3 =";
        LPRINT USING "######.###"; L3; :   LPRINT " "; UL$
        IF W3 = 0 GOTO 840
        LPRINT TAB(31); "w3 ="; USING "######.###"; W3; :   LPRINT " "; UW$
840     IF P3 = 0 GOTO 890
        LPRINT TAB(31); "P3 ="; USING "######.###"; P3; :   LPRINT " "; UF$
        LPRINT TAB(31); "a3 ="; USING "######.###"; A3; :   LPRINT " "; UL$
'
' Compute the reaction components at the fixed supports
890     B0 = L0 - A0:    B1 = L1 - A1:    B2 = L2 - A2:    B3 = L3 - A3
        K = -W1 * L1 ^ 3 / 4 - W2 * L2 ^ 3 / 4
        K = K - P1 * A1 * (L1 ^ 2 - A1 ^ 2) / L1
        K = K - P2 * B2 * (L2 ^ 2 - B2 ^ 2) / L2
        MA = -P0 * B0 - W0 * L0 ^ 2 / 2: MC = -P3 * A3 - W3 * L3 ^ 2 / 2
        MB = (K - MA * L1 - MC * L2) / 2 / (L1 + L2)
        RA = P0 * (L1 + B0) + W0 * L0 * (L1 + L0 / 2)
        RA = (RA + P1 * B1 + W1 * L1 ^ 2 / 2 + MB) / L1
        RC = P3 * (L2 + A3) + W3 * L3 * (L2 + L3 / 2)
        RC = (RC + P2 * A2 + W2 * L2 ^ 2 / 2 + MB) / L2
        RB = P0 + P1 + P2 + P3 + W0 * L0 + W1 * L1 + W2 * L2
        RB = RB + W3 * L3 - RA - RC

        LPRINT :   LPRINT "Support moments and reactions are:"
        DR1$ = " (upward)":    DR2$ = " (downward)"
        LPRINT TAB(31); "MA ="; USING "######.###"; MA; :   LPRINT " "; UM$
        LPRINT TAB(31); "MB ="; USING "######.###"; MB; :   LPRINT " "; UM$
        LPRINT TAB(31); "MC ="; USING "######.###"; MC; :   LPRINT " "; UM$
        LPRINT TAB(31); "RA ="; USING "######.###"; RA; :   LPRINT " "; UF$;
        IF RA > 0 THEN LPRINT TAB(49); DR1$
        IF RA < 0 THEN LPRINT TAB(49); DR2$
        LPRINT TAB(31); "RB ="; USING "######.###"; RB; :   LPRINT " "; UF$;
        IF RB > 0 THEN LPRINT TAB(49); DR1$
        IF RB < 0 THEN LPRINT TAB(49); DR2$
        LPRINT TAB(31); "RC ="; USING "######.###"; RC; :   LPRINT " "; UF$;
        IF RC > 0 THEN LPRINT TAB(49); DR1$
        IF RC < 0 THEN LPRINT TAB(49); DR2$
    END
```


C17-2 (a)

```
ANALYSIS OF TWO-SPAN CONTINUOUS BEAM WITH OVERHANGS

Solution to EXAMPLE 17-9

The input data:
  The left overhang:            L0 =     3.000 m
                                P0 =    10.000 kN
                                a0 =     0.000 m
  The first span:               L1 =     8.000 m
                                P1 =    20.000 kN
                                a1 =     4.000 m
  The second span:              L2 =     8.000 m
                                w2 =     5.000 kN/m

Support moments and reactions are:
                                MA =   -30.000 kN-m
                                MB =   -27.500 kN-m
                                MC =     0.000 kN-m
                                RA =    20.313 kN  (upward)
                                RB =    33.125 kN  (upward)
                                RC =    16.563 kN  (upward)
```

C17-2 (b)

```
ANALYSIS OF TWO-SPAN CONTINUOUS BEAM WITH OVERHANGS

Solution to Problem 17-23

The input data:
  The first span:               L1 =    10.000 ft
                                P1 =     6.000 kip
                                a1 =     5.000 ft
  The second span:              L2 =    10.000 ft
                                w2 =     2.000 kip/ft

Support moments and reactions are:
                                MA =     0.000 kip-ft
                                MB =   -18.125 kip-ft
                                MC =     0.000 kip-ft
                                RA =     1.188 kip (upward)
                                RB =    16.625 kip (upward)
                                RC =     8.188 kip (upward)
```

C17-2 (c)

```
ANALYSIS OF TWO-SPAN CONTINUOUS BEAM WITH OVERHANGS

Solution to Problem 17-25

The input data:
  The first span:               L1 =     5.000 m
                                w1 =    10.000 kN/m
  The second span:              L2 =     5.000 m
                                w2 =    10.000 kN/m
  The right overhang:           L3 =     2.000 m
                                P3 =    20.000 kN
                                a3 =     2.000 m

Support moments and reactions are:
                                MA =     0.000 kN-m
                                MB =   -21.250 kN-m
                                MC =   -40.000 kN-m
                                RA =    20.750 kN  (upward)
                                RB =    50.500 kN  (upward)
                                RC =    48.750 kN  (upward)
```

Solution to Computer Program Assignment C18-1

```
' * C18-1 *    Stresses on inclined planes
'
' Clear screen, compute pi and degree and radian conversion factors
      CLS :    PI = 4 * ATN(1):     DR = PI / 180:  RD = 1 / DR
'
' Display the introductory remarks
      PRINT "Want to see the introductory remarks?"
      INPUT "-- Y / N "; RMK$
      IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100   PRINT "REMARKS: (1) This program can be used to compute the normal"
      PRINT "             and shear stresses on inclied planes."
      PRINT "         (2) The input data include the normal stresses in"
      PRINT "             the vertical and horizontal planes, the shear"
      PRINT "             stress on the vertical plane, and the angle"
      PRINT "             measured from the vertical to the inclined plane."
      PRINT "         (3) The input data and the computed results are"
      PRINT "             printed on the printer immediately after"
      PRINT "             data are entered. Make sure your printer is on"
      PRINT "             before entering data."
'
' Input problem I.D.
200   PRINT "Enter problem designation:"
      INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
      PRINT "Select the system of units:"
      PRINT "      1 -- SI units"
      PRINT "      2 -- US customary units"
      INPUT "-- 1 / 2"; U
      IF U = 1 THEN US$ = "MPa"
      IF U = 2 THEN US$ = "ksi"
'
' Print problem I.D.
      LPRINT "STRESSES ON INCLINED PLANES IN AN ELEMENT"
      LPRINT "SUBJECTED TO PLANE STRESS"
      LPRINT :   LPRINT "Solution to "; ID$
'
' Enter data for the element
      PRINT."Normal stress on the vertical plane:"
      PRINT "-- Snx ("; US$; ")"; :   INPUT SNX
      PRINT "Normal stress on the horizontal plane:"
      PRINT "-- Sny ("; US$; ")"; :   INPUT SNY
      PRINT "Shear stress on the vertical plane:"
      PRINT "-- Ssx ("; US$; ")"; :   INPUT SSX
'
' Print the input data for the element
      LPRINT :   LPRINT "The given state of plane stress:":    LPRINT
      LPRINT TAB(3); "Normal stress on vert. plane:"; TAB(33); "Snx =";
      LPRINT USING "######.###"; SNX; :   LPRINT " "; US$
      LPRINT TAB(3); "Normal stress on hori. plane:"; TAB(33); "Sny =";
      LPRINT USING "######.###"; SNY; :   LPRINT " "; US$
      LPRINT TAB(3); "Shear stress on vert. plane:"; TAB(33); "Ssx =";
      LPRINT USING "######.###"; SSX; :   LPRINT " "; US$
'
      PRINT "The direction of the incline:"
      PRINT "-- theta (degrees, ccw as +)"; :   INPUT AD
'
' print heading for output results
      LPRINT :   LPRINT "Stresses on inclined planes:"
      LPRINT "________________________________________"
      LPRINT :   LPRINT "Inclined plane  Normal stress  Shear stress"
```


Solution to Computer Program Assignment C18-1 Continued

```
      LPRINT TAB(5); "degrees"; TAB(22); US$; TAB(37); US$
      LPRINT "from vert(ccw+)"
      LPRINT "________________________________________"
      LPRINT
'
' Compute the stresses on the inclined plane
      A = (SNX + SNY) / 2:  B = (SNX - SNY) / 2:  C = SSX
      R = SQR(B * B + C * C)
600   DAR = 2 * AD * DR
      SN = A + B * COS(DAR) + C * SIN(DAR)
      SS = -B * SIN(DAR) + C * COS(DAR)
      LPRINT USING "#######.##": AD;
      LPRINT USING "###########.###"; SN;
      LPRINT USING "###########.###"; SS
      PRINT "-- theta (degrees, enter 0 to quit)"; :   INPUT AD
      IF AD = 0 GOTO 670 ELSE GOTO 600
670   LPRINT "________________________________________"
   END
```

C18-1 (a)

```
STRESSES ON INCLINED PLANES IN AN ELEMENT
SUBJECTED TO PLANE STRESS

Solution to EXAMPLE 18-9

The given state of plane stress:

  Normal stress on vert. plane: Snx =     -3.000 ksi
  Normal stress on hori. plane: Sny =      5.000 ksi
  Shear stress on vert. plane:  Ssx =     -4.000 ksi

Stresses on inclined planes:
________________________________________

Inclined plane  Normal stress  Shear stress
    degrees          ksi            ksi
from vert(ccw+)
________________________________________

     -30.00          2.464         -5.464
________________________________________
```

C18-1 (b)

```
STRESSES ON INCLINED PLANES IN AN ELEMENT
SUBJECTED TO PLANE STRESS

Solution to Problem 18-29

The given state of plane stress:

  Normal stress on vert. plane: Snx =     50.000 MPa
  Normal stress on hori. plane: Sny =    -50.000 MPa
  Shear stress on vert. plane:  Ssx =      0.000 MPa

Stresses on inclined planes:
________________________________________

Inclined plane  Normal stress  Shear stress
    degrees          MPa            MPa
from vert(ccw+)
________________________________________

     -45.00         -0.000         50.000
________________________________________
```

C18-1 (c)

STRESSES ON INCLINED PLANES IN AN ELEMENT
SUBJECTED TO PLANE STRESS

Solution to Problem 18-31

The given state of plane stress:

```
  Normal stress on vert. plane: Snx =     -4.000 ksi
  Normal stress on hori. plane: Sny =      6.000 ksi
  Shear stress on vert. plane:  Ssx =     -5.000 ksi
```

Stresses on inclined planes:

Inclined plane degrees from vert(ccw+)	Normal stress ksi	Shear stress ksi
-30.00	2.830	-6.830

C18-1 (d)

STRESSES ON INCLINED PLANES IN AN ELEMENT
SUBJECTED TO PLANE STRESS

Solution to Problem 18-32

The given state of plane stress:

```
  Normal stress on vert. plane: Snx =    -70.000 MPa
  Normal stress on hori. plane: Sny =    -70.000 MPa
  Shear stress on vert. plane:  Ssx =     30.000 MPa
```

Stresses on inclined planes:

Inclined plane degrees from vert(ccw+)	Normal stress MPa	Shear stress MPa
-50.00	-99.544	-5.209

Solution to Computer Program Assignment C18-2

```
' * C18-2 *   Analysis of plane stresses
'
' Clear screen, compute pi and conversion factors for degree and radian
      CLS :   PI = 4 * ATN(1):    DR = PI / 180:  RD = 1 / DR
'
' Display the introductory remarks
      PRINT "Want to see the introductory remarks?"
      INPUT "-- Y / N "; RMK$
      IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 210
100   PRINT "REMARKS: (1) This program can be used to compute the principal"
      PRINT "             stresses, orientation of principal planes, the"
      PRINT "             maximum and minimum shear stresses and shear"
      PRINT "             planes, and stresses on inclined planes."
      PRINT "         (2) The input data include the normal stresses in the"
      PRINT "             vertical and horizontal planes, the shear stress"
      PRINT "             on the vertical plane, and angles from vertical"
      PRINT "             to the inclined plane."
      PRINT "         (3) The input data and the computed results are"
      PRINT "             printed on the printer immediately after"
      PRINT "             data are entered. Make sure your printer is on"
      PRINT "             before entering data."
'
' Input problem I.D.
210   PRINT "Enter problem designation:"
      INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
      PRINT "Select the system of units:"
      PRINT "        1 -- SI units"
      PRINT "        2 -- US customary units"
      INPUT "-- 1 / 2"; U
      IF U = 1 THEN US$ = "MPa"
      IF U = 2 THEN US$ = "ksi"
'
' Print problem I.D.
      LPRINT "ANALYSIS OF PLANE STRESSES"
      LPRINT :   LPRINT "Solution to "; ID$
'
' Enter data for the element
      PRINT "Normal stress on the vertical plane:"
      PRINT "-- Snx ("; US$; ")"; :   INPUT SNX
      PRINT "Normal stress on the horizontal plane:"
      PRINT "-- Sny ("; US$; ")"; :   INPUT SNY
      PRINT "Shear stress on the vertical plane:"
      PRINT "-- Ssx ("; US$; ")"; :   INPUT SSX
'
' Print the input data for the element
      LPRINT :   LPRINT "The given state of plane stress:":    LPRINT
      LPRINT TAB(3); "Normal stress on vert. plane:"; TAB(33); "Snx =";
      LPRINT USING "######.###"; SNX; :   LPRINT " "; US$
      LPRINT TAB(3); "Normal stress on hori. plane:"; TAB(33); "Sny =";
      LPRINT USING "######.###"; SNY; :   LPRINT " "; US$
      LPRINT TAB(3); "Shear stress on vert. plane:"; TAB(33); "Ssx =";
      LPRINT USING "######.###"; SSX; :   LPRINT " "; US$
'
' Compute the principal stresses
      A = (SNX + SNY) / 2:  B = (SNX - SNY) / 2:  C = SSX
      R = SQR(B * B + C * C)
      SN1 = A + R:    SN2 = A - R:    DA1 = ATN((R - B) / C) * RD
      DA2 = DA1 + 90
      IF ABS(DA2) > 90 THEN DA2 = DA1 - 90
```

Solution to Computer Program Assignment C18-2 Continued

```
        SS = 0
'
' Print the principal stresses
        LPRINT :   LPRINT "The principal stresses and the principal planes:"
        LPRINT "_______________________________________________________________"
        LPRINT
        LPRINT "Principal plane  Normal stress  Shear stress     Remark"
        LPRINT TAB(5); "degrees"; TAB(23); US$; TAB(38); US$
        LPRINT "from vert(ccw+)"
        LPRINT "---------------------------------------------------------------"
        LPRINT USING "#######.##"; DA1;
        LPRINT USING "###########.###"; SN1;
        LPRINT USING "###########.###"; SS;
        LPRINT TAB(46); "Max. Normal Str."
        LPRINT USING "#######.##"; DA2;
        LPRINT USING "###########.###"; SN2;
        LPRINT USING "###########.###"; SS;
        LPRINT TAB(46); "Min. Normal Str."
        LPRINT "_______________________________________________________________"
'
' Compute the maximum and minimum shear stresses
        SS1 = R:     SS2 = -R:       SN = A
        DAS1 = -ATN(B / (R + C)) * RD: DAS2 = DAS1 + 90
        IF ABS(DAS2) > 90 THEN DAS2 = DAS1 - 90
'
' Print the maximim and minimum shear stresses
        LPRINT :   LPRINT
        LPRINT "The max. & min. shear stresses and shear planes:"
        LPRINT "_______________________________________________________________"
        LPRINT
        LPRINT " Max. or min.    Normal stress  Shear stress     Remark"
        LPRINT "  Shear plane"
        LPRINT TAB(5); "degrees"; TAB(23); US$; TAB(38); US$
        LPRINT "from vert(ccw+)"
        LPRINT "---------------------------------------------------------------"
        LPRINT USING "#######.##"; DAS1;
        LPRINT USING "###########.###"; SN;
        LPRINT USING "###########.###"; SS1;
        LPRINT TAB(46); "Max. Shear Str."
        LPRINT USING "#######.##"; DAS2;
        LPRINT USING "###########.###"; SN;
        LPRINT USING "###########.###"; SS2;
        LPRINT TAB(46); "Min. Shear Str."
        LPRINT "_______________________________________________________________"
'
        PRINT "The direction of the incline:"
        PRINT "-- theta (degrees, ccw as +, enter 0 to quit this function)"; :
INPUT AD
        IF AD = 0 GOTO 1120
'
' Print stresses on inclined planes
        LPRINT :   LPRINT :   LPRINT "Stresses on inclined planes:"
        LPRINT "____________________________________________"
        LPRINT :   LPRINT "Inclined plane   Normal stress  Shear stress"
        LPRINT TAB(5); "degrees"; TAB(23); US$; TAB(38); US$
        LPRINT "from vert(ccw+)"
        LPRINT "--------------------------------------------"
'
' Compute the stresses on the inclined plane
1020    DAR = 2 * AD * DR
        SN = A + B * COS(DAR) + C * SIN(DAR)
```


Solution to Computer Program Assignment C18-2 Continued

```
        SS = -B * SIN(DAR) + C * COS(DAR)
'
' Print stresses on the incline
        LPRINT USING "#######.##"; AD;
        LPRINT USING "###########.###"; SN;
        LPRINT USING "###########.###"; SS
        PRINT "-- theta (degrees, ccw as +, enter 0 to quit this function)";
        INPUT AD
        IF AD = 0 GOTO 1110 ELSE GOTO 1020
1110    LPRINT "____________________________________________"
1120 END
```

C18-2 (a)

```
ANALYSIS OF PLANE STRESSES

Solution to EXAMPLE 18-11

The given state of plane stress:

  Normal stress on vert. plane: Snx =     -2.000 ksi
  Normal stress on hori. plane: Sny =      6.000 ksi
  Shear stress on vert. plane:  Ssx =     -3.000 ksi

The principal stresses and the principal planes:
_______________________________________________________________

Principal plane  Normal stress  Shear stress     Remark
    degrees           ksi            ksi
from vert(ccw+)
---------------------------------------------------------------
     -71.57          7.000          0.000     Max. Normal Str.
      18.43         -3.000          0.000     Min. Normal Str.
_______________________________________________________________

The max. & min. shear stresses and shear planes:
_______________________________________________________________

 Max. or min.    Normal stress  Shear stress     Remark
  Shear plane
    degrees           ksi            ksi
from vert(ccw+)
---------------------------------------------------------------
      63.43          2.000          5.000     Max. Shear Str.
     -26.57          2.000         -5.000     Min. Shear Str.
_______________________________________________________________
```

C18-2 (b)

ANALYSIS OF PLANE STRESSES

Solution to Problem 18-44

The given state of plane stress:

```
  Normal stress on vert. plane: Snx =    -4.000 MPa
  Normal stress on hori. plane: Sny =     8.000 MPa
  Shear stress on vert. plane:  Ssx =    -8.000 MPa
```

The principal stresses and the principal planes:

Principal plane degrees from vert(ccw+)	Normal stress MPa	Shear stress MPa	Remark
-63.43	12.000	0.000	Max. Normal Str.
26.57	-8.000	0.000	Min. Normal Str.

The max. & min. shear stresses and shear planes:

Max. or min. Shear plane degrees from vert(ccw+)	Normal stress MPa	Shear stress MPa	Remark
71.57	2.000	10.000	Max. Shear Str.
-18.43	2.000	-10.000	Min. Shear Str.

C18-2 (c)

ANALYSIS OF PLANE STRESSES

Solution to Problem 18-45

The given state of plane stress:

```
  Normal stress on vert. plane: Snx =   -15.000 MPa
  Normal stress on hori. plane: Sny =    -5.000 MPa
  Shear stress on vert. plane:  Ssx =    12.000 MPa
```

The principal stresses and the principal planes:

Principal plane degrees from vert(ccw+)	Normal stress MPa	Shear stress MPa	Remark
56.31	3.000	0.000	Max. Normal Str.
-33.69	-23.000	0.000	Min. Normal Str.

The max. & min. shear stresses and shear planes:

Max. or min. Shear plane degrees from vert(ccw+)	Normal stress MPa	Shear stress MPa	Remark
11.31	-10.000	13.000	Max. Shear Str.
-78.69	-10.000	-13.000	Min. Shear Str.

Solution to Computer Program Assignment C19-1

```
    ' * C19-1 *   AISC allowable compressive stresses for steel columns
    '
    ' Clear screen, compute pi
        CLS :    PI = 4 * ATN(1)
        DIM SALL(200)
    '
    ' Display the introductory remarks
        PRINT "Want to see the introductory remarks?"
        INPUT "-- Y / N "; RMK$
        IF RMK$ = "Y" OR RMK$ = "y" GOTO 110 ELSE GOTO 200
110     PRINT "REMARKS: (1) This program can be used to produce a table"
        PRINT "             listing the AISC allowable compressive stresses"
        PRINT "             for slenderness ratios varying from 1 to 200 at"
        PRINT "             steps of 1 for a given yield strength."
        PRINT "         (2) The user needs only to input the yield strength"
        PRINT "             of the column."
        PRINT "         (3) The input data and the computed results are"
        PRINT "             printed on the printer immediately after"
        PRINT "             data are entered. Make sure your printer is on"
        PRINT "             before entering data."
    '
    ' Select system of units
200     PRINT "Select the system of units:"
        PRINT "        1 -- SI units"
        PRINT "        2 -- US customary units"
        INPUT "-- 1 / 2"; U
        IF U = 2 GOTO 260
        US$ = "MPa":    UE$ = "GPa":     GOTO 290
260     US$ = "ksi":    UE$ = "ksi"
    '
    ' Input data
290     PRINT "Yield strength of the column:"
        PRINT "-- Sy ("; US$; ")"; :    INPUT SY
    '
    ' Compute the AISC allowable compressive stresses for steel columns
        IF U = 1 THEN E = 200000!
        IF U = 2 THEN E = 29000
        CC = SQR(2 * PI ^ 2 * E / SY)
      FOR S = 1 TO 200
        SCC = S / CC
        IF S < CC GOTO 400
        SALL(S) = PI ^ 2 * E / S ^ 2 / 1.92
        GOTO 420
400     FS = 5 / 3 + 3 * SCC / 8 - SCC ^ 3 / 8
        SALL(S) = (1 - SCC ^ 2 / 2) * SY / FS
420   NEXT S
    '
        LPRINT "AISC ALLOWABLE COMPRESSIVE STRESSES FOR STEEL COLUMNS"
        LPRINT "for Sy ="; USING "####"; SY; :    LPRINT " "; US$
        LPRINT "_____________________________________________________________";
        LPRINT "_______"
        LPRINT
        LPRINT "kL/r   Sa      kL/r   Sa       kL/r   Sa        kL/r   Sa     ";
        LPRINT "kL/r   Sa"
        IF U = 1 GOTO 450
        LPRINT "       (ksi)          (ksi)           (ksi)            (ksi)   ";
        LPRINT "       (ksi)"
        GOTO 470
450     LPRINT "       (MPa)          (MPa)           (MPa)            (MPa)   ";
        LPRINT "       (MPa)"
470     LPRINT "_____________________________________________________________";
```

Solution to Computer Program Assignment C19-1 Continued

```
      LPRINT "_______"
      LPRINT
    FOR N = 1 TO 40
      LPRINT USING "###"; N; :   LPRINT USING "#####.##"; SALL(N);
      LPRINT USING "######"; N + 40;
      LPRINT USING "#####.##"; SALL(N + 40);
      LPRINT USING "######"; N + 80;
      LPRINT USING "#####.##"; SALL(N + 80);
      LPRINT USING "#######"; N + 120;
      LPRINT USING "####.##"; SALL(N + 120);
      LPRINT USING "#######"; N + 160;
      LPRINT USING "####.##"; SALL(N + 160)
      IF N = 40 GOTO 580
      IF N = (N \ 5) * 5 THEN LPRINT
580 NEXT N
      LPRINT "_____________________________________________________________";
      LPRINT "_______"
      LPRINT :   LPRINT "Note:  Cc ="; :   LPRINT USING "####.#"; CC
  END
```


C19-1 (a)

AISC ALLOWABLE COMPRESSIVE STRESSES FOR STEEL COLUMNS
for Sy = 60 ksi

kL/r	Sa (ksi)	kL/r	Sa (ksi)	kL/r	Sa (ksi)	kL/r	Sa (ksi)	kL/r	Sa (ksi)
1	35.92	41	30.15	81	20.65	121	10.18	161	5.75
2	35.83	42	29.95	82	20.37	122	10.02	162	5.68
3	35.74	43	29.75	83	20.09	123	9.85	163	5.61
4	35.64	44	29.55	84	19.80	124	9.70	164	5.54
5	35.54	45	29.35	85	19.51	125	9.54	165	5.48
6	35.44	46	29.15	86	19.22	126	9.39	166	5.41
7	35.34	47	28.94	87	18.93	127	9.24	167	5.35
8	35.23	48	28.73	88	18.63	128	9.10	168	5.28
9	35.12	49	28.52	89	18.34	129	8.96	169	5.22
10	35.01	50	28.31	90	18.04	130	8.82	170	5.16
11	34.89	51	28.09	91	17.73	131	8.69	171	5.10
12	34.77	52	27.87	92	17.43	132	8.56	170	5.04
13	34.65	53	27.66	93	17.12	133	8.43	173	4.98
14	34.52	54	27.43	94	16.81	134	8.30	174	4.92
15	34.40	55	27.21	95	16.50	135	8.18	175	4.87
16	34.27	56	26.98	96	16.19	136	8.06	176	4.81
17	34.13	57	26.76	97	15.87	137	7.94	177	4.76
18	34.00	58	26.53	98	15.52	138	7.83	178	4.70
19	33.86	59	26.29	99	15.21	139	7.72	179	4.65
20	33.71	60	26.06	100	14.91	140	7.61	180	4.60
21	33.57	61	25.82	101	14.61	141	7.50	181	4.55
22	33.42	62	25.58	102	14.33	142	7.39	182	4.50
23	33.27	63	25.34	103	14.05	143	7.29	183	4.45
24	33.12	64	25.10	104	13.78	144	7.19	184	4.40
25	32.96	65	24.86	105	13.52	145	7.09	185	4.36
26	32.81	66	24.61	106	13.27	146	6.99	186	4.31
27	32.65	67	24.36	107	13.02	147	6.90	187	4.26
28	32.48	68	24.11	108	12.78	148	6.81	188	4.22
29	32.32	69	23.86	109	12.55	149	6.71	189	4.17
30	32.15	70	23.60	110	12.32	150	6.63	190	4.13
31	31.98	71	23.34	111	12.10	151	6.54	191	4.09
32	31.81	72	23.08	112	11.88	152	6.45	192	4.04
33	31.63	73	22.82	113	11.67	153	6.37	193	4.00
34	31.45	74	22.56	114	11.47	154	6.29	194	3.96
35	31.28	75	22.29	115	11.27	155	6.20	195	3.92
36	31.09	76	22.02	116	11.08	156	6.13	196	3.88
37	30.91	77	21.75	117	10.89	157	6.05	197	3.84
38	30.72	78	21.48	118	10.71	158	5.97	198	3.80
39	30.53	79	21.21	119	10.53	159	5.90	199	3.76
40	30.34	80	20.93	120	10.35	160	5.82	200	3.73

Note: Cc = 97.7

C19-1 (b)

AISC ALLOWABLE COMPRESSIVE STRESSES FOR STEEL COLUMNS
for Sy = 345 MPa

kL/r	Sa (MPa)	kL/r	Sa (MPa)	kL/r	Sa (MPa)	kL/r	Sa (MPa)	kL/r	Sa (MPa)
1	206.56	41	177.26	81	129.77	121	70.22	161	39.66
2	206.10	42	176.27	82	128.38	122	69.07	162	39.17
3	205.62	43	175.28	83	126.97	123	67.95	163	38.69
4	205.13	44	174.27	84	125.56	124	66.86	164	38.22
5	204.62	45	173.25	85	124.13	125	65.80	165	37.76
6	204.10	46	172.22	86	122.69	126	64.76	166	37.31
7	203.56	47	171.19	87	121.24	127	63.74	167	36.86
8	203.01	48	170.14	88	119.79	128	62.75	168	36.43
9	202.44	49	169.08	89	118.32	129	61.78	169	36.00
10	201.86	50	168.01	90	116.84	130	60.83	170	35.57
11	201.27	51	166.92	91	115.35	131	59.91	171	35.16
12	200.65	52	165.83	92	113.85	132	59.00	172	34.75
13	200.03	53	164.73	93	112.33	133	58.12	173	34.35
14	199.39	54	163.62	94	110.81	134	57.26	174	33.96
15	198.74	55	162.50	95	109.28	135	56.41	175	33.57
16	198.07	56	161.36	96	107.73	136	55.58	176	33.19
17	197.39	57	160.22	97	106.17	137	54.78	177	32.82
18	196.69	58	159.07	98	104.60	138	53.98	178	32.45
19	195.98	59	157.91	99	103.02	139	53.21	179	32.09
20	195.26	60	156.73	100	101.43	140	52.45	180	31.73
21	194.53	61	155.55	101	99.83	141	51.71	181	31.38
22	193.78	62	154.36	102	98.21	142	50.99	182	31.04
23	193.02	63	153.15	103	96.59	143	50.28	183	30.70
24	192.25	64	151.94	104	94.95	144	49.58	184	30.37
25	191.46	65	150.72	105	93.29	145	48.90	185	30.04
26	190.66	66	149.48	106	91.63	146	48.23	186	29.72
27	189.85	67	148.24	107	89.80	147	47.58	187	29.40
28	189.03	68	146.99	108	88.14	148	46.94	188	29.09
29	188.20	69	145.72	109	86.53	149	46.31	189	28.78
30	187.35	70	144.45	110	84.97	150	45.69	190	28.48
31	186.49	71	143.16	111	83.44	151	45.09	191	28.18
32	185.62	72	141.87	112	81.96	152	44.50	192	27.89
33	184.73	73	140.57	113	80.51	153	43.92	193	27.60
34	183.84	74	139.25	114	79.11	154	43.35	194	27.32
35	182.93	75	137.93	115	77.74	155	42.79	195	27.04
36	182.02	76	136.60	116	76.40	156	42.25	196	26.76
37	181.09	77	135.25	117	75.10	157	41.71	197	26.49
38	180.15	78	133.90	118	73.84	158	41.18	198	26.22
39	179.19	79	132.53	119	72.60	159	40.67	199	25.96
40	178.23	80	131.16	120	71.39	160	40.16	200	25.70

Note: Cc = 107.0

Solution to Computer Program Assignment C19-2

```
' * C19-2 *    AISC allowable compressive load for steel columns
'
' Clear screen, compute pi
      CLS :    PI = 4 * ATN(1)
'
' Display the introductory remarks
      PRINT "Want to see the introductory remarks?"
      INPUT "-- Y / N "; RMK$
      IF RMK$ = "Y" OR RMK$ = "y" GOTO 100 ELSE GOTO 200
100   PRINT "REMARKS: (1) This program can be used to compute the AISC"
      PRINT "             allowable compressive load for a steel column."
      PRINT "         (2) The input data include the length, the area, the"
      PRINT "             least radius of gyration, the yield strength, and"
      PRINT "             various supporting conditions of the column."
      PRINT "         (3) The input data and the computed results are"
      PRINT "             printed on the printer immediately after data are"
      PRINT "             entered. Make sure the printer is on before"
      PRINT "             entering data."
'
' Input problem I.D.
200   PRINT "Enter problem designation:"
      INPUT "-- Prob. I.D. "; ID$
'
' Select system of units
      PRINT "Select the system of units:"
      PRINT "        1 -- SI units"
      PRINT "        2 -- US customary units"
      INPUT "-- 1 / 2"; U
      IF U = 2 GOTO 300
      UL$ = "m": UR$ = "m": UA$ = "m^2": UF$ = "kN": US$ = "MPa"
      GOTO 330
300   UL$ = "ft": UR$ = "in": UA$ = "in^2": UF$ = "kip": US$ = "ksi"
'
' Print problem I.D.
330   LPRINT "AISC ALLOWABLE COMPRESSIVE LOAD FOR STEEL COLUMNS"
      LPRINT :   LPRINT "Solution to "; ID$
'
' Input data
      PRINT "Length of column:"
      PRINT "-- L ("; UL$; ")"; :    INPUT L
      PRINT "Cross-sectional area:"
      PRINT "-- A ("; UA$; ")"; :    INPUT A
      PRINT "Least radius of gyration:"
      PRINT "-- r min. ("; UR$; ")"; :    INPUT R
      PRINT "Yield strength (stress):"
      PRINT "-- Sy ("; US$; ")"; :    INPUT SY
'
' Print input data
      LPRINT :   LPRINT "The input data for the column are:":    LPRINT
      LPRINT TAB(3); "Length of column:"; TAB(36); "L =";
      LPRINT USING "#####.###"; L; :    LPRINT " "; UL$
      LPRINT TAB(3); "Cross-sectional area:"; TAB(36); "A =";
      IF U = 2 GOTO 490
      LPRINT USING "#####.###"; A * 1000; : LPRINT " "; UA$; "/1000"
490   LPRINT USING "#####.###"; A; :    LPRINT " "; UA$
      LPRINT TAB(3); "Least radius of gyration:"; TAB(36); "r =";
      LPRINT USING "#####.###"; R; :    LPRINT " "; UR$
      LPRINT TAB(3); "Yield strength:"; TAB(35); "Sy =";
      LPRINT USING "#####.###"; SY; :    LPRINT " "; US$
      LPRINT :   LPRINT "The computed results are:"
'
```

Solution to Computer Program Assignment C19-2 Continued

```
      ' Specify E and change unit of L
          IF U = 1 THEN E = 200000!
          IF U = 2 THEN E = 29000
          IF U = 2 THEN L = L * 12
      '
          PRINT "Type of support:"
          PRINT "        1 -- Pinned ends"
          PRINT "        2 -- Fixed ends"
          PRINT "        3 -- Fixed-pinned ends"
          PRINT "        4 -- Fixed-free ends"
660       INPUT "Support -- 1/2/3/4 (Enter 0 to quit)"; TS
          IF TS = 0 GOTO 1020
          IF TS = 1 THEN K = 1
          IF TS = 2 THEN K = .65
          IF TS = 3 THEN K = .8
          IF TS = 4 THEN K = 2.1
          IF TS = 1 THEN TS$ = "pinned ends"
          IF TS = 2 THEN TS$ = "fixed ends"
          IF TS = 3 THEN TS$ = "fixed-pinned ends"
          IF TS = 4 THEN TS$ = "fixed-free ends"
      '
      ' Compute the AISC allowable compressive load for the steel column
          S = K * L / R
          CC = SQR(2 * PI ^ 2 * E / SY)
          SCC = S / CC
          IF S < CC GOTO 840
          SALL = PI ^ 2 * E / S ^ 2 / 1.92
          GOTO 860
840       FS = 5 / 3 + 3 * SCC / 8 - SCC ^ 3 / 8
          SALL = (1 - SCC ^ 2 / 2) * SY / FS
860       PALL = SALL * A
      '
      ' Print results
          LPRINT
          LPRINT TAB(3); "For the "; TS$; " supports:"; TAB(36); "k =";
          LPRINT USING "#####.###"; K
          LPRINT TAB(3); "Slenderness ratio:"; TAB(33); "kL/r =";
          LPRINT USING "#####.###"; S
          LPRINT TAB(3); "Transition slenderness ratio:"; TAB(35); "Cc =";
          LPRINT USING "#####.###"; CC
          LPRINT TAB(3); "AISC allowable compr. stress:"; TAB(35); "Sa =";
          LPRINT USING "#####.###"; SALL; :   LPRINT " "; US$
          LPRINT TAB(3); "AISC allowable compr. load:"; TAB(35); "Pa =";
          IF U = 2 GOTO 1000
          LPRINT USING "#####.###"; PALL * 1000;
          LPRINT " "; UF$:    GOTO 660
1000      LPRINT USING "#####.###"; PALL; :   LPRINT " "; UF$:    GOTO 660
      '
1020 END
```


C19-2 (a)

```
AISC ALLOWABLE COMPRESSIVE LOAD FOR STEEL COLUMNS

Solution to EXAMPLE 19-4

The input data for the column are:

  Length of column:                    L =    10.000 ft
  Cross-sectional area:                A =     4.750 in^2
  Least radius of gyration:            r =     0.870 in
  Yield strength:                     Sy =    36.000 ksi

The computed results are:

  For the pinned ends supports:        k =     1.000
  Slenderness ratio:                kL/r =   137.931
  Transition slenderness ratio:       Cc =   126.099
  AISC allowable compr. stress:       Sa =     7.836 ksi
  AISC allowable compr. load:         Pa =    37.219 kip

  For the fixed ends supports:         k =     0.650
  Slenderness ratio:                kL/r =    89.655
  Transition slenderness ratio:       Cc =   126.099
  AISC allowable compr. stress:       Sa =    14.246 ksi
  AISC allowable compr. load:         Pa =    67.667 kip
```

C19-2 (b)

```
AISC ALLOWABLE COMPRESSIVE LOAD FOR STEEL COLUMNS

Solution to EXAMPLE 19-5

The input data for the column are:

  Length of column:                    L =    15.000 m
  Cross-sectional area:                A =    11.380 m^2/1000
    0.011 m^2
  Least radius of gyration:            r =     0.109 m
  Yield strength:                     Sy =   345.000 MPa

The computed results are:

  For the fixed ends supports:         k =     0.650
  Slenderness ratio:                kL/r =    89.450
  Transition slenderness ratio:       Cc =   106.972
  AISC allowable compr. stress:       Sa =   117.654 MPa
  AISC allowable compr. load:         Pa =  1338.901 kN
```

C19-2 (c)

```
AISC ALLOWABLE COMPRESSIVE LOAD FOR STEEL COLUMNS

Solution to Problem 19-31

The input data for the column are:

  Length of column:                  L =     9.000 m
  Cross-sectional area:              A =    21.200 m^2/1000
    0.021 m^2
  Least radius of gyration:          r =     0.068 m
  Yield strength:                   Sy =   250.000 MPa

The computed results are:

  For the fixed-pinned ends supports:
                                     k =     0.800
  Slenderness ratio:              kL/r =   105.727
  Transition slenderness ratio:     Cc =   125.664
  AISC allowable compr. stress:     Sa =    84.665 MPa
  AISC allowable compr. load:       Pa =  1794.887 kN
```

C19-2 (d)

```
AISC ALLOWABLE COMPRESSIVE LOAD FOR STEEL COLUMNS

Solution to Problem 19-35

The input data for the column are:

  Length of column:                  L =    25.000 ft
  Cross-sectional area:              A =    11.760 in^2
  Least radius of gyration:          r =     3.670 in
  Yield strength:                   Sy =    36.000 ksi

The computed results are:

  For the pinned ends supports:      k =     1.000
  Slenderness ratio:              kL/r =    81.744
  Transition slenderness ratio:     Cc =   126.099
  AISC allowable compr. stress:     Sa =    15.160 ksi
  AISC allowable compr. load:       Pa =   178.283 kip
```

INSTRUCTOR'S NOTES

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